A Study on the Calculation Model for the Emissivities of Carbon Dioxide and Water Vapor

Byung-Ki Hur
Department of Biological Engineering, Inha University
Nam-Jin Kim
Graduate School, Inha University
Tae-Beom Seo, Chong-Bo Kim*
Department of Mechanical Engineering, Inha University

The main mode of heat transfer of combustion gases at high temperature is thermal radiation of the participating gases, which are mainly carbon dioxide and water vapor. Therefore, the information of the emissivities of carbon dioxide and water vapor would be very important in the thermal performance analysis of a furnace. In this study, an exponential model for the emissivities of carbon dioxide and water vapor is derived as a function of the product of the partial pressure and the characteristic length and a polynomial of reciprocal of temperature. Error analysis of the calculated values from the present model is performed for the temperature ranges of 555.6~2777.8 K and the partial-pressure-length product ranges of 0.09144~609.6 cm-atm. For carbon dioxide, the difference between the values from the present model and the Hottel's chart is less than 2.5% using a polynomial in 1/T of degree of 4. For water vapor, the model can predict the emissivity within 2.5% difference using a polynomial in 1/T of degree of 3.

Key Words : Emissivity, Radiation Intensity, Carbon Dioxide

Nomenclature

a : Weighting factor
c : Speed of light (2.9978×10^{-8} m/s)
C : Coefficient in equation (16)
F : Fraction of total emissive power
k : Absorption coefficients ((cm-atm)$^{-1}$)
h : Plack's constant (6.625×10^{-34} Js)
I : Radiation intensity (W/m^2)
L : Pathlength (cm)
P : Pressure (atm)
T : Temperature (K)
W : Emissive power (W/m^2)
V : Variable defined in equation (10)
β, $\bar{\beta}$: Wavelength of both ends of arbitrary radiation bands
γ : Coefficient of a polynomial of 1/T
ε : Emissivity of combustion gases
λ : Wavelength (μm)
σ : Stefan-Boltzmann constant (5.670×10^{-8} W/m^{2}K^4)

Subscripts

B : Blackbody
C : Calculated value
g : Gas
L : Calculated at L
λ : Spectral
λT : Evaluated at λT
0 : Incident or initial

* Corresponding Author,
E-mail : cbkim@inha.ac.kr
TEL : +82-32-860-7313 ; **FAX** : +82-32-868-1716
Mechanical Engineering, Inha University, Inchon 402-751, Korea. (Manuscript **Received** April 24, 2000; **Revised** November 27, 2000)

1. Introduction

The internal and external combustion engines use thermal energy of combustion gases produced by fossil fuels. Because this combustion gases reach a high temperature, radiation heat transfer becomes the main mode of heat transfer to the surroundings. More than 90% in weight of fossil fuels is carbon and hydrogen so that combustion gases consist mainly of carbon dioxide and water vapor. Since carbon dioxide and water vapor are participating media, the emissivities of these gases should be known exactly in order to perform efficiently the thermal and material design of the combustion engine.

Based on the theoretical work and the experimental data, Hottel and Sarofil (1967) and Mcadams (1954) made the total emissivity charts for carbon dioxide and water vapor as a function of the wide range of temperature, pathlength, and partial pressure. Because it is reasonably accurate and easy to be used for a variety of engineering applications, the charts are widely used for the performance design and analysis of radiation heat transfer equipment. If the geometry of combustion equipment is complicated and the temperature is not uniform in the equipment, however, distribution of temperature and the heat transfer rate throughout the combustion area should be investigated in order to predict the thermal performance of the system accurately. Therefore, it is very difficult to use the Hottel's Charts to calculate radiation heat transfer from the combustion gas.

In order to properly consider the spectral characteristics of the participating gases, many researchers (Farag, 1976; Nakara and Smith, 1977; Sarofim *et al*., 1978; Farag and Allam, 1981; Felske and Charalam, 1982; Ha and Hur, 1986) have used an exponential model to express the total emissivity of the combustion gases as a sum of gray gases of absorption coefficient k_i and its weighting factor a_i as follows:

$$\varepsilon=\sum_{i=1}^{M}a_i[1-e^{-k_iP_gL}] \tag{1}$$

where M represents the total number of gray bands, a_i the energy fraction over the spectral region in which the gray band of absorption coefficient k_i exists, P_g the partial pressure of combustion gas, and L the pathlength. Even though it was known that this model could not give the accurate solution for the detailed spectral calculations with small number of gray bands [9], it is still useful for numerous engineering applications because of its simplicity. In order to simulate the spectral absorptivity of the nongray gases using this model, it was presented that a number of gray bands have to be considered, and Kim (1997) obtained good results with more than fifteen gray bands after painful calculations. Fortunately, the calculated spectral properties from a weighted sum of gray gases with a sufficient number of gray bands are valuable with a powerful computer although the amount of the calculation for radiation heat transfer becomes enormous. However, the detailed spectral calculation is too complicated to be applied for relatively simple problems in the industry. Therefore, the correlation to predict the total emissivities of the major gases is required in the industry.

In the present study, the total emissivities of carbon dioxide and water vapor are obtained as a function of the absorption properties and polynomial of reciprocal of temperature. And, the results of the present model are compared to those of Hottel in order to confirm the accuracy of the equation developed.

2. Calculation Model

The main products of combustion of fossil fuels, which are carbon dioxide and water vapor, absorb and scatter the incident radiation energy. Thus, if radiation energy is incident in the medium of combustion gases, the intensity is reduced due to the absorption, neglecting the scattering, as follows:

$$-\frac{dI_\lambda}{dx}=k_\lambda' I_\lambda \tag{2}$$

where I_λ is the monochromatic radiation intensity of electromagnetic wave of wavelength λ, k_λ the absorption coefficient. Because the absorption coefficient is proportional to the partial pressure of gases, Eq. (2) can be expressed as a function of the partial pressure.

$$-\frac{dI_\lambda}{dx}=k_\lambda P_g I_\lambda \tag{3}$$

After an electromagnetic wave of a wavelength λ passes through the gas layer L, the spectral radiation intensity is given by

$$I_{L,\lambda}=I_{o,\lambda}e^{-k_\lambda P_g L} \tag{4}$$

where $I_{0,\lambda}$ represents the incident spectral radiation intensity of a wavelength λ into the gas layer, $I_{l,\lambda}$ the spectral radiation intensity after passing through the gas layer, and L the thickness of gas layer. Under the local thermodynamic equilibrium, the medium must emit an amount of energy equal to that to be absorbed. Thus the spectral emissive power in the gas layer L is expressed as

$$W_\lambda=I_{o,\lambda}-I_{L,\lambda} \tag{5}$$

The total emissive power is obtained by integrating Eq. (5) with respect to λ and then it is given by

$$W_g=\int_0^\infty W_\lambda d\lambda=\int_0^\infty (I_{o,\lambda}-I_{L,\lambda})d\lambda$$
$$=\int_0^\infty I_{o,\lambda}(1-e^{-k_\lambda P_g L})d\lambda \tag{6}$$

The total emissivity is defined as the ratio of the emissive power of the gas to that of the blackbody at the same temperature.

$$\varepsilon_g=\frac{W_g}{W_b}=\frac{\int_0^\infty I_{o,\lambda}(1-e^{-k_\lambda P_g L})d\lambda}{\sigma T^4} \tag{7}$$

From the Planck's law, $I_{o,\lambda}$ is expressed as

$$I_{o,\lambda}=2\pi hc^2\lambda^{-5}/[e^{\frac{hc}{k\lambda T}}-1] \tag{8}$$

Substituting Eq. (8) into Eq. (7), Eq. (7) becomes

$$\varepsilon_g=\frac{1}{\sigma T^4}\int_0^\infty \frac{2\pi hc^2}{\lambda^5(e^{\frac{hc}{k\lambda T}}-1)}(1-e^{-k_\lambda P_g L})d\lambda \tag{9}$$

Defining a variable V as

$$V=\frac{hc}{k\lambda T} \tag{10}$$

Eq. (9) becomes

$$\varepsilon_g=\frac{15}{\pi^4}\int_0^\infty \frac{V^3}{e^V-1}(1-e^{-k_\lambda P_g L})d\lambda \tag{11}$$

Combustion gases do not emit radiation energy uniformly in the entire spectrum range because the radiation property is a strong function of the wavelength. Thus, Eq. (11) can be changed to the sum of integration for each absorption gray band.

$$\varepsilon_g=\frac{15}{\pi^4}\int_0^\infty \frac{V^3}{e^V-1}(1-e^{-k_\lambda P_g L})d\lambda$$
$$=\frac{15}{\pi^4}(1-e^{-k_\lambda P_g L})\int_{\bar\beta_1}^{\bar\beta_1}\frac{V^3}{e^V-1}d\lambda \tag{12}$$
$$+\frac{15}{\pi^4}(1-e^{-k_\lambda P_g L})\int_{\bar\beta_2}^{\bar\beta_2}\frac{V^3}{e^V-1}d\lambda$$
$$+\frac{15}{\pi^4}(1-e^{-k_\lambda P_g L})\int_{\bar\beta_3}^{\bar\beta_3}\frac{V^3}{e^V-1}d\lambda+\Lambda$$

Pivovonsky et al. (1961) and Wiebeit (1967) derived the following equation to express the fraction of the total emissive power at some wavelengths from the distribution of radiation energy according to the variation of wavelength for the blackbody and gray gas.

$$F_{o,\lambda T}=\frac{15}{\pi^4}\sum_{m=1}\frac{e^{-mV}}{m^4}\{[(mV+3)mV+6]$$
$$+6\},\ V\geq 2 \tag{13}$$
$$F_{o,\lambda T}=1-\frac{15}{\pi^4}V^3\left[\frac{1}{3}-\frac{V}{8}+\frac{V^2}{60}-\frac{V^4}{5040}\right.$$
$$\left.+\frac{V^6}{272{,}160}-\frac{V^8}{13{,}305{,}600}\right],\ V<2 \tag{14}$$

Expanding the exponential function of Eq. (13) as a Taylor series and arranging all terms, Eq. (13) becomes

$$F_{o,\lambda T}=\frac{15}{\pi^4}\sum_{m=1}\left[\frac{1}{m^4}+\frac{V}{m^2}+\frac{V^2}{2}+\Lambda\right]$$
$$[(m^2V^2+3mV+6)mV+6] \tag{15}$$

Eq. (14) and (15) for the fraction of the total emissive power can be generalized as follows:

$$F_{o,\lambda T}=\frac{15}{\pi^4}[C_0+C_1V+C_2V^2+\Lambda+C_nV^n]$$
$$=\frac{15}{\pi^4}\left[\bar C_o+\frac{\bar C_1}{T}+\frac{\bar C_2}{T_2}+A+\frac{\bar C_n}{T^n}\right] \tag{16}$$

Combining Eqs. (12) and (16), the fraction of the total emissive power emitted by the arbitrary radiation band between the wavelengths of β and $\bar\beta$ can be expressed as follows:

$$\frac{15}{\pi^4}(1-e^{-k_\lambda P_g L})\int_{\beta_i}^{\bar\beta_i}\frac{V^3}{e^V-1}dV=[F_{O,\bar\beta_i T}$$
$$-F_{O,\beta_i,T}]=\left[\gamma_0+\frac{\gamma_1}{T}+\frac{\gamma_2}{T^2}+\Lambda+\frac{\gamma_n}{T^n}\right] \tag{17}$$

The exponential function in Eq. (17) is only a function of $P_g\ L$, but the integral function of V is only a function of temperature. Combining Eq. (17) and Eq. (12), the emissivity of gray gas can be expressed as

$$\varepsilon_g=\sum_{i=1}^{k}a_i[1-e^{-k_i P_g L}]$$

$$=\frac{15}{\pi^4}\sum_{i=1}^{k}[1-e^{-k_iP_gL})\int_{\beta_i}^{\bar{\beta}_i}\frac{V^3}{e^V-1}dV \quad (18)$$
$$=\sum_{i=1}^{k}\left[\sum_{j=0}^{n}\frac{\gamma_{ij}}{T^j}\right][1-e^{-k_iP_gL}]$$

2.1 Calculation procedure of coefficient γ_{ij}

In order to obtain the value of a coefficient a_i for P_gL at temperature T, Eq. (18) is changed as follows:

$$\varepsilon_g=\sum_{i=1}^{k}a_iX_i \quad (19)$$

where

$$X_i=1-e^{-k_iP_gL} \quad (20)$$

Using a least square method, Eq. (19) can be transformed to the following system of linear equations.

$$a_1(T)\sum_{j=1}^{N}(X_{1j})^2+a_2(T)\sum_{j=1}^{N}X_{1j}X_{2j}+\cdots$$
$$+a_m(T)\sum_{j=1}^{N}X_{1j}X_{mj}=\sum_{j=1}^{N}\varepsilon_jX_{1j}$$
$$a_1(T)\sum_{j=1}^{N}X_{1j}X_{2j}+a_2(T)\sum_{j=1}^{N}(X_{2j})^2+\cdots$$
$$+a_m(T)\sum_{j=1}^{N}X_{2j}X_{mj}=\sum_{j=1}^{N}\varepsilon_jX_{2j} \quad (21)$$
$$a_1(T)\sum_{j=1}^{N}X_{1j}X_{mj}+a_2(T)\sum_{j=1}^{N}X_{2j}X_{mj}+\Lambda$$
$$+a_m(T)\sum_{j=1}^{N}(X_{mj})^2=\sum_{j=1}^{N}\varepsilon_jX_{mj}$$

where

$$X_{ij}=1-e-{k_aP_gL_j} \quad (22)$$

and ε_j is the emissivity of each gray band. The values of reference [3] and the Hottel's Chart are used as those of absorption coefficients k_i and emissivities ε_j to calculate the α_1, $\alpha_2\cdots\alpha_m$. in the above system of linear equations, respectively. In this calculation, a starting temperature is 555.6K (1,000R) and a temperature is increased by 138.9K (250R) until it reaches to 2777.8K (5000R).

In order to compute the values of the coefficient γ_{ij} in Eq. (18), the function of temperature in Eq. (18) and $a_i(T_k)$ obtained from Eq. (21) are combined and changed into the following system of linear equations by application of a least square method.

$$\gamma_{io}\sum_{k=1}^{l}(Y_{1k})^2+\gamma_{i1}\sum_{k=1}^{l}Y_{1k}Y_{2k}+\cdots$$
$$+\gamma_{i(n-1)}\sum_{k=1}^{l}Y_{1k}Y_{(n-1)k}=\sum_{k=1}^{l}a_i(T_k)Y_{ol}$$
$$\gamma_{io}\sum_{k=1}^{l}Y_{1k}Y_{2h}+\gamma_{i1}\sum_{k=1}^{l}(Y_{2k})^2+\cdots$$
$$+\gamma_{i(n-1)}\sum_{k=1}^{l}Y_{2k}Y_{(n-1)k}=\gamma_{i(n-1)}\sum_{k=1}^{l}a_i(T_K)Y_{1l}$$
$$(23)$$
$$\gamma_{io}\sum_{k=1}^{l}Y_{1k}Y_{(n-1)k}+\gamma_{i1}\sum_{k=1}^{l}Y_{2k}Y_{(n-1)k}+\Lambda$$
$$+\gamma_{i(n-1)}\sum_{k=1}^{l}(Y_{(n-1)k})^2=\sum_{k=1}^{l}a_i(T_k)Y_{(n-1)l}$$

where

$$Y_{ik}=\frac{1}{(T^j)_k},\ j=1\sim n,\ k=1\sim l \quad (24)$$

The coefficient γ_{ij} of the weighting factor a_i are obtained from the solution of this system of linear equations.

3. Results

3.1 Calculation results of coefficient γ_{ij}

Tables 1~5 show γ_{ij} obtained from the present model and absorption coefficient k_i given in the reference. (Farag and Allam, 1981) Table 1 shows

Table 1 Values of the constants $\gamma_{i,j}$ and k_i in Eq. (19) for carbon dioxide

$$a_i=\gamma_{i,o}+\frac{\gamma_{i1}}{\tau}+\frac{\gamma_{i2}}{\tau^2},\ \tau=\frac{T}{1000},\ P_gL=cm\cdot atm$$

I \ j	$\gamma_{i,o}$	$\gamma_{i,1}$	$\gamma_{i,2}$	k_i
1	0.4114771		0.02654507	0.0003647
2	−0.00570993	0.1751408	−0.0850763	0.0036330
3	−0.0114724	0.1439534	−0.06216872	0.0310
4	−0.00539995	0.07096846	−0.027158105	0.1496
5	−0.03176933	0.1269945	−0.05171511	1.036
6	−0.00797345	0.02744371	−0.00596318	7.806

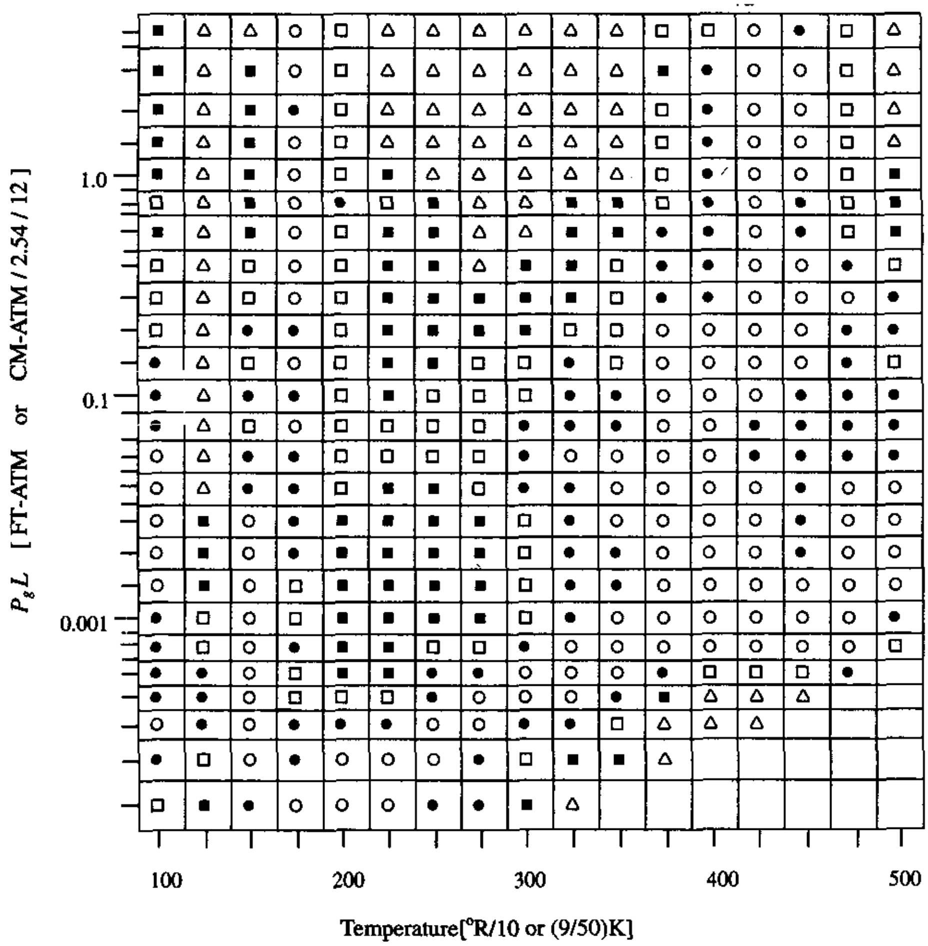

Fig. 1 Results of error analysis between the values of the Hottel's chart and the calculated values of Eq. (25) for the emissivities of carbon dioxide. (see Table 6 for the explanation of the symbols)

the results for a weighting factor a_i of carbon dioxide using a polynomial in $\frac{1}{T}$ of degree 2.

$$a_i = \gamma_{io} + \frac{\gamma_{i1}}{\tau} + \frac{\gamma_{i2}}{\tau^2} \tag{25}$$

where

$$\tau = \frac{T}{1000} \tag{26}$$

Table 2 shows the results for a weighting factor a_i of carbon dioxide using a polynomial in $\frac{1}{T}$ of degree 3.

$$a_i = \gamma_{io} + \frac{\gamma_{i1}}{\tau} + \frac{\gamma_{i2}}{\tau^2} + \frac{\gamma_{i3}}{\tau^3} \tag{27}$$

Also, the results for a weighting factor a_i of carbon dioxide using a polynomial $\frac{1}{T}$ of degree 4 are shown in Table 3.

$$a_i = \gamma_{io} + \frac{\gamma_{i1}}{\tau} + \frac{\gamma_{i2}}{\tau^2} + \frac{\gamma_{i3}}{\tau^3} + \frac{\gamma_{i4}}{\gamma^4} \tag{28}$$

Using Eq. (25) and (27), the coefficients of the polynomial for a weighting factor a_i of water vapor are given in Table 4 and 5, respectively.

3.2 Error analysis

From the following error analysis, we can estimate the accuracy of the present model for the emissivities of carbon dioxide and water vapor. The error is defined as follows:

$$Error(\%) = \left| \frac{\varepsilon_r - \varepsilon_c}{\varepsilon_r} \right| \times 100 \tag{29}$$

where ε_r represents the reference emissivity and

Table 2 Values of the constants $\gamma_{i,j}$ and k_i in Eq. (19) for carbon dioxide

$$a_i=\gamma_{i,0}+\frac{\gamma_{i1}}{\tau}+\frac{\gamma_{i2}}{\tau^2}+\frac{\gamma_{i,3}}{\tau^3},\ \tau=\frac{T}{1000},\ P_gL=cm\cdot atm$$

I \ j	$\gamma_{i,0}$	$\gamma_{i,1}$	$\gamma_{i,2}$	$\gamma_{i,3}$	k_i
1	0.4114711	−0.11911224	0.0265407	—	0.0003647
2	−0.1824832	0.8685437	−0.8188864	0.2253813	0.003633
3	−0.07770912	0.4130442	−0.3665191	0.09878845	0.031
4	−0.0294800	0.1702040	−0.1423403	0.0381592	0.1496
5	−0.03768278	0.1527180	−0.08346745	0.01091601	1.036
6	−0.00346328	0.006393768	0.02257021	−0.01061336	7.806

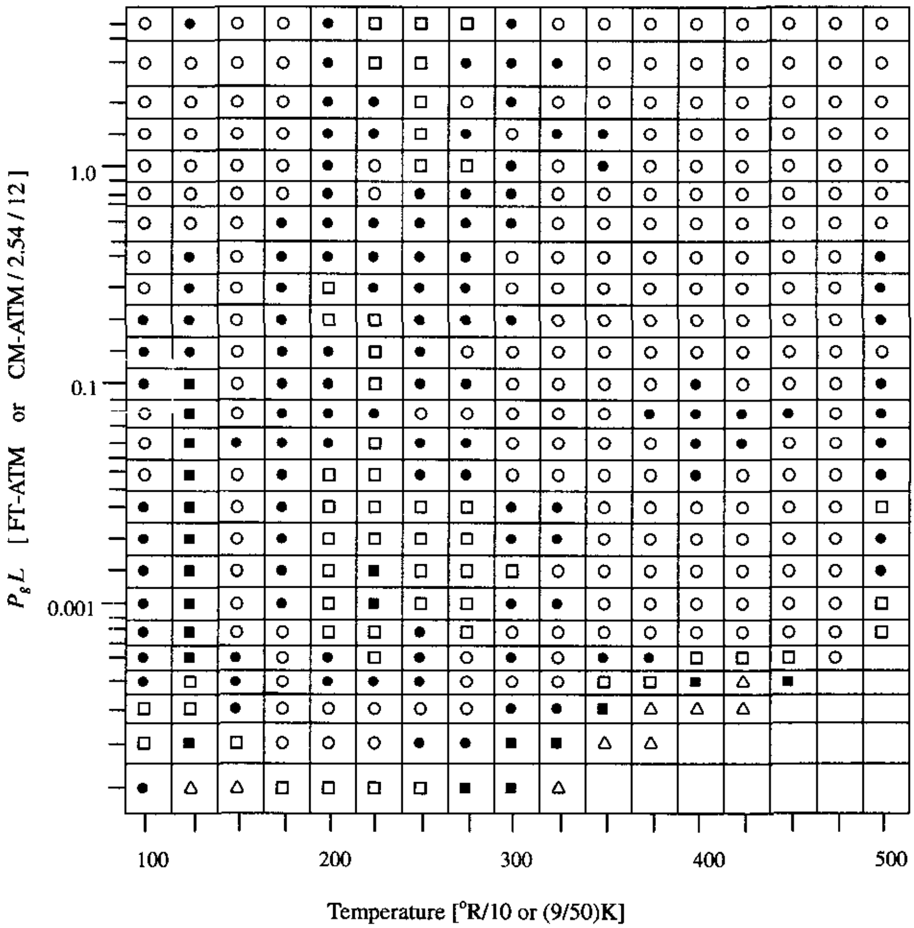

Fig. 2 Results of error analysis between the values of the Hottel's chart and the calculated values of Eq. (27) for the emissivities of carbon dioxide. (see Table 6 for the explanation of the symbols)

ε_c the calculated emissivity. The present study performs the error analysis between the calculated values from the present model and the values of the Hottel's Chart for the temperature range of 555.6~2777.8 K (1000~5000 R) and a partial-pressure-length product range of 0.09144~609.6 cm-atm (0.003~20 ft-atm). In Table 6, the symbols representing error percent range are shown.

Table 3 Values of the constants $\gamma_{i,j}$ and k_i in Eq. (19) for carbon dioxide

$$a_i=\gamma_{i,o}+\frac{\gamma_{i1}}{\tau}+\frac{\gamma_{i2}}{\tau^2}+\frac{\gamma_{i,3}}{\tau^3}+\frac{\gamma_{i,4}}{\tau^4},\ \tau=\frac{T}{1000},\ P_gL=cm\cdot atm$$

I \ j	$\gamma_{i,o}$	$\gamma_{i,1}$	$\gamma_{i,2}$	$\gamma_{i,3}$	$\gamma_{i,4}$	k_i
1	0.4114771	−0.1191224	0.0265451	—	—	0.0003647
2	−0.01874957	0.8950949	−0.8657413	0.2581601	−0.0078304	0.0003633
3	−0.1736316	0.9294681	−1.295731	0.7644727	−0.16244	0.0310
4	−0.00337878	0.0315129	0.1090557	−0.1429214	0.044528	0.1496
5	−0.0139446	0.0181269	0.1719281	−0.1793469	0.0477589	1.0361
6	0.0066593	−0.0538770	0.1438112	−0.1064498	0.0253044	7.806

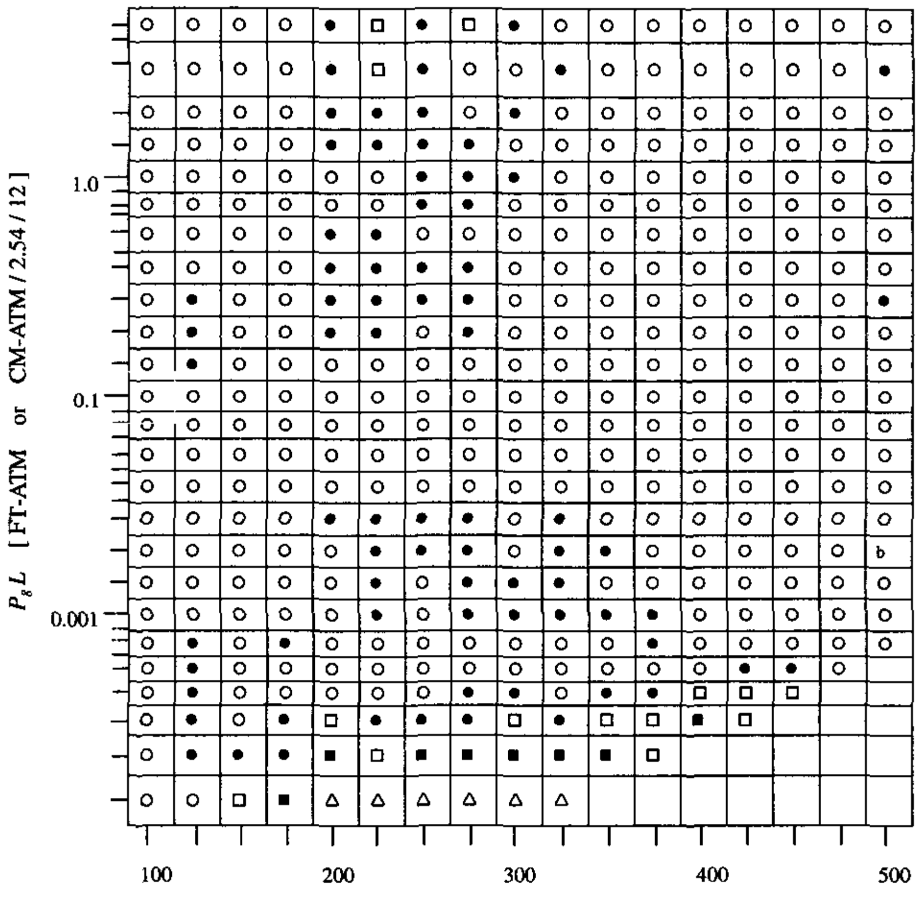

Fig. 3 Results of error analysis between the values of the Hottel's chart and the calculated values of Eq. (28) for the emissivities of carbon dioxide. (see Table 6 for the explanation of the symbols)

Figure 1 shows the difference of the results between the Hottel's Chart and the calculation from Eq. (25) for the emissivity of carbon dioxide using the coefficients of Table 1. It is shown that the results of the calculation are close to those of the Hottel's Chart in the wide ranges of P_gL and temperature. Furthermore, the calculated values from Eq. (25) for both low P_gL and high temper-

Table 4 Values of the constants $\gamma_{i,j}$ and k_i in Eq. (19) for water vapor

$$a_i=\gamma_{i,o}+\frac{\gamma_{i1}}{\tau}+\frac{\gamma_{i2}}{\tau^2},\ \tau=\frac{T}{1000},\ P_gL=cm\cdot atm$$

I \ j	$\gamma_{i,o}$	$\gamma_{i,1}$	$\gamma_{i,2}$	k_i
1	0.0141073	0.5928425	−0.22584130	0.0082352
2	−0.05889870	0.2642632	−0.08552752	0.071972
3	−0.00201229	0.02520694	0.00894706	0.50574
4	−0.01077463	0.01068194	−0.000238125	4.1788

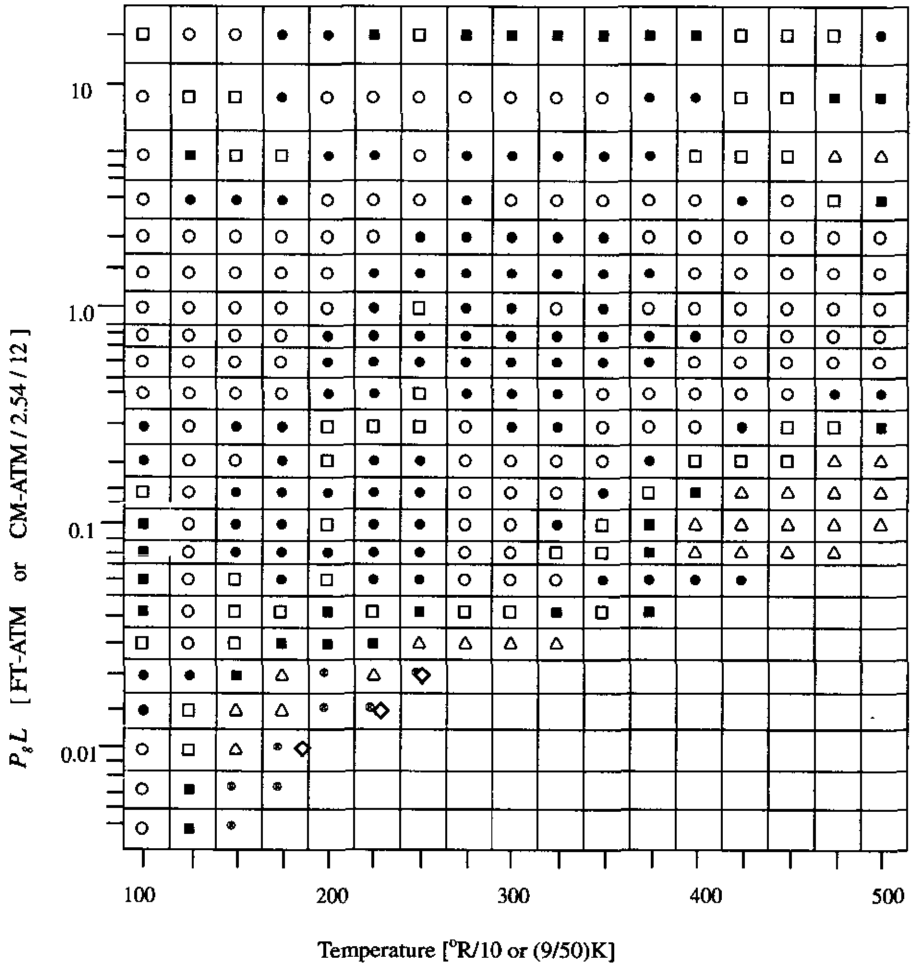

Fig. 4 Results of error analysis between the values of the Hottel's chart and the calculated values of Eq. (25) for the emissivities of carbon dioxide. (see Table 6 for the explanation of the symbols)

ature also agree well with the values of the Hottel's Chart. If the emissivity of carbon dioxide is calculated from Eq. (25) with the coefficients in Table 2, the errors are generally less than 10%. And, the maximum error is less than 15%.

Figure 2 shows the errors between the values of the Hottel's Chart and the calculated values of Eq. (27) using the data in Table 2. These results show that the errors are generally less than 5% in the entire ranges for P_gL and temperature except for very low temperature and small P_gL. Therefore, it is known that Eq. (27) can predict the total emissivities of carbon dioxide more accurately than Eq. (25).

The differences for the emissivity of carbon dioxide between the Hottel's Chart and the calculation from Eq. (28) and Table 3 are presented in Fig. 3. The emissivities obtained from Eq. (28) and Table 3 are closest to the values of the Hottel's Chart. The errors are less than 2.5% except for extremely low P_gL and temperature. Thus, Eq. (28) with the coefficients in Table 3 can be used very effectively and accurately for the computer simulation to require iterative calculations for the thermal performance analysis of combustion engines.

Figure 4 shows the difference distribution between the emissivities from the Hottel's Chart and those from Eq. (25) and Table 4 for water vapor. These results show that the errors are less than 5% for the entire ranges of P_gL and temperature except for very low values of P_gL. The weighting factors are expressed as a polynomial in T of degree 2 in the reference and in $1/T$ of the present calculation.

The comparison of the results from the Hottel's Chart and Eq. (27) with the coefficients given in Table 5 is shown in Fig. 5. The errors are very small in the temperature ranges of 555.6~2777.8K(1000~5000R) and P_gL ranges of 0.09144

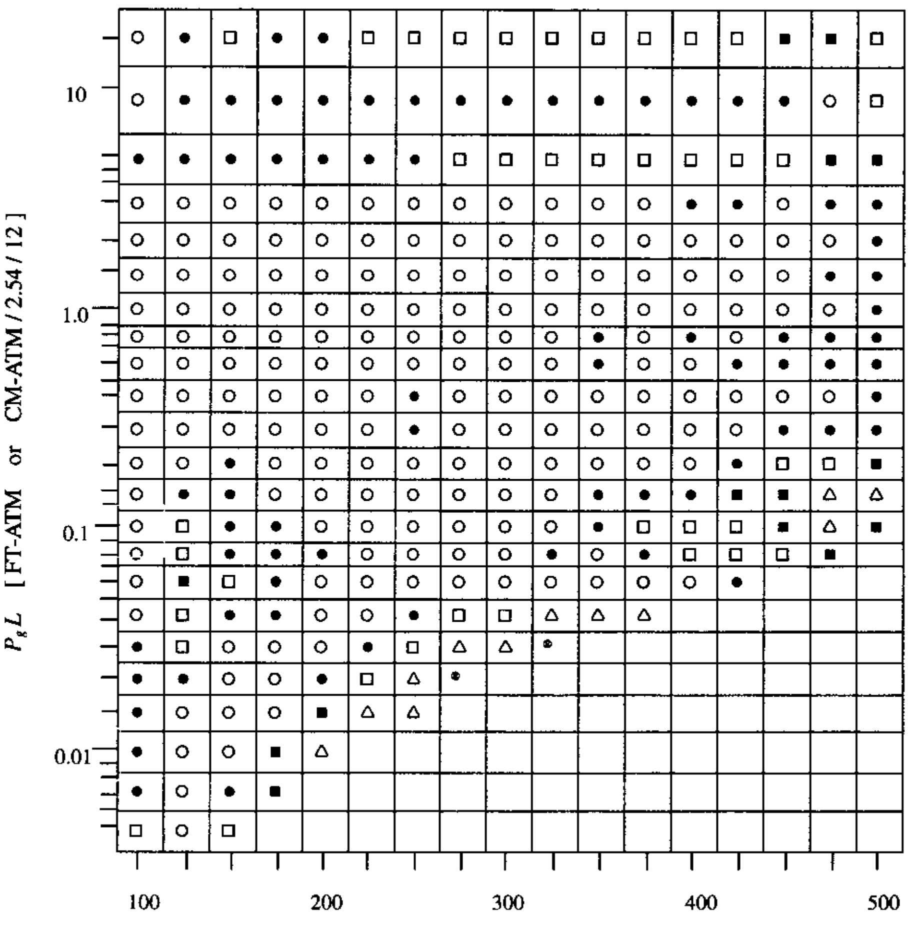

Fig. 5 Results of error analysis between the values of the Hottel's chart and the calculated values of Eq. (27) for the emissivities of carbon dioxide. (see Table 6 for the explanation of the symbols)

Table 5 Values of the constants $\gamma_{i,j}$ and k_i in Eq. (19) for water vapor

$$a_i=\gamma_{i,0}+\frac{\gamma_{i1}}{\tau}+\frac{\gamma_{i2}}{\tau^2}+\frac{\gamma_{j,3}}{\tau^3},\ \tau=\frac{T}{1000},\ P_gL=cm\cdot atm$$

I \ j	$\gamma_{i,0}$	$\gamma_{i,1}$	$\gamma_{i,2}$	$\gamma_{i,3}$	k_i
1	−0.0591638	0.8932018	−0.5719211	0.1145219	0.0082352
2	−0.1198696	0.5108355	−0.3838696	0.1006553	0.071972
3	0.02965407	−0.1167801	0.1932530	−0.06682821	0.50574
4	−0.01225271	0.01525528	−0.00468856	0.00151498	4.1788

Table 6 Symbols representing error percent range

Symbols	Error range
○	$0\leq$Error(%)≤ 2.5
●	$2.5\leq$Error(%)≤ 5.0
□	$5.0\leq$Error(%)≤ 7.5
■	$7.5\leq$Error(%)≤ 10.0
®	$10.0\leq$Error(%)≤ 15
◇	$20\leq$Error(%)≤ 30
◆	$30\leq$Error(%)≤ 50
/	$50\leq$Error(%)≤ 100
×	$100\leq$Error(%)

~609.6 cm-atm (0.003~20 ft-atm) except for very low values of P_gL. Because the errors are less than 2.5%, we can conclude that Eq. (27) and the coefficients given in Table 5 can be used for the calculation of the total emissivities of water vapor without significant discrepancy.

4. Conclusions

The present study shows that the emissivities of carbon dioxide and water vapor can be expressed as an exponential function of P_gL and a polynomial of reciprocal of temperature. The following results are obtained by comparing the results of present model with the Hottel's Chart for the emissivities of carbon dioxide and water vapor. The results from the present model agree well with those from the Hottel's Chart.

For carbon dioxide, the error between the calculated values by the present model and the values from the Hottel's Chart was distributed within 2.5% using a polynomial in $1/T$ of degree of 4. For water vapor, the model has less than 2.5% error using a polynomial in $1/T$ of degree 3. It is convenient to use the results of the present study for the numerical simulation because the radiation properties of carbon dioxide and water vapor can be accessible as simple polynomials.

Acknowledgements

The authors acknowledge financial support of their work by Korea Science and Engineering Foundation

References

Farag, I. H., 1976, "Radiation Heat Transmission From Non-Luminous Gases, Computational Study of the Emissivities of Water Vapor and Carbon Dioxide," SC. D. Thesis, MIT., Cambridge.

Farag, I. H. and Allam, T. A., 1981, "Gray-Gas Approximation of Carbon Dioxide and Standard Emissivity," *J. of Heat Transfer*, Vol. 103, pp. 403~405

Felske, J. D. and Charalam Populos, T. T., 1982, "Gray Gas Weighting Coefficients for Arbitrary Gas-Soot Mixtures," *Int. J. of Heat and mass transfer*, Vol. 25, No. 12, pp. 1849~1855.

Ha, M. Y. and Hur, B. K., 1986, "Calculation of the Absorption Coefficient and Weighting Factor Expressing the Total Emissivity of Flame," *Trans. KSME*, Vol. 10, No. 1, pp. 121~130.

Hottel, H. C. and Sarofim, A. F., 1967, *Radiative Heat Transfer*, Mcgraw-Hill, New York.

Kim, O. J., 1997, "Spectral Weighted-Sum-of-Gray-Gases Modeling for Narrow Bands," Ph. D.

Thesis, Korea Advanced Institute of Science and Technology.

Mcadams, W. H., 1954, *Heat Transmission*, Mcgraw-Hill.

Nakara, N. K. and Smith, T. S., 1977, "Combined Radiation-Convection for a Real gas," *J. of Heat Transfer*, pp. 60~65.

Pivovonsky et al., 1961, "Tables of Blackbody Radiation Functions, Macmillian Company," New York.

Sarofim, A. F., Farag, I. H., and Hottel, H. C., 1978, "Radiative Heat Transmission From Non-luminous Gases, Computational Study of the Emissivity of Carbon Dioxide," *Presented at the AIAA-ASME Thermodynamics & Heat Transfer Conference*, Palo, Alto, Calf.

Taine, J., Soufiani, A., Riviere, P., and M. Perrin, 1998, Recent Developments in Modeling the Infrared Radiative Properties of Hot Gases, Heat Transfer 1998, Vol. 1, pp. 175~187.

Wiebelt, J. A., 1966, "Engineering Radition Heat Transfer, Holt, Riehart and Winston," Inc., New York