Crack Problem at Interface of Piezoelectric Strip Bonded to Elastic Layer Under Anti-Plane Shear

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Using the theory of linear piezoelectricity, the problem of two layered strip with a piezoelectric ceramic bonded to an elastic material containing a finite interface crack is considered. The out-of-plane mechanical and in-plane electrical loadings are simultaneously applied to the strip. Fourier transforms are used to reduce the problem to a pair of dual integral equations, which is then expressed in terms of a Fredholm integral equation of the second kind. The stress intensity factor is determined, and numerical analyses for several materials are performed and discussed.

Key Words : Piezoelectric Material, Interface Crack, Anti-Plane Shear, Stress Intensiy Factor

Nomenclature

ϕ : Electric potential
E_i : Electric field vector
D_i : Electric displacement vector
c_{ijkl} : Elastic stiffness
e_{kij} : Piezoelectric constant
ε_{ij} : Dielectric permittivity

1. Introduction

Since the brothers Curie discovered the electro-mechanical coupling phenomenon of piezoelectric material in 1880, it has been used in various applications of the electro-mechanical devices such as actuators, sensors, and transducers. In practical structures, many piezoelectric devices are consisted of both piezoelectric and structural layers, and the failure of the piezoelectric layers due to their brittle nature can influence the performance of such devices. Therefore, the electro-elastic behavior of layered piezoelectric structures have been studied by several researchers (Parton, 1976 ; Sosa and Pak, 1990 ; Beom and Atluri, 1996 ; Chen et al., 1997 ; Narita and Shindo, 1998 ; Narita et al., 1999). Also, Kim and Jones (1996) reported that the brittle fracture of the piezoelectric composite structures could be initiated from the interface of dissimilar materials.

We consider the problem of two layered strip with a piezoelectric ceramic bonded to an elastic material containing a finite crack at the interface under both out-of-plane mechanical and in-plane electrical loads. Fourier transforms are used to reduce the problem to a pair of dual integral equations, which is then expressed to a Fredholm integral equation of the second kind. The stress intensity factor is determined, and numerical analyses for several materials are obtained and discussed.

2. Problem Formulation

Consider an interface crack of a piezoelectric ceramic strip that is bonded to an elastic strip layer as shown in Fig. 1. The present model is simplified the structure of shear type piezoelectric accelerometers. A finite crack of length $2a$ is located between a piezoelectric layer of thickness h_1 and an elastic layer of thickness h_2. The Cartesian coordinates is set at the center of a crack for reference. The piezoelectric medium is

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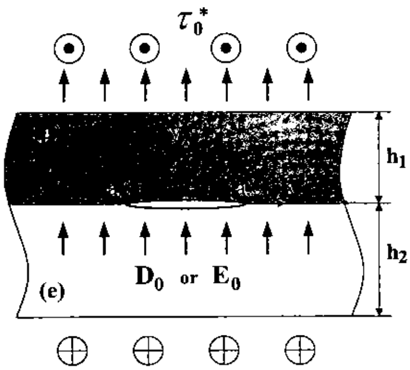

Fig. 1 A layered piezoelectric strip with an interface crack

considered to be transversely isotropic with hexagonal symmetry, which has an isotropic basal plane of the xy-plane and a poling direction of the z-axis.

The piezoelectric boundary value problem is simplified under the out-of-plane displacements and in-plane electric fields such that

$$u_x^{(p)}=u_y^{(p)}=0,\ u_z^{(p)}=w^{(p)}(x, y) \quad (1)$$
$$E_x=E_x(x, y),\ E_y=E_y(x, y),\ E_z=0 \quad (2)$$
$$u_x^{(e)}=u_y^{(e)}=0,\ u_z^{(e)}=w^{(e)}(x, y) \quad (3)$$

where the superscripts (p) and (e) imply the piezoelectric and elastic layers, respectively.

In this case, the basic relations become

$$\tau_{zi}^{(p)}=c_{44}\gamma_{zi}^{(p)}-e_{15}E_i,\ \tau_{zi}^{(e)}=c_{44}^{(e)}\gamma_{zi}^{(e)} \quad (4)$$
$$E_i=-\phi_{,i}, \qquad D_i=e_{15}\gamma_{zi}^{(p)}+\varepsilon_{11}E_i \quad (5)$$

where ϕ is the electric potential, τ_{zi}, γ_{zi}, E_i and $D_i(i=x,\ y)$ are the components of the stress, strain, electric field and electric displacement vectors, respectively. Also, c_{44}, e_{15} and ε_{11} are the elastic stiffness of piezoelectric material measured in a constant electric field, the piezoelectric constant and the dielectric permittivity measured at a constant strain, respectively. And $c_{44}^{(e)}$ is the shear modulus of elastic material.

The governing equations are simplified to

$$c_{44}\nabla^2 w^{(p)}+e_{15}\nabla^2\phi=0 \quad (6)$$
$$e_{15}\nabla^2 w^{(p)}-\varepsilon_{11}\nabla^2\phi=0 \quad (7)$$
$$\nabla^2 w^{(e)}=0 \quad (8)$$

The following boundary conditions are considered,

$$\tau_{zy}^{(p)}(x, 0^+)=\tau_{zy}^{(e)}(x, 0^-)=0 \quad (|x|<a) \quad (9)$$
$$w^{(p)}(x, 0)=w^{(e)}(x, 0) \quad (|x|\geq a) \quad (10)$$
$$\tau_{zy}^{(p)}(x, 0)=\tau_{zy}^{(e)}(x, 0) \quad (|x|\geq a) \quad (11)$$
$$(Case\ 1)\ \begin{array}{l}\tau_{zy}^{(p)}(x, h_1)=\tau_{zy}^{(e)}(x, -h_2)=\tau_0\\ D_y(x, 0)=D_y(x, h_1)=D_0 \quad (|x|<\infty)\end{array} \quad (12)$$
$$(Case\ 2)\ \begin{array}{l}\tau_{zy}^{(p)}(x, h_1)=\tau_{zy}^{(e)}(x, -h_2)=\tau_0\\ E_y(x, 0)=E_y(x, h_1)=E_0 \quad (|x|<\infty)\end{array} \quad (13)$$

where τ_0 is the constant shear stress combined by the purely mechanical shear stress τ_0^* and uniform electric displacement D_0 or uniform electric field E_0 in the forms,

$$\tau_0=\begin{cases}\dfrac{\widetilde{c_{44}}}{c_{44}}\tau_0^*-\dfrac{e_{15}}{\varepsilon_{11}}D_0 & (Case\ 1)\\ \tau_0^*-e_{15}E_0 & (Case\ 2)\end{cases} \quad (14)$$

and $\widetilde{c_{44}}=c_{44}+e_{15}^2/\varepsilon_{11}$.

In this interface crack problem between the piezoelectric and elastic materials, it does not need to consider the electrical boundary condition at the crack surface, which is under heavy debate (Deeg, 1980 ; Pak, 1990 ; Hao and Shen, 1994 ; Duun, 1994 ; Park and Sun, 1995 ; Sosa and Khutoryansky, 1996 ; Kumar and Singh, 1997).

3. Solution Procedure

Applying Fourier transforms to Eqs. (6)~(8), the solutions are formed as follows,

$$w^{(p)}(x, y)=\frac{2}{\pi}\int_0^\infty\{A_1(s)\exp(sy)+A_2(s)\exp(-sy)\}\cos(sx)ds+a_0y \quad (15)$$
$$\phi(x, y)=\frac{2}{\pi}\int_0^\infty\{B_1(s)\exp(sy)+B_2(s)\exp(-sy)\}\cos(sx)ds-b_0y \quad (16)$$
$$w^{(e)}(x, y)=\frac{2}{\pi}\int_0^\infty\{C_1(s)\exp(sy)+C_2(s)\exp(-sy)\}\cos(sx)ds+c_0y \quad (17)$$

where $A_j(s)$, $B_j(s)$ and $C_j(s)$ $(j=1, 2)$ are the unknown functions, and a_0, b_0 and c_0 are the real constants to be solved.

The field components of the piezoelectric and elastic layers can be obtained from the relations (4) and (5), and then applying the boundary conditions, the unknown constants and functions are determined as follows,

$$(Case\ 1)\quad a_0=\frac{\varepsilon_{11}\tau_0+e_{15}D_0}{c_{44}\varepsilon_{11}+e_{15}^2},\ b_0=\frac{c_{44}D_0+e_{15}\tau_0}{c_{44}\varepsilon_{11}+e_{15}^2}$$

$$c_0=\frac{\tau_0}{c_{44}^{(e)}} \tag{18}$$

$$(Case\ 2)\quad a_0=\frac{\tau_0+e_{15}E_0}{c_{44}},\ b_0=E_0,\ c_0=\frac{\tau_0}{c_{44}^{(e)}} \tag{19}$$

$$A_1(s)=\exp(-2sh_1)A_2(s) \tag{20}$$

$$B_1(s)=\begin{cases}\frac{e_{15}}{\varepsilon_{11}}\exp(-2sh_1)A_2(s) & (Case\ 1)\\ 0 & (Case\ 2)\end{cases} \tag{21}$$

$$B_2(s)=\begin{cases}\frac{e_{15}}{\varepsilon_{11}}A_2(s) & (Case\ 1)\\ 0 & (Case\ 2)\end{cases} \tag{22}$$

$$C_1(s)=-\frac{\hat{\mu}}{c_{44}^{(e)}}\frac{1-\exp(-2sh_1)}{1-\exp(-2sh_2)}A_2(s) \tag{23}$$

$$C_2(s)=-\frac{\hat{\mu}}{c_{44}^{(e)}}\frac{1-\exp(-2sh_1)}{1-\exp(-2sh_2)}\exp(-2sh_2)A_2(s) \tag{24}$$

where

$$\hat{\mu}=\begin{cases}\widetilde{c_{44}}=c_{44}+\frac{e_{15}^2}{\varepsilon_{11}} & (Case\ 1)\\ c_{44} & (Case\ 1)\end{cases} \tag{25}$$

By Eqs. (18)~(25), the mixed boundary conditions, Eqs. (9) and (10), lead to a set of dual integral equations in the forms,

$$\int_0^\infty s\{1-\exp(-2sh_1)\}A_2(s)\cos(sx)ds=\frac{\pi}{2}\frac{\tau_0}{\hat{\mu}}\quad (0\le x<a) \tag{26}$$

$$\int_0^\infty \Omega(s)A_2(s)\cos(sx)ds=0(x\ge a) \tag{27}$$

where

$$\Omega(s)=\{1+\exp(-2sh_1)\}+\frac{\hat{\mu}}{c_{44}^{(e)}}\coth(sh_2)\{1-\exp(-2sh_1)\} \tag{28}$$

Equations (26) and (27) may be solved by using the new function $\Phi(\xi)$ defined by

$$A_2(s)=\frac{\pi}{2}\frac{\hat{\mu}+c_{44}^{(e)}}{\hat{\mu}c_{44}^{(e)}}\frac{\tau_0 a^2}{\Omega(s)}\int_0^1\sqrt{\xi}\,\Phi(\xi)J_0(sa\xi)d\xi \tag{29}$$

where $J_0(\ )$ is the zero-order Bessel function of the first kind.

Inserting Eq. (29) into Eqs. (26) and (27), we can find that $\Phi(\xi)$ is given by a Fredholm integral equation of the second kind in the form,

$$\Phi(\xi)+\int_0^1 K(\xi,\eta)\Phi(\eta)d\eta=\sqrt{\xi} \tag{30}$$

where

$$K(\xi,\eta)=\sqrt{\xi\eta}\int_0^\infty s\left[F\left(\frac{s}{a}\right)-1\right]J_0(s\xi)J_0(s\eta)ds \tag{31}$$

$$F\left(\frac{s}{a}\right)=\frac{\hat{\mu}+c_{44}^{(e)}}{c_{44}^{(e)}}\frac{1-\exp\left(-2s\frac{h_1}{a}\right)}{\Omega\left(\frac{s}{a}\right)} \tag{32}$$

The mode III stress intensity factor is defined and determined in the form,

$$K_{III}\equiv\lim_{x\to a}\sqrt{2\pi(x-a)}\,\tau_{zy}^{(k)}(x,0)=\tau_0\sqrt{\pi a}\,\Phi(1)\qquad (k=p,\ e) \tag{33}$$

The strain intensity factor as well as the electric field and electric displacement intensity factors, which are considered in the crack problems of the piezoelectric materials, do not exist in the present problem.

4. Numerical Results and Discussions

The piezoelectric ceramic is considered to be the PZT-4 or PZT-5H, and the elastic layer to be the aluminum or epoxy. Material properties used in the examples are given in Table 1 (Narita et al., 1999).

For the structure consisted of PZT-4 and epoxy under Case 1 loading condition, the normalized stress intensity factor $K_{III}/\tau_0\sqrt{\pi a}$ with the variance of the normalized crack length $2a/h_1$ is shown for various layer thickness ratios h_1/h_2 in Fig. 2. Figure 3 shows the result for PZT-5H and aluminum under Case 2 loading condition. The stress intensity factor increases with the increases of both $2a/h_1$ and h_1/h_2. Also, the normalized stress intensity factor $K_{III}/\tau_0\sqrt{\pi a}$ approaches to unity with the decrease of $2a/h_1$ for all h_1/h_2.

Figures 4 and 5 show the stress intensity factor normalized by the purely mechanical shear stress, $K_{III}/\tau_0^*\sqrt{\pi a}$ as a function of the normalized crack length $2a/h_1$ under various normalized electrical

Table 1 Material properties

	PZT-4	PZT-5H	Al	Epoxy
c_{44} or $c_{44}^{(e)}$ ($\times10^{10}N/m^2$)	2.56	2.3	2.65	0.176
e_{15} (C/m^2)	12.7	17.0	0	0
ε_{11} ($\times10^{-10}C/Vm$)	64.6	150.4	-	-

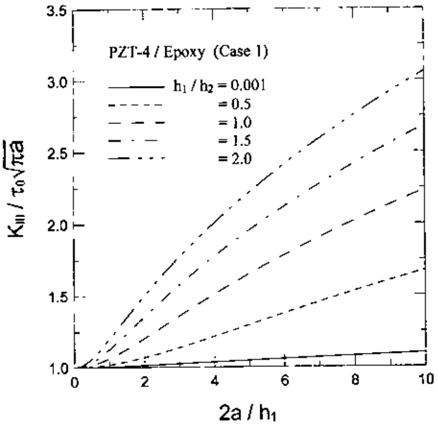

Fig. 2 Normalized stress intensity factor vs. $2a/h_1$ for PZT-4/Epoxy (Case 1)

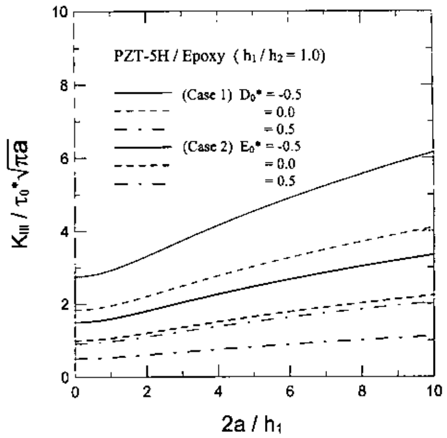

Fig. 5 Normalized stress intensity factor vs. $2a/h_1$ for PZT-5H/Epoxy under normalized electrical loads

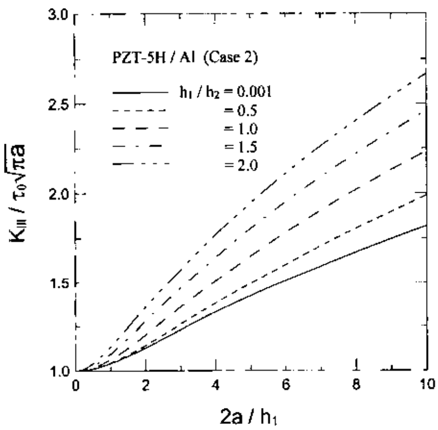

Fig. 3 Normalized stress intensity factor vs. $2a/h_1$ for PZT-5H/Aluminum (Case 2)

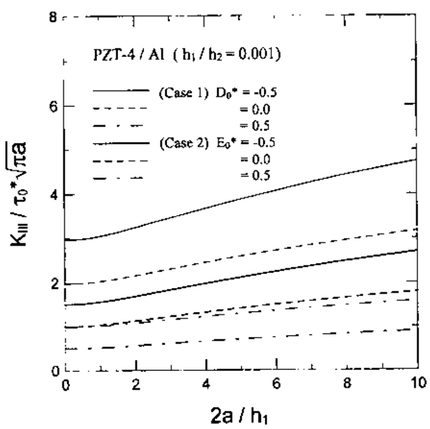

Fig. 4 Normalized stress intensity factor vs. $2a/h_1$ for PZT-4/Aluminum under normalized electrical loads

loads for PZT-4/aluminum and PZT-5H/epoxy, respectively. Here, the normalized electric displacement is defined as $D_0^*=(c_{44}e_{15}D_0)/(\tilde{c}_{44}\epsilon_{11}\tau_0^*)$ and the normalized electric field as $E_0^*=(e_{15}E_0)/\tau_0^*$, respectively. Figure 4 agrees with the result of a piezoelectric ceramic layer bonded to an elastic half plane by Narita and Shindo (1998). The increase of positive normalized electrical loads decreases the stress intensity factor, otherwise, the increase of negative normalized electrical loads increases the stress intensity factor. For unit normalized electrical loads, the stress intensity factor approaches to zero.

5. Conclusion

A theoretical analysis was performed for the problem of two layered strip with a piezoelectric ceramic bonded to an elastic material containing a finite crack at the interface. In the present interface crack problem, the fracture criterion is determined only by the stress intensity factor. The numerical results on the stress intensity factor depend on the normalized crack length, the thickness ratio of the piezoelectric ceramic to elastic layers, and the material properties such as stiffness and constant as well as the electrical loading conditions.

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