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A Study on the Performance of a Submerged Breakwater by Using the Singularity Distribution Method
특이점 분포법에 의한 잠수된 방파제의 성능 해석

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Abstract ☐ In this study, a submerged plate-type breakwater is considered, which is supported by elastic foundation. This breakwater makes use of wave phase interaction among the incident, diffracted and radiated waves. We apply a three-dimensional singularity distribution method within the linear potential theory in order to describe the wave field. The submerged plate is assumed to be rigid and the elastic support be a linear spring with constant stiffness. A typical rectangle plate is exemplified for numerical calculation. The thickness of the plate is carefully selected in order to guarantee the solution to be stable by checking the condition number of the system matrix. A parametric study is carried out for examining the effect of the stiffness of the elastic support on performance of the breakwater. We also examine the effect of the submerged depth.

Keywords : Submerged Plate, Elastic Support, Wave Attenuation, Singularity Distribution

要 旨 : 본 연구에서는 탄성 지지된 잠수 평판으로 이루어진 방파제의 파랑감쇠 성능을 고찰하였다. 이 방파제는 파랑감쇠를 위하여 입사파, 산란파 그리고 방사파의 위상 상호작용을 이용한다. 유동장을 기술하기 위하여 선형포텐셜 이론을 도입하였으며, 특이점 분포방법을 적용하였다. 잠수된 평판은 강체라고 가정하였으며, 탄성지지는 일정한 탄성계수를 가진 선형 스프링으로 간주하였다. 수치계산을 위하여 직사각형 평판을 고려하였다. 수치해의 안정성을 위하여 영향행렬의 조건수를 조사하여 평판의 두께를 결정하였다. 탄성지지의 강성이 방파제의 성능에 미치는 영향을 살펴보았다. 한편 잠수 깊이가 파랑감쇠에 미치는 영향을 조사하였다. 계산결과에 의하면 방파제의 공진 진동수가 입사파의 진동수보다 약간 작게 설정되어야 파랑감쇠가 최대로 일어남을 확인하였다.

핵심용어 : 탄성지지, 파랑감쇠, 특이점 분포법

1. INTRODUCTION

In phase with the increasing activity of ocean space utilization, much attention has been paid to breakwaters. For example, the wave-induced motion of a very large floating structure(VLFS) has to be reduced for safe landing and taking off. In order to guarantee the safety, a VLFS must be protected from severe wave attacks by means of breakwaters.

Surface waves may be characterized by four physical quantities - wave height, wave direction, wave period and phase lag - and controlled by energy dissipation, phase interaction, reflection, change in direction and frequency (Sawaragi, 1995). Breakwaters may be classified into several types according to the above wave control mechanism. In this study, a submerged plate-type breakwater with elastic foundation is considered. This breakwater makes use of wave phase interaction among the incident, diffracted and radiated waves.

There are a number of studies that deal with floating or submerged breakwaters for wave attenuation. Srokosz (1979) considered a submerged sphere, which absorbs the power from incident waves. Evans and Linton(1991) showed how a submerged body can, if properly tuned to

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incoming waves, reflect an appreciable amount of the incident wave energy by creating waves through its own motion using multipole expansion method. Yip and Chwang(1996, 1997) analyzed the performance of a pitching plate and a pitching porous plate based on the matched eigen-function expansion method in two dimensions. Kim *et al.*(1998) carried out an experimental and numerical investigation on the performance of the floating breakwaters.

We apply the singularity distribution method within linear potential theory in order to describe the wave field. The submerged plate is assumed to be rigid and the elastic support be a linear spring with constant stiffness. In order to validate the numerical analysis, computations are made for a submerged sphere and the results are examined. Finally, a typical rectangle plate is exemplified for numerical calculation. The thickness of the plate is carefully selected in order to guarantee the solution to be stable by checking the condition number of the system matrix. A parametric study is carried out to investigate the effect of the stiffness of the elastic support on the performance of the breakwater. The effect of the submerged depth on the wave attenuation is also discussed.

2. MATHEMATICAL FORMULATION

Consider a submerged plate with elastic supports as shown in Fig. 1. In order to describe the fluid motion and the equation of motion of the submerged plate, the global coordinate and the body fixed coordinate system are introduced, where their origins are on the undisturbed free surface and the plate center, respectively. The positive z-axis points vertically upward in both cases.

It is assumed that the wave height is small compared to the body dimension so that the viscous effects may be neglected. The wave steepness is so small that the linear wave theory may be used. The fluid is assumed to be incompressible and inviscid, and the flow be irrotational. Thus the velocity potential can be introduced in order to describe the flow field.

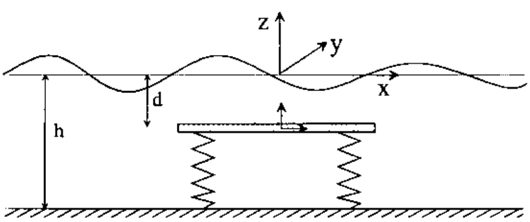

Fig. 1. Schematic diagram of a submerged breakwater.

The velocity potential for harmonic motion may be written as follows:

$$\Phi(x;t) = Re[\phi(x)e^{i\omega t}] \tag{1}$$

where $\boldsymbol{x}$ is referred to as the global coordinate, Re denotes the real part and ω is the angular frequency.

$\phi(\boldsymbol{x})$ satisfies

$$\nabla^2\phi(x) = 0 \text{ in the fluid} \tag{2}$$

$$-\omega^2\phi + g\frac{\partial\phi}{\partial z} = 0\,,\ z = 0 \tag{3}$$

$$\frac{\partial\phi}{\partial z} = 0\,,\ z = -h \tag{4}$$

$$\frac{\partial\phi}{\partial n} = \boldsymbol{U}\cdot\boldsymbol{n} \text{ on the body surface} \tag{5}$$

In equation (5), $\boldsymbol{n}$ is the generalized unit vector normal to the body surface.

If the Green theorem is applied, the velocity potential turns out to be a solution of the following Fredholm integral equation of the second kind:

$$\alpha\phi(x) + \int_S \phi(\xi)\frac{\partial G(x;\xi)}{\partial n}dS = \int_S \frac{\partial\phi(\xi)}{\partial n}G(x;\xi)dS \tag{6}$$

where α is the solid angle, and S the boundary surface of the body. $G(\boldsymbol{x};\xi)$ is the appropriate Green function (Wehausen and Laitone, 1960).

For convenience, the velocity potential ϕ is subdivided into the incident potential ϕ_I, the diffraction potential ϕ_D and the radiation potentials ϕ_j as follows:

$$\phi = \phi_I + \phi_D + \sum_{j=1}^{6}\xi_j\phi_j \tag{7}$$

where ξ_j is the complex amplitude of the j-th motion mode. The incident velocity potential is given by

$$\phi_I = -\frac{gA}{w}\frac{\cosh k(z+h)}{\cos kh}e^{-i(k\cos\beta x + k\sin\beta y) + i\omega t} \tag{8}$$

$$\text{with } \omega^2 = gk\tanh kh \tag{9}$$

where A is the incident wave amplitude, k the wave number and β the incident wave angle.

If the velocity potential is determined, the hydrodynamic coefficients and the wave loading on the body can be calculated by means of the linearized Bernoulli equation.

$$p = -\rho\frac{\partial \Phi}{\partial t} = -i\omega\rho\phi \tag{10}$$

where p and ρ are the fluid pressure and density, respectively.

The total force on the body is evaluated by integrating the fluid pressure over the wetted surface in its mean position. The added mass and the wave damping coefficient of the body can be defined as follows:

$$A_{ij} = \frac{\rho}{\omega}\iint_S \mathrm{Im}(\phi_j n_i)dS \tag{11}$$

$$B_{ij} = -\rho\iint_S \mathrm{Re}(\phi_j n_i)dS \tag{12}$$

The wave exciting force takes the form:

$$F_i = -i\omega\rho\iint_S(\phi_I + \phi_D)n_i dS \tag{13}$$

By applying Newtons second law of motion, the equations of motion for the submerged body yield to

$$\sum_{j=1}^{6}[(m_{ij} + A_{ij})\ddot{x}_j + B_{ij}\dot{x}_j + C_{ij}x_j] = F_i,\ i, j = 1, 2, \cdots, 6 \tag{14}$$

where m_{ij} is the mass and C_{ij} the restoring stiffness resulting from the elastic support. Assuming harmonic oscillations of the body, the above equation can be solved in frequency domain.

When the complex amplitude of the body motion is determined, the free surface profile can be described by

$$\zeta(x,y,t) = -\frac{1}{g}\frac{\partial \Phi}{\partial t}\bigg|_{z=0} = -\frac{i\omega}{g}\phi(x,y,0)e^{i\omega t} \tag{15}$$

The transmission coefficient T is defined by the following equation in average sense.

$$T = \frac{1}{S_f}\int_{S_f}\left|\frac{\zeta}{A}\right|dS \tag{16}$$

where S_f denotes a prescribed zone at far down-stream.

3. NUMERICAL RESULTS

3.1 Numerical Verification

In order to validate the numerical procedure, the added mass for a submerged sphere is calculated in frequency domain. The radius of the sphere is 100 m and its surface divided by 450 panels. The water depth is 500 m. The submerged depth is taken to be 150 m, 200 m and 400 m.

The added mass is normalized by the displacement of the sphere and plotted in Fig. 2. As the submergence depth increases, the added mass in heave mode approaches to the value of hemisphere. This is already expected because the non-dimensional added-mass of a sphere oscillating in an infinite fluid with no free surface is 1/2(Newman, 1977).

In order to examine the behaviour of the condition

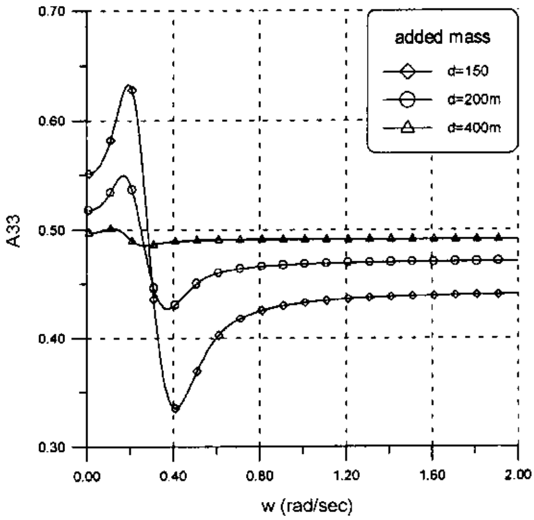

Fig. 2. Added mass for the heaving submerged sphere.

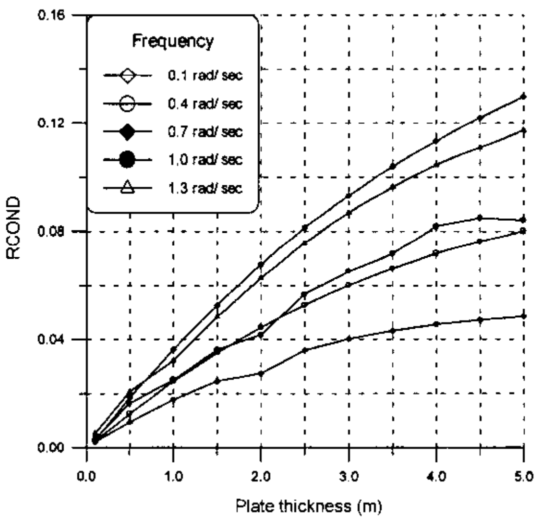

Fig. 3. Condition number of the influence matrix for the plate with different thickness.

number of the influence matrix for different plate thickness, the condition number of the matrix was computed by applying LINPACK reciprocal condition estimator (RCOND). If RCOND is near 1.0, it implies that the influence matrix is well-conditioned. Generally, a matrix is ill-conditioned if its RCOND approaches the machines floating-point precision(for example, less than 10^{-6} for single precision). Fig. 3 shows the condition number for the plate with different thickness. As expected, the influence matrix becomes ill-conditioned when the plate thickness decreases. Based on this analysis, we select the thickness of the submerged plate as 1.0 m.

3.2 Submerged Plates

Now we consider a submerged plate-type breakwater in a water depth of 50 m. The length and breadth of the plate are 20 m and 100 m, respectively. Its surface is represented by 208 panels for hydrodynamic calculations. It is assumed that the surge, sway and yaw motions are restricted.

Fig. 4 shows the added mass for the heaving plate at several submergence depths. As the submerged depth increases, the added mass coefficient becomes insensitive to frequency. It can be observed that the added mass varies strongly in shallow submerged depths. The effect of the plate thickness on the added mass is given in Fig. 5. The change in the added mass is primarily due to the effective submerged depth. As the thickness of the plate increases, the submerged depth of the plate top relatively decreases. However, it is found that the change is not significant.

Fig. 6 and Fig. 7 show the heave and pitch motions of the breakwater. The stiffness of the elastic support is taken to be 0.5 K_o, 1.0 K_o, 2.0 K_o and 3.0 K_o. Here the reference stiffness K_o is defined by

$$K_o = \frac{M}{\omega_o^2},\ \omega_o^2 = g\left(\frac{2\pi}{L}\right) \tag{17}$$

where M and L are the mass and the length of breakwater,

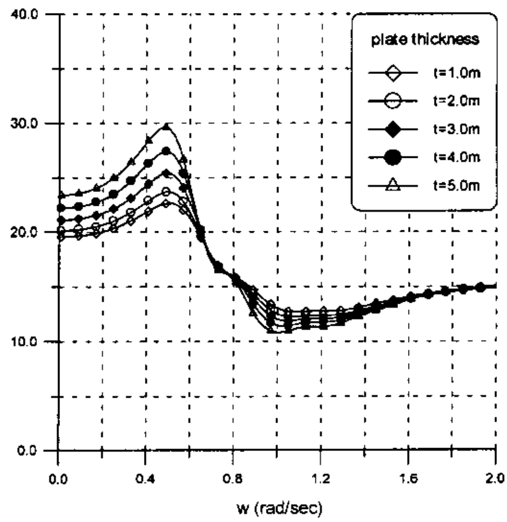

Fig. 5. Added mass of the heaving plate with different plate thickness (d=10 m).

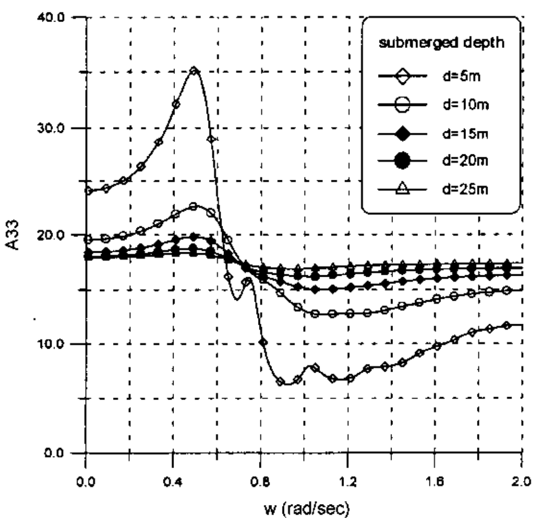

Fig. 4. Added mass of the heaving plate with different submergence (t=1.0 m).

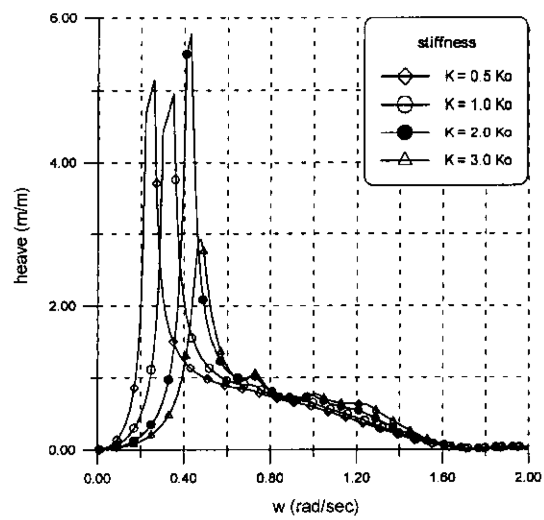

Fig. 6. Heave motion response of breakwater with different stiffness (d=5 m).

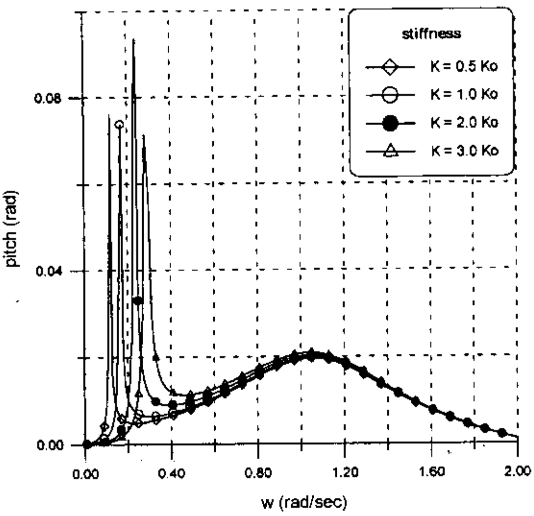

Fig. 7. Pitch motion response of breakwater with different stiffness (d=5 m).

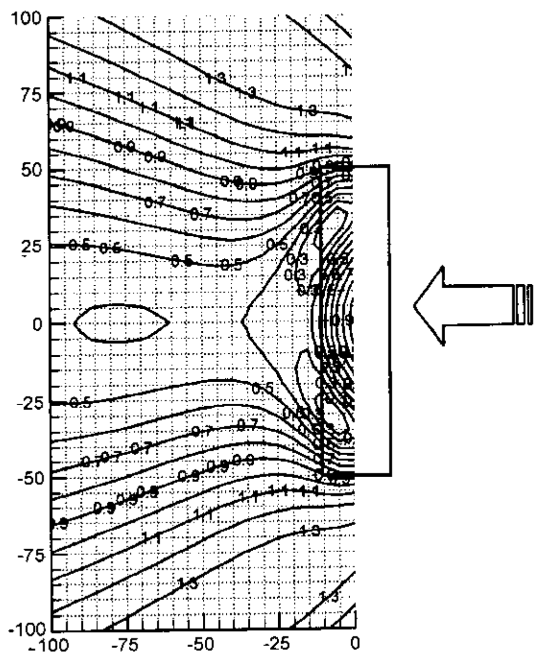

Fig. 8. Wave field around breakwater (d=10 m, ω=0.65 rad/sec, K=2.0 K_o).

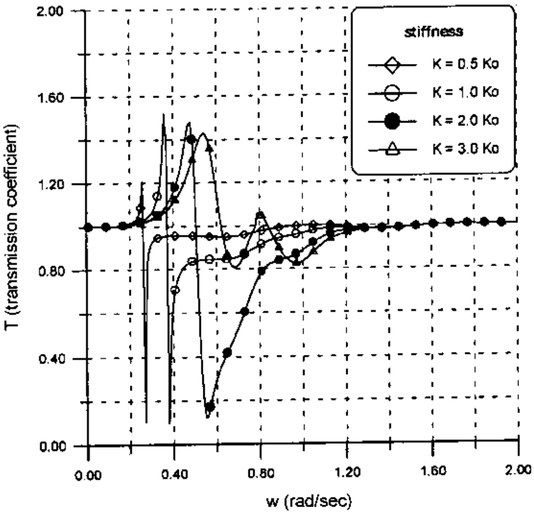

Fig. 9. Transmission coefficient of breakwater with different stiffness (d=10 m).

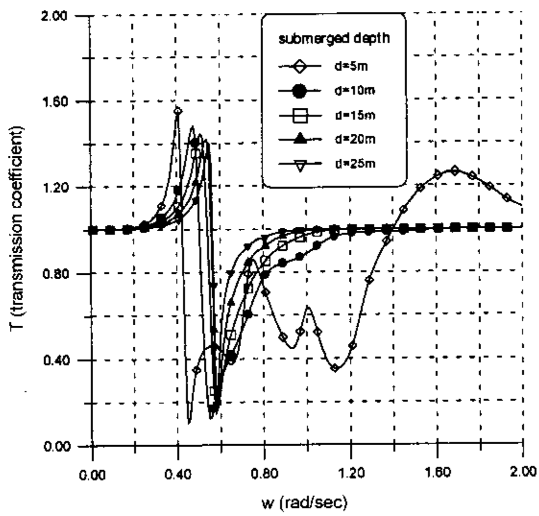

Fig. 10. Transmission coefficient of breakwater with different submergence (K=2.0 K_o).

respectively. It is expected that the heave motion governs the wave transmission because this motion is dominant. Fig. 8 shows the wave field resulting from the interaction between the incident, diffracted and radiated waves in the case of d=10 m, ω = 0.65 rad/sec and K=2.0 K_o.

Fig. 9 and Fig. 10 show the transmission coefficient for floating breakwaters with different stiffnesses of elastic supports and submerged depths. It is clearly observed that the transmission coefficient is strongly affected by the stiffness and by the submerged depth, too. As the stiffness becomes stronger, the breakwater shows a larger wave attenuation and has the narrow bandwidth of attenuated wave frequency to some degree. Thus it seems there exists an optimal stiffness. The effect of the submerged depth is similar to that of the stiffness. It is also found that the wave height can be amplified by the submerged breakwater, if the incident wave frequency is lower than the resonant frequency of heave motion. Thus it is desirable that the resonant frequency is slightly smaller than that of incident waves for a good performance of breakwaters. In order to meet this condition, the added mass coefficient should be proportional to ω^2. However, the added mass curve does not satisfy this condition based on the three dimensional hydrodynamic analysis. Because the added mass for three-dimensional bodies has a finite value at low frequency region unlike two-dimensional bodies(Evans and Linton, 1991). It is to note that the three-dimensional submerged breakwater can amplify low frequency waves such as swells. Thus the breadth of the submerged breakwater must be as wide as possible in order to avoid the amplification of low frequency waves.

The reflection coefficient is given in Fig. 11. In the case of the breakwater with strong stiffness, the reflected wave has high amplitude. Fig. 12 shows the effect of radiation waves on the transmission coefficients. If the plate-type breakwater is fixed, the attenuation of wave cannot be expected in the sense of the wave control mechanism of phase interactions.

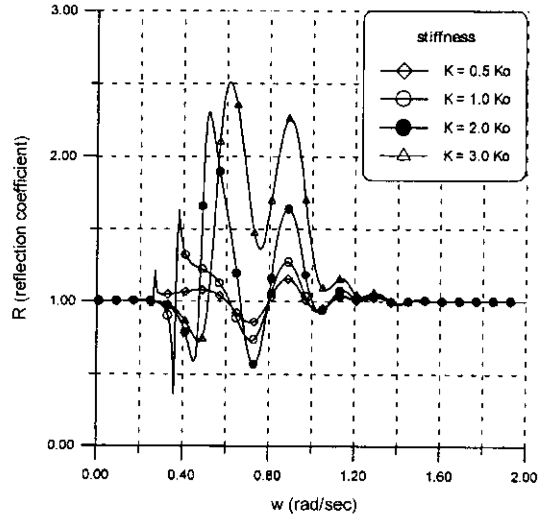

Fig. 11. Reflection coefficient of breakwater with different stiffness.

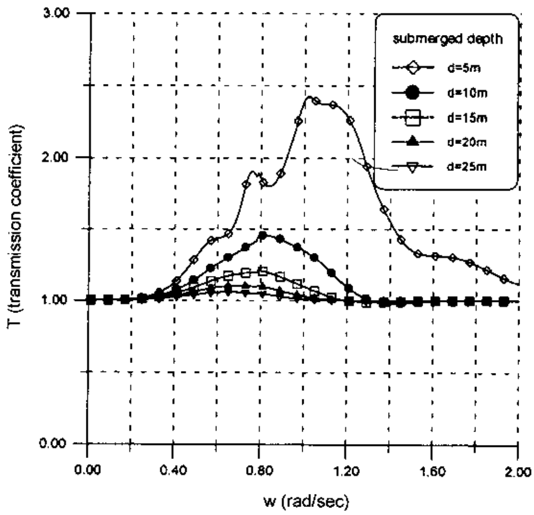

Fig. 12. Transmission coefficient for fixed breakwater with different submergence.

4. SUMMARY AND CONCLUSION

A performance analysis program for submerged plate-type breakwaters has been developed based on three-dimensional singularity distribution method within the linear potential theory. The performance of a typical submerged plate breakwater has been analyzed and the effects of the elastic support and the submerged depth on the wave attenuation are investigated.

Based on the numerical results, it is found that the performance of plate-type breakwaters in three dimensions is quite different from that in two dimensions. That is, the breakwater can amplify low frequency waves. The performance of the submerged breakwater is strongly governed by the stiffness of elastic support and the submergence depth. For a good performance, the resonance frequency of the breakwater has to be selected slightly lower than the incident wave frequency. The breadth of breakwaters must be as wide as possible in order to avoid the amplification of low frequency waves such as swells.

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