한국공작기계학회논문집 Vol.10 No.1 2001. 2.
Transactions of the Korean Society of Machine Tool Engineers

Configuration sensitivity analysis of mechanical dynamics

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Abstract

Design sensitivity is an important device in improving a mechanical system design. A continuum design consists of the shape and orientation design. This research develops the shape and orientation design sensitivity method. The configuration design variables of multibody systems define the shape and orientation changes. The equations of motion are directly differentiated to obtain the governing equations for the design sensitivity. The governing equation of the design sensitivity is formulated as an overdetermined differential algebraic equation and treated as ordinary differential equations on manifolds. The material derivative of a domain functional is performed to obtain the sensitivity due to shape and orientation changes. The configuration design sensitivities of a fly-ball governor system and a spatial four bar mechanism are obtained using the proposed method and are validated against those obtained from the finite difference method.

1. Introduction

Design sensitivity analysis methods for multibody systems have appeared in several pieces of literature.[1-3] There were basically two different approaches. One is the direct differentiation method[1] and the other is the adjoint variable method.[4] The adjoint variable method was employed from the area of optimal control and involves forward numerical integration for dynamic analysis and backward numerical integration for sensitivity analysis. Since backward numerical integration incurs some numerical error and large data storage requirements, the direct differentiation method is used in this paper.

Configuration design sensitivity analysis methods are well developed in the area of the structural mechanics. Sensitivity analyses of the static response and eigenvectors

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due to configuration design change are presented in Refs.[5-7]. Twu and Choi[7] developed a continuum-based configuration design sensitivity analysis method for static responses and eigenvalues, using the material derivative idea developed for shape design sensitivity analysis. Shape and orientation design changes are separated. Two basic assumptions are made throughout the development of the orientation design sensitivity analysis: (1) the design component rotates without shape changes and (2) only a small design perturbation is considered. Line and surface design components are considered. In contrast to the structural design area, research in sensitivity analysis for the mechanical system design area has been limited to relatively simple mechanical systems, since the system responses are highly nonlinear and the design problems have not been well-defined.

Design propagation analysis due to a design change of mechanical systems has been presented in Ref.[8], where the size of a rigid body of a mechanical system is perturbed. The design perturbation of one body influences the positions and orientations of the rest of the bodies and the state variables are modified to satisfy all kinematic admissibility conditions. However, the modification process may not yield a unique state. To avoid the non-unique state problem, a configuration design change method can be used.

If all points and orientations on a body are taken as design variables, there are too many design variables for a practical design consideration and thus many design constraints must be imposed among these variables to satisfy kinematic admissibility conditions. Therefore, the configuration design change of a body is proposed in this paper. Benefits of the configuration design variable are twofold. First, the number of the design variables will be significantly reduced. Second, the velocity field can be selected so that kinematic admissibility conditions are satisfied, which eliminates the modification process of state variables after a design change. The sensitivity analysis method in Ref.[7] was applied for the configuration design sensitivity analysis of kinematic responses in Ref.[10] In this paper, the same theory is then applied to configuration design sensitivity analysis of multibody system dynamics. The kinematics and concept of configuration design changes of a body are introduced in Section 2. Section 3 derives the governing equations of design sensitivity due to a configuration change and a solution method, using an implicit integration method. The numerical examples are presented in Section 4. Finally, conclusions are drawn in Section 5.

2. Configuration design change of a body

Consider the rigid body to be designed in Fig. 1. The body is considered as a continuum during a design stage. The x'- y'-z' frame is the body reference frame and the X-Y-Z frame is the inertial reference frame. The design reference frame on which a design is defined must be specified. The body reference frame x'-y'-z' is chosen as the design reference frame in this paper.

Since the body shown in Fig. 1 is considered as a continuum, the location and the orientation of all points on the domain of the body are treated as design parameters to be determined in this paper. Consequently, the point reference frame of the $x'' - y'' - z''$ is assigned at all points on the domain. The location design parameter, $\mathbf{s}_0$, and the orientation design parameter, θ_0, of the $x'' - y'' - z''$ frame with respect to the design reference frame of the x' -y' -z' frame are treated independently. The orientation design parameter can be parameterized in many different ways and the 3-1-3 Euler angles (θ_1, θ_2, θ_3)[9] are used in this paper. It is assumed that the $x_\tau'' - y_\tau'' - z_\tau''$ frame remains an orthogonal frame after a design change.

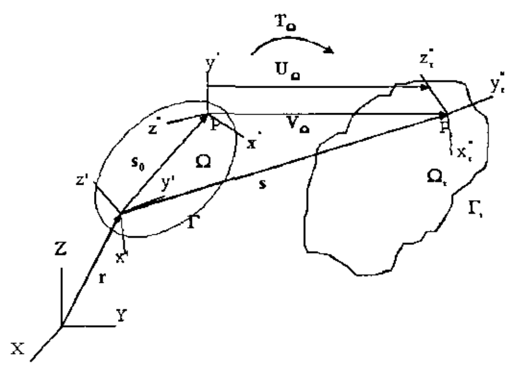

Fig. 1 Configuration change of a body by mapping T Ω

Suppose that only one parameter, τ, defines the transformation $\mathbf{T}_\Omega$, as shown in Fig. 1, where Ω_τ, Γ_τ, and the $x''_\tau - y''_\tau - z''_\tau$ frame denote the changes of Ω, Γ, and the $x'' - y'' - z''$ frame by the mapping $\mathbf{T}_\Omega$, respectively. The mapping $\mathbf{T}_\Omega : \begin{bmatrix} \mathbf{s}_0 \\ \theta_0(\mathbf{s}_0) \end{bmatrix} \rightarrow \begin{bmatrix} \mathbf{s} \\ \theta \end{bmatrix}$, $\mathbf{s}_0 \in \Omega$, is given by

$$\begin{bmatrix} \mathbf{s} \\ \theta \end{bmatrix} = \mathbf{T}_\Omega(\mathbf{s}_0, \tau) = \begin{bmatrix} \mathbf{s}_0 \\ \theta_0(\mathbf{s}_0) \end{bmatrix} + \tau \begin{bmatrix} \mathbf{V}_\Omega \\ \mathbf{U}_\Omega \end{bmatrix} \tag{1}$$

$$\Omega_\tau \equiv \mathbf{T}_\Omega(\Omega, \tau) \tag{2}$$

where the subscript "o" denotes the original configuration and components of θ denote the orientation of the $x'' - y'' - z''$ frame. In Eq. 1, $\mathbf{V}_\Omega$ and $\mathbf{U}_\Omega$ are the shape and orientation design velocity fields, respectively, and are defined by:

$$\mathbf{V}_\Omega \equiv \frac{d\mathbf{s}}{d\tau} \tag{3}$$

$$\mathbf{U}_\Omega \equiv \frac{d\theta}{d\tau} \tag{4}$$

Note that in this paper $\mathbf{V}_\Omega$ and $\mathbf{U}_\Omega$ are assumed to be independent. When a mechanical system undergoes a configuration design change, the kinematic admissibility conditions at a joint interface must be preserved so that the kinematic constraints are satisfied after configuration design changes. To achieve this goal, the velocity fields must satisfy some geometric conditions at the joint interface. Joints have been characterized by a set of elementary constraint.[10] The velocity fields must be given such that kinematic admissibility conditions are satisfied after a design change. Conditions for the velocity fields are formulated for each kinematic joint in Ref.[10].

3. Governing equations of design sensitivity

3.1 Implicit numerical integration of equations of motion

The equations of motion and constraints for a constrained mechanical system have been obtained in Ref.[10] as follows:

$$\mathbf{F} = \mathbf{M}\ddot{\mathbf{q}} + \Phi_{\mathbf{q}}^{\mathrm{T}}\lambda - \mathbf{Q} = 0 \tag{5}$$

$$\Phi(\mathbf{q}) = 0 \tag{6}$$

where

$$\mathbf{M} = \mathrm{diag}[\mathbf{M}_1,\ \mathbf{M}_2, \ldots,\ \mathbf{M}_{nbd}] \tag{7}$$

$$\mathbf{Q} = [\mathbf{Q}_1^{\mathrm{T}},\ \mathbf{Q}_2^{\mathrm{T}}, \ldots,\ \mathbf{Q}_{nbd}^{\mathrm{T}}]^{\mathrm{T}} \tag{8}$$

$$\ddot{\mathbf{q}} = [\ddot{\mathbf{q}}_1^{\mathrm{T}},\ \ddot{\mathbf{q}}_2^{\mathrm{T}}, \ldots,\ \ddot{\mathbf{q}}_{nbd}^{\mathrm{T}}]^{\mathrm{T}} \tag{9}$$

The vectors $\mathbf{q}$ and $\mathbf{Q}$ denote the generalized coordinate vector and generalized force vector, respectively. The vectors λ and Φ represent the Lagrange multiplier and the position level constraint, respectively. The mass matrix $\mathbf{M}$ includes the mass and moment of inertia of all bodies. The nbd denotes the number of bodies in a system.

The equations of motion can be implicitly rewritten by introducing $\mathbf{v} \equiv \dot{\mathbf{q}}$ as:

$$\mathbf{F}(\mathbf{q}, \mathbf{v}, \dot{\mathbf{v}}, \lambda) = \mathbf{0} \tag{10}$$

Successive differentiations of the position level constraint in Eq. 6 yield

$$\dot{\Phi}(\mathbf{q}, \mathbf{v}) = \dot{\Phi}_{\mathbf{v}}\mathbf{v} - \nu = \mathbf{0} \tag{11}$$

$$\ddot{\Phi}(\mathbf{q}, \mathbf{v}, \dot{\mathbf{v}}) = \ddot{\Phi}_{\dot{\mathbf{v}}}\dot{\mathbf{v}} - \gamma = \mathbf{0} \tag{12}$$

Equation 10 and all levels of the constraints comprise a system of overdetermined differential algebraic equations. An algorithm for the backward differentiation formula (BDF) to solve the overdetermined differential algebraic equations is given in Ref. [12] as:

$$\mathbf{H}(\mathbf{x})=\begin{bmatrix}\mathbf{F}(\mathbf{x})\\ \mathbf{\Phi}\\ \dot{\mathbf{\Phi}}\\ \ddot{\mathbf{\Phi}}\\ \mathbf{U}_0^T(h'\mathbf{R}_1)\\ \mathbf{U}_0^T(h'\mathbf{R}_2)\end{bmatrix}=\begin{bmatrix}\mathbf{F}(\mathbf{q},\ \mathbf{v},\ \dot{\mathbf{v}},\ \lambda)\\ \mathbf{\Phi}(\mathbf{q})\\ \dot{\mathbf{\Phi}}_v\mathbf{v}-\nu\\ \ddot{\mathbf{\Phi}}_{\dot{v}}\dot{\mathbf{v}}-\gamma\\ \mathbf{U}_0^T(h'\mathbf{v}-\mathbf{q}-\varsigma_1)\\ \mathbf{U}_0^T(h'\dot{\mathbf{v}}-\mathbf{v}-\varsigma_2)\end{bmatrix}=\mathbf{0} \quad (13)$$

where $h'\equiv\frac{h}{c_0}$, $\varsigma_1=\frac{1}{c_0}\sum_{i=1}^{k}c_i\mathbf{q}_{(-i)}$, and $\varsigma_2=\frac{1}{c_0}\sum_{i=1}^{k}c_i v_{(-i)}$, in which k is the order of integration; $\mathbf{q}_{(-i)}$ and $v_{(-i)}$ are $\mathbf{q}$ and v at past integration knot points. c_i's are the BDF coefficients, and $\mathbf{x}\equiv[\mathbf{q}^T,\mathbf{v}^T,\dot{\mathbf{v}}^T\lambda^T]^T$. $\mathbf{U}_0$ is chosen as $\begin{bmatrix}\Phi_q\\ \mathbf{U}_0^T\end{bmatrix}$, the inverse of which exists. The LU decomposition method is used to obtain $\mathbf{U}_0$ in this paper. The solution method has been presented in Ref.[13]

The number of equations and the number of unknowns in Eq. 13 are the same, so Eq. 13 can be solved for $\mathbf{x}$. Newton's numerical method can be applied to obtain the solution $\mathbf{x}$ as:

$$\mathbf{H}_x\Delta\mathbf{x}=-\mathbf{H} \quad (14)$$

$$\mathbf{x}^{i+1}=\mathbf{x}^i+\Delta\mathbf{x} \qquad i=1,2,... \quad (15)$$

where

$$\mathbf{H}_x=\begin{bmatrix}\mathbf{F}_q & \mathbf{F}_v & \mathbf{F}_{\dot{v}} & \mathbf{F}_\lambda\\ \mathbf{\Phi}_q & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \dot{\mathbf{\Phi}}_q & \dot{\mathbf{\Phi}}_v & \mathbf{0} & \mathbf{0}\\ \ddot{\mathbf{\Phi}}_q & \ddot{\mathbf{\Phi}}_v & \ddot{\mathbf{\Phi}}_{\dot{v}} & \mathbf{0}\\ -\mathbf{U}_0^T & h'\mathbf{U}_0^T & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & -\mathbf{U}_0^T & h'\mathbf{U}_0^T & \mathbf{0}\end{bmatrix} \quad (16)$$

3.2 Implicit numerical integration of configuration design sensitivity equations

A mechanical system consists of bodies, joints, and force elements. Physical properties of these elements are described by various parameters. As an example, the geometric properties of a joint, the inertial properties of a body, and the compliance characteristics of a force element are candidates for design parameters. The design parameters can be related by a configuration design variable τ.

Differentiating Eq. 13 with respect to the design variable τ and appending the BDF integration formulas yield the following equations of design sensitivity:

$$\Psi(\mathbf{x},\mathbf{x}_\tau)=\begin{bmatrix}\frac{dF}{d\tau}\\ \frac{d\Phi}{d\tau}\\ \frac{d\dot{\Phi}}{d\tau}\\ \frac{d\ddot{\Phi}}{d\tau}\\ h'\mathbf{U}_0^T(\mathbf{R}_3)\\ h'\mathbf{U}_0^T(\mathbf{R}_4)\end{bmatrix}=\begin{bmatrix}\mathbf{F}_q\mathbf{q}_\tau+\mathbf{F}_v\mathbf{v}_\tau+\mathbf{F}_{\dot{v}}\dot{\mathbf{v}}_\tau+\mathbf{F}_\lambda\lambda_\tau+\mathbf{F}_\tau\\ \Phi_q\mathbf{q}_\tau+\Phi_\tau\\ \dot{\Phi}_q\mathbf{q}_\tau+\dot{\Phi}_v\mathbf{v}_\tau+\dot{\Phi}_\tau\\ \ddot{\Phi}_q\mathbf{q}_\tau+\ddot{\Phi}_v\mathbf{v}_\tau+\ddot{\Phi}_{\dot{v}}\dot{\mathbf{v}}_\tau+\ddot{\Phi}_\tau\\ \mathbf{U}_0^T(h'\mathbf{v}_\tau-\mathbf{q}_\tau-\varsigma_3)\\ \mathbf{U}_0^T(h'\dot{\mathbf{v}}_\tau-\nu_\tau-\varsigma_4)\end{bmatrix}=\mathbf{0} \quad (17)$$

where ζ_3 and ζ_4 are collections of all previous values at integration knot points for $\mathbf{v}_\tau$ and $\mathbf{q}_\tau$ in the BDF and $\mathbf{x}_\tau\equiv[\mathbf{q}_\tau^T,\mathbf{v}_\tau^T,\dot{\mathbf{v}}_\tau^T,\lambda_\tau^T]^T$. Equation 17 comprises the same number of equations as the unknowns and is solved for $\mathbf{x}_\tau$ as

$$\Psi_{\mathbf{x}_\tau}\mathbf{x}_\tau=-\eta \quad (18)$$

$\eta=\left[\mathbf{F}_\tau^T,\Phi_\tau^T,\dot{\Phi}_\tau^T,\ddot{\Phi}_\tau^T,-(\mathbf{U}_0^T\zeta_3)^T,-(\mathbf{U}_0^T\zeta_4)^T\right]^T$. The computational method for $\mathbf{F}_\tau$, Φ_τ, $\dot{\Phi}_\tau$ and $\ddot{\Phi}_\tau$ are presented in Section 3.3. Since $\Psi_{\mathbf{x}_\tau}=\mathbf{H}_\mathbf{x}$, $\Psi_{\mathbf{x}_\tau}$ does not need to be implemented. An implementation algorithm for Eqs. 13 and 17 is shown in Fig. 2

3.3 Material derivative for configuration design sensitivity analysis

Consider a domain functional, defined as an integral over Ω_τ

$$\xi=\int_{\Omega_\tau}f_\tau(s)d\Omega_\tau \quad (19)$$

where f_τ is a regular function defined on Ω_τ. If Ω has C^k regularity, then the material derivative of the ξ has been obtained in Ref.[5] as

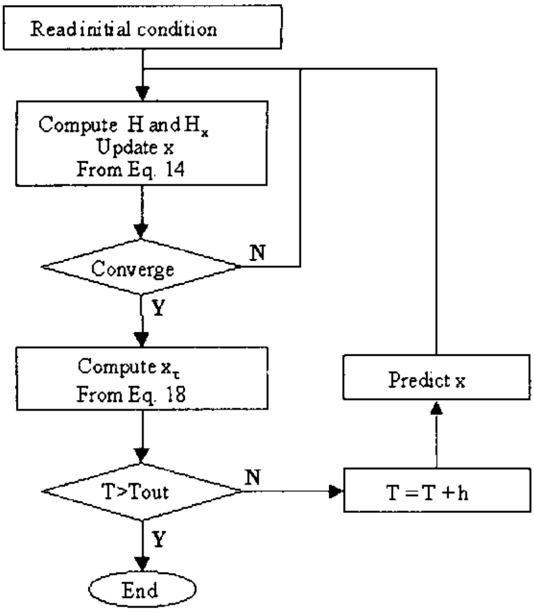

Fig. 2 Solution algorithm for sensitivity analysis

$$\frac{\partial \xi}{\partial \tau} = \int_\Omega \left[\frac{\partial f}{\partial \tau} + \mathrm{div}\{f\mathbf{V}\} \right] \mathrm{d}\Omega$$
$$= \frac{\partial f}{\partial \tau} \int_\Omega \mathrm{d}\Omega + \int_\Gamma f\mathbf{V}^{\mathrm{T}} \mathbf{n} \mathrm{d}\Gamma \qquad (20)$$

div $(f\mathbf{V}) \equiv \sum_{i=1}^{3} \frac{\partial (f\mathbf{V})_i}{\partial x_i}$, $\mathbf{n}$ is a unit normal vector to the infinitesimal area $d\Gamma$, and $\mathbf{V}$ is the velocity field vector.

$\mathbf{F}_\tau \left(= \frac{\partial \mathbf{F}}{\partial_\tau} \right)$ must be computed to generate η in Eq. 18. In $\mathbf{F}$ of the equations of motion, the mass and moment of inertia of a body are affected by the domain integral.

The mass and moment of inertia of a body are expressed as:

$$\mathrm{m}(\tau) = \int_\Omega \mathrm{dm} = \int_\Omega \rho \mathrm{d}\Omega \qquad (21)$$

$$\mathbf{J}(\tau) = -\int_\Omega \rho \tilde{\mathbf{s}} \tilde{\mathbf{s}} \mathrm{d}\Omega \qquad (22)$$

where ρ is the material density and the vector $\mathbf{s}$ is defined in Fig. 1. A tilde operator associated with vectors $\mathbf{a}$ and $\mathbf{b}$ to denote the vector cross product is defined as follows.

$$\tilde{\mathbf{a}}\mathbf{b} = \mathbf{a} \times \mathbf{b} \qquad (23)$$

Partial derivatives of Eqs. 21 and 22 with respect to τ are obtained by applying Eq. 20 to Eqs. 21 and 22 as:

$$\frac{\partial \mathrm{m}}{\partial \tau} = \int_\Gamma \rho \left(\mathbf{V}^{\mathrm{T}} \mathbf{n} \right) \mathrm{d}\Gamma \qquad (24)$$

$$\frac{\partial \mathbf{J}}{\partial \tau} = -\rho \int_\Omega \left(\tilde{\mathbf{s}}_0 \tilde{\mathbf{V}} + \tilde{\mathbf{s}}_0 \tilde{\mathbf{V}} \right) \mathrm{d}\Omega - \rho \int_\Gamma \tilde{\mathbf{s}}_0 \tilde{\mathbf{s}}_0 \left(\mathbf{V}^{\mathrm{T}} \mathbf{n} \right) \mathrm{d}\Gamma \qquad (25)$$

Once $\frac{\partial \mathrm{m}}{\partial \tau}$ and $\frac{\partial \mathbf{J}}{\partial \tau}$ are obtained, $\mathbf{F}_\tau$ is computed by

$$\mathbf{F}_\tau = \mathbf{F}_\mathrm{m} \mathrm{m}_\tau + \sum_{i=1}^{3} \sum_{j=1}^{3} \mathbf{F}_{\mathbf{J}_{ij}} \mathbf{J}_{ij_\tau} + \mathbf{F}_\mathrm{k} \mathbf{k}_\tau + \mathbf{F}_\mathrm{c} \mathrm{c}_\tau + \mathbf{F}_\mathrm{s} \mathbf{V} + \mathbf{F}_\theta \mathbf{U} \qquad (26)$$

where the vectors $\mathbf{V}$ and $\mathbf{U}$ are the velocity fields of the force reference frame.

Consider the joint whose reference frame is affected by a configuration design variable defined in Eq. 1. Since the location and orientation of the reference frame affect $\mathbf{\Phi}$, $\dot{\mathbf{\Phi}}$ and $\ddot{\mathbf{\Phi}}$, their derivatives with respect to τ are obtained as:

$$\mathbf{\Phi}_\tau = \mathbf{\Phi}_S \mathbf{V} + \mathbf{\Phi}_\theta \mathbf{U} \qquad (27)$$

$$\dot{\mathbf{\Phi}}_\tau = \dot{\mathbf{\Phi}}_S \mathbf{V} + \dot{\mathbf{\Phi}}_\theta \mathbf{U} \qquad (28)$$

$$\ddot{\mathbf{\Phi}}_\tau = \ddot{\mathbf{\Phi}}_S \mathbf{V} + \ddot{\mathbf{\Phi}}_\theta \mathbf{U} \qquad (29)$$

where the vectors $\mathbf{V}$ and $\mathbf{U}$ are the velocity fields of the joint reference frame.

4. Numerical examples

4.1 Fly-ball governor

A three dimensional fly-ball governor system consists of two balancing balls, couplers, a collar, and a shaft, as shown in Fig. 3. The balancing balls generate centrifugal forces as the shaft rotates. An equilibrium position is

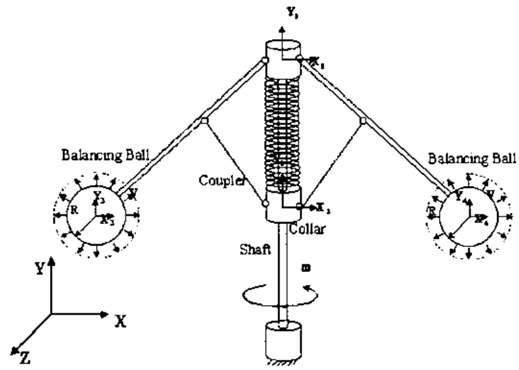

Fig. 3 A fly-ball governor system

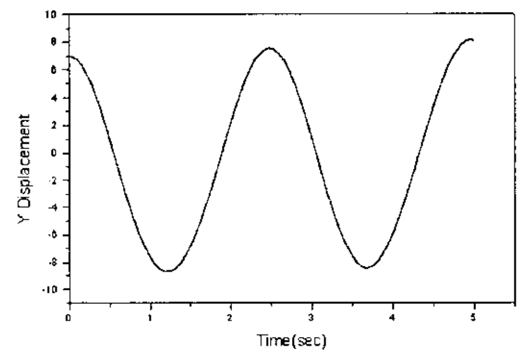

Fig. 4 The y-displacement of left balancing ball

reached when the centrifugal force as well as the spring and gravitational forces are balanced. A dynamic analysis is carried out for 10.0 seconds with an initial angular velocity of the shaft of ω = 1.0 rad/sec. The Y-displacement of a left balancing ball is given in Fig. 4. The shape of the balancing balls are selected as the design variable. The shape design velocity fields are given along the boundary, as shown in Fig. 3. The mass and moment of inertia of the ball are expressed in terms of the radius R as

$$m = \frac{4}{3}\pi R^3 \rho \tag{30}$$

$$J = \begin{bmatrix} J_{xx} & 0 & 0 \\ 0 & J_{yy} & 0 \\ 0 & 0 & J_{zz} \end{bmatrix} \tag{31}$$

$$J_{xx} = J_{yy} = J_{zz} = \frac{8}{15}\pi R^5 \rho \tag{32}$$

where ρ is the density and 7.87 m g/cm^3. Consequently, $\frac{\partial m}{\partial \tau}$ and $\frac{\partial J}{\partial \tau}$ of the ball are analytically obtained as

$$\frac{\partial m}{\partial \tau} = 4\pi R^2 \rho V \tag{33}$$

$$\frac{\partial J_{xx}}{\partial \tau} = \frac{\partial J_{yy}}{\partial \tau} = \frac{\partial J_{zz}}{\partial \tau} = \frac{8}{3}\pi R^4 \rho V \tag{34}$$

In Eqs. 33 and 34 , V is the norm of **V** and is given to be 1 for this example.

The proposed sensitivity analysis is carried out for 10 seconds. The sensitivity of the Y-displacement of the left balancing ball is obtained, as shown in Fig. 5. The analytic and FDM sensitivities are shown to be close in Fig. 5, which validates the proposed method.

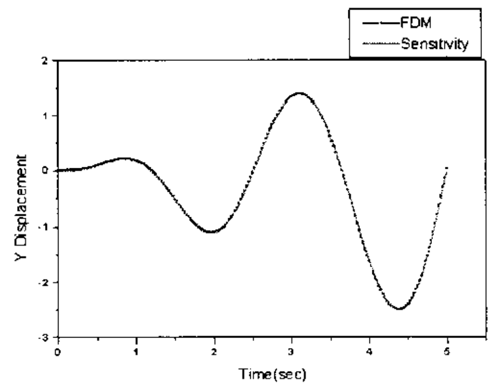

Fig. 5 The sensitivity of y-displacement of left balancing ball

4.2 Spatial four bar mechanism

Configuration sensitivity analysis of a spatial four bar mechanism, shown in Fig. 6, is carried out. The mechanism consists of three bodies, two revolute joints, and two ball joints. The shapes of bodies 2 and 4 are chosen as the configuration design variables. The velocity fields **U** and **V** are given in Fig. 6.

The sensitivity analysis is carried out for 10.0 seconds with an initial angular velocity of ω = 1.0 rad/sec of a crank shaft. The Y-displacement of body 2 and its sensitivity due to the configuration change are shown in Figs. 7 and 8, respectively. The analytic sensitivity and FDM sensitivity are shown to be very close in Fig. 8, which validates the proposed method.

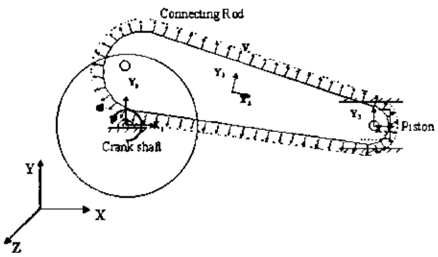

Fig.6 The spatial four-bar mechanism

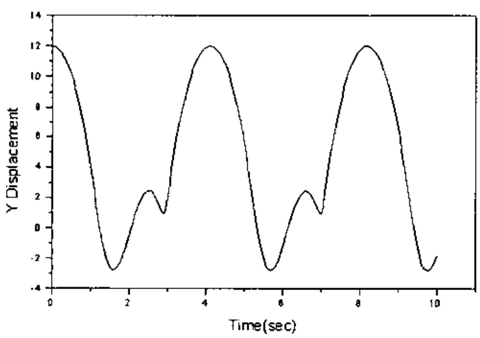

Fig.7 The y-displacement of body 2

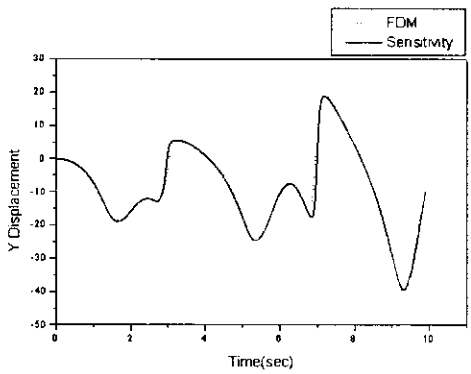

Fig.8 The sensitivity of y-displacement of body 2

5. Conclusions

To compute the sensitivity for mechanical design, the finite difference method(FDM) has been used. But the method has always had a problem which is how to determine the variations of the design variables. These variations are closely affected the stability and accuracy for numerical analysis. Since the equations of configuration design sensitivity are analytically formulated in this paper, the problem is swept away. A continuum-based configuration design sensitivity analysis method for dynamic responses of mechanical systems is proposed in this paper. The configuration design variable for the mechanical systems is defined. The equations of configuration design sensitivity are formulated as the overdetermined differential algebraic equation, using the direct differentiation method. The material derivatives of the mass and moment of inertia are calculated to obtain the right hand side of the governing equations of the configuration design sensitivity. Design sensitivity analyses of a fly-ball governor system and a spatial four bar mechanism due to a configuration design change are successfully performed and their results are validated against those from the finite difference method.

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