Robust H_∞ FIR Filtering for Uncertain Time-Varying Sampled-Data Systems

Hee-Seob Ryu, Byung-Moon Kwon and Oh-Kyu Kwon

Abstract - This paper considers the problem of robust H_∞ filter with FIR (Finite Impulse Response) structure for linear continuous time-varying systems with sampled-data measurements. It is assumed that the system is subject to real time-varying uncertainty which is represented by the state-space model having parameter uncertainty. The robust H_∞ FIR filter is derived by using the equivalence relationship between the FIR filter and the recursive filter, that would be guarantee a prescribed H_∞ performance in the continuous-time context, irrespective of the parameter uncertainty and unknown initial states.

Key Words - H_∞ FIR filter, continuous-time linear filtering, robustness, sampled-data systems

1. Introduction

In practical systems, it is often the case that the plant dynamics is represented by continuous-time processes but the output signal is measured by digital devices. Digital filtering devices tend to fail when the sampling frequency is too low and the system dynamics are relatively too fast because the inter-sampling behavior of the system is not considered. So, in the estimation problems for the continuous-time system, it is required to produce a continuous-time estimate of an analogue signal based on sampled-data measurements. In this situation, the performance measure should be defined directly in terms of the continuous-time signals. This filtering approach is referred as the sampled-data filtering. The important feature of sampled-data filtering is that we deal directly with a continuous-time model of the signal generating mechanism which is highly desirable, in particular when the model is subject to parameter uncertainty as is often the case. A state space approach to sampled-data filtering in an H_∞ formulation has been proposed in [8,9]. However, the conventional H_∞ filters proposed so far are mainly limited to time-invariant systems. Therefore they can not be applied to general time-varying systems on the infinite horizon $[0,\infty)$ since one of two Riccati differential equations required to solve the problem can not be computed on the infinite horizon [6].

This paper deals with the robust H_∞ filtering problem for a class of continuous time-varying uncertain systems under sampled-data measurements on the infinite horizon. The class of uncertain systems is described by a linear state space model with real time-varying norm-bounded parameter uncertainty in the state and output matrices. Here attention is focused on the design of linear filters for time-varying systems which guarantee a prescribed performance, irrespective of the uncertainty. The performance measure is defined directly in the continuous-time context and is of an H_∞ type. The basic idea of the current paper is to formulate the robust H_∞ filtering problem on the continuous-time moving horizon $[t-T, t]$ and to adopt the FIR (Finite Impulse Response) filter structure.

FIR filters are widely used in the signal processing area, and they were utilized in the estimation problem as the optimal FIR filters [3,4,5]. Note that IIR (Infinite Impulse Response) or recursive filter structure requires the initial conditions on each horizon, which is an impractical assumption, but that FIR filter structure does not requires the initial conditions. The optimal FIR filters are, however, presented so far not in the H_∞ setting but in the minimum variance formulation.

The estimator of the current paper is rather a one-step-ahead predictor than a filter. This filtering problem is referred to as robust H_∞ FIR sampled-data filtering in the sense that it is an H_∞ sampled-data filter with the FIR structure for uncertain systems. We show that the robust H_∞ FIR sampled-data filtering problem can be solved in terms of two Riccati equation.

One of the main contributions of the current paper is that the H_∞ FIR sampled-data filter always has a solution if the standard H_∞ sampled-data filter exists on the finite horizon. Therefore, it is noted that the sampled-data filter

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proposed works on the time-varying systems with sampled-data measurements.

2. Problem formulation

Let us consider the following class of uncertain sampled-data time-varying systems

$$\dot{x}(t) = [A(t)+\Delta A(t)]x(t)+B(t)w(t),\ x(0)=x_0 \quad (1)$$

$$z(t) = L(t)x(t) \quad (2)$$

$$y(i) = [C(i)+\Delta C(i)]x(i)+D(i)v(i), \quad (3)$$

where $x(t)\in\mathcal{R}^n$ is the state, x_0 is unknown initial state, $w(t)\in\mathcal{R}^q$ is the process noise, $y(i)\in\mathcal{R}^m$ is the sampled measurement, $v(i)\in\mathcal{R}^r$ is the measurement noise, $z(t)\in\mathcal{R}^p$ is a linear combination of state variables to be estimated over a moving horizon $[t-T,t]$, i is an integer, $A(t)$, $B(t)$, $C(i)$, $D(i)$ and $L(t)$ are known real time-varying bounded matrices of appropriate dimensions with $A(t)$, $B(t)$ and $L(t)$ being piecewise continuous, and $\Delta A(t)$ and $\Delta C(i)$ are real-valued matrix functions which represent real time-varying parameter uncertainties in $A(t)$ and $C(i)$, respectively. These uncertainties are assumed to be of the form

$$\Delta A(t) = H(t)F(t)E(t) \quad (4)$$

$$\Delta C(i) = H_d(i)F_d(i)E_d(i), \quad (5)$$

where $F(t)\in\mathcal{R}^{a\times b}$ and $F_d(i)=\mathcal{R}^{a\times\beta}$ are unknown time-varying matrix functions satisfying

$$F^T(t)F(t) < I,\quad \forall t \quad (6)$$

$$F_d^T(i)F_d(i) < I,\quad \forall i \quad (7)$$

with the elements of $F(t)$ being Lebesgue measurable, and $E(t), E_d(i), H(t)$ and $H_d(i)$ are known real time-varying bounded matrices of appropriate dimensions with $E(t)$ and $H(t)$ being piecewise continuous. For the sake of notation simplification, in the sequel the dependence on t or i for all matrices will be omitted.

In the current paper, the FIR filter is defined by the form

$$\hat{x}(i+1|i;T) = \sum_{k=i-T}^{i} M(i,k,T)y(k)$$

$$\hat{z}(i+1|i;T) = L(i+1)\hat{x}(i+1|i;T),$$

where $M(i,\cdot;T)$ is the finite impulse response with the finite duration T. This FIR filter is a kind of the one-step-ahead predictor since it estimates the state or the output at the time point $i+1$ based on the observation on $[i-T,i]$. The H_∞ FIR filter is obtained by constructing its impulse response from that of the H_∞ filter on the finite moving horizon $[i-T,i]$

The following assumption for the system (1)-(2) is to be assumed:

Assumption 1.

(a) $[D(i)\quad H_d(i)]$ is of full the row rank for all $i\in(i-T,i)$;

(b) $DB^T=0$.

Assumption 1(a) means that the robust filtering problem is 'non-singular'. We observe that when there is no parameter uncertainty in the output matrix, i.e. $H_d(i)=0$ over the moving horizon $(i-T,i)$, which corresponds to a standard non-singularity condition in the H_∞ filtering problem for the nominal system (1)-(3). Assumption 1(b) means that the system is un-correlated. But if the system is correlated, the state equation should be modified in order to apply the FIR filter to the system since it requires the system be un-correlated.

Note that parameter uncertainty structure as in (4)-(5) has been widely used in the problems of robust control and filtering of uncertain systems [2] and can represent parameter uncertainty in many practical cases. Also, it should be observed that the uncertainty matrices $F(t)$ and $F_d(i)$ are allowed to be state dependent, i.e. $F(t)=F(t,x)$ and $F_d(i)=F_d(i,x)$ as long as (6)-(7) are satisfied. Furthermore, any possible parameter uncertainties in $B(t)$ and $D(i)$ are assumed to be absorbed in w and v, respectively.

In this paper we are concerned with obtaining an estimate $\hat{z}(t)$ of $z(t)$ over a moving horizon $[t-T,t]$ via a linear causal filter using the measurements $\{y(i), i-T<i\le i\}$, and where no a priori estimate of the initial state of (1)-(3) is assumed. The filter is required to provide a uniformly small estimation error, $e(t)=z(t)-\hat{z}(t)$ for any $w\in L_2[0,\infty)$, $v\in l_2(0,\infty)$ and for all admissible uncertainties. More specifically, the robust filtering problem we address is as follows:

Given a prescribed level of noise attenuation $\gamma>0$ and an initial state weighting matrix $R=R^T>0$, find a linear causal filter such that the estimation error dynamics in the infinite horizon, $z(t)-\hat{z}(t)$, is exponentially stable and

$$\|z-\hat{z}\|^2 < \gamma^2\{\|w\|^2_{[t-T,t]}+\|v\|^2_{(t-T,t)}+x_0^TRx_0\} \quad (8)$$

holds for any non-zero $(w,v,x_0)\in L_2[0,\infty)\oplus l_2(0,\infty)\oplus R^n$ and for all admissible uncertainties with initial state $x_0=x(t-T)$.

The initial weighting matrix R is a measure of the uncertainty in the initial state of (1)-(3) relative to the uncertainty in w and v. A large value of R reflects that the initial state is certain to be close to zero. In the case when the initial state of the system (1)-(3) is known to be zero, (8) will be replaced

$$\|z-\hat{z}\|^2 < \gamma^2\{\|w\|^2_{[t-T,t]} + \|v\|^2_{(i-T,i)}\} \tag{9}$$

It should be noted that (9) can be viewed as the limit of (8) as the smallest eigenvalue of R approaches infinity.

3. Preliminaries

In the sequel the bounded real lemma for linear time-varying systems with finite discrete jumps will be recalled. It will be fundamental in the derivation of our main results.

Consider the following linear time-varying system with finite discrete jumps:

$$(\Sigma_J):\ \begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)w(t),\ t \neq i,\ x(t-T) = x_o \\ x(i) &= A_d(i)x(i^-) + B_d(i)v(i), \\ z(t) &= C(t)x(t) \end{aligned}$$

where $x \in \mathcal{R}^n$ is the state, $w \in \mathcal{R}^q$ and $v \in \mathcal{R}^r$ are input signals which belong to $L_2[0,\infty)$ and $l_2(0,\infty)$, respectively. $z \in R^p$ is the output, i is the integer and $A(t)$, $A_d(i)$, $B(t)$, $B_d(i)$ and $C(t)$ are known real time-varying bounded matrices of appropriate dimensions with $A(t)$, $B(t)$ and $C(t)$ being piecewise continuous.

Next, introduce the following worst-case performance index for (Σ_J):

$$J(\Sigma_J, R, T) = \sup\left\{\left[\frac{\|z\|^2}{\|w\|^2_{[t-T,t]} + \|v\|^2_{(i-T,i)} + x_0^T R x_0}\right]^{1/2}\right\}, \tag{10}$$

where $R = R^T > 0$ is given weighting matrix for x_0.

We now present the bounded real lemma on finite moving horizon $[0, T]$ for systems of the form of (Σ_J) with the performance measure (10).

Lemma 1 [7]. Consider the system (Σ_J) with the performance measure (10) and let $\gamma > 0$ be a given scalar. Then, the following statements are equivalent over a moving horizon $[0, T]$:

(a) $J(\Sigma_J, R, T) < \gamma$;

(b) There exists a bounded matrix function $P(t) = P^T(t) \geq 0,\ \forall t \in [0, T]$, satisfying

$$-\dot{P} = A(t)^T P + PA(t) + \gamma^{-2} PB(t)B^T(t)P + C^T(t)C(t), \quad t \neq i \tag{11}$$

$$P(T) = 0 \tag{12}$$

$$\gamma^2 I - B_d^T(i)P(i^+)B_d(i) > 0 \tag{13}$$

$$P(i) = A_d(i)^T P(i^+)A_d(i) + A_d^T(i)P(i^+)B_d(i) \cdot [\gamma^2 I - B_d^T(i)P(i^+)B_d(i)]^{-1} B_d^T(i)P(i^+)A_d(i) \tag{14}$$

$$P(0) < \gamma^2 R\ ; \tag{15}$$

(c) There exists a bounded matrix function $Q(t) = Q^T(t) > 0,\ \forall t \in [0, T]$ satisfying

$$-\dot{Q} > A^T(t)Q + QA(t) + \gamma^{-2} QB(t)B^T(t)Q + C^T(t)C(t), \tag{16}$$

$$Q(T) > 0 \tag{17}$$

$$\gamma^2 I - B_d^T(i)Q(i^+)B_d(i) > 0 \tag{18}$$

$$Q(i) > A_d^T(i)Q(i^+)A_d(i) + A_d^T(i)Q(i^-)B_d(i) \cdot [\gamma^2 I - B_d^T(i)Q(i^+)B_d(i)]^{-1} B_d^T(i)Q(i^+)A_d(i) \tag{19}$$

$$Q(0) < \gamma^2 R. \tag{20}$$

■

Note that (12) and (17) are terminal conditions and (15) and (20) are initial conditions over the moving horizon $[0, T]$, and when the initial state of (Σ_J) is known to be zero, the condition of (15) and (20) in Lemma 1 will no longer be required as an initial state which is certain to be zero corresponds to choosing a 'very large' value of matrix R.

We end this section by recalling a matrix inequality that will be need in the proof of our main results.

Lemma 2. [1] Let A, E, F, H and M be real matrices of appropriate dimensions with M being symmetric. Then, there exists a matrix $P = P^T > 0$ such that

$$[A + HFE]^T P[A + HFE] + M < 0 \tag{21}$$

for all matrices F satisfying $F^T F < I$, if there exists some $\varepsilon > 0$ such that the following conditions are satisfied

(a) $\varepsilon^{1/2} H^T PH < I$

(b) $A^T PA + A^T PH[\varepsilon I - H^T PH]^{-1} H^T PA + \varepsilon E^T E + M < 0$.

■

4. Robust H_∞ FIR Sampled-Data Filters

In this section a Riccati equation approach is proposed for solving the robust H_∞ FIR sampled-data filtering problem for system (1)-(3). The following result provides a solution to the robust H_∞ FIR filtering problem over a finite moving horizon.

Theorem 1. Consider the system (1)-(3) satisfying (4)-(7) and Assumption 1. Given a scalar $\gamma > 0$ and an initial state weighting matrix $R = R^T > 0$, the robust H_∞ FIR sampled-data filtering problem over a moving horizon $[0, T]$ is solvable if for some non-zero ε and ν, the following conditions are satisfied:

(a) There exists a bounded solution $P(t)$ over $[0, T]$ to the Riccati differential equation with jumps

$$-\dot{P}(t) = A^T(t)P(t) + P(t)A(t) + \gamma^{-2}P(t)\hat{B}\hat{B}^TP(t) + \varepsilon^2E^TE, \quad t \neq i \tag{22}$$

$$P(i) = P(i^+) + \nu^2E_d^TE_d, \quad \forall i \in (0, T) \tag{23}$$

with terminal condition $P(T) = 0$ and such that $P(0) < \gamma^2R$, where

$$\hat{B} = \left[B \ \ \frac{\gamma}{\varepsilon}H\right].$$

(b) There exists a bounded solution $S(t)$ over $[0, T]$ to the Riccati differential equation with jumps

$$\dot{S}(t) = \hat{A}S(t) + S(t)\hat{A}^T + \gamma^{-2}S(t)L^T(t)L(t)S(t) + \hat{B}\hat{B}^T, \quad t \neq i \tag{24}$$

$$S(i) = [S^{-1}(i^-) + C^T(i)V^{-1}C(i)]^{-1}, \tag{25}$$

with initial condition $S(0) = [R - \gamma^{-2}P(0)]^{-1}$, where $\hat{A}(t) = A(t) + \gamma^{-2}\hat{B}\hat{B}^TP(t)$, $V(i) = \hat{D}(i)\hat{D}^T(i)$ and

$$\hat{D} = \left[D(i) \frac{\gamma}{\varepsilon}H_d\right].$$

Moreover, if conditions (a) and (b) are satisfied, a suitable filter is given by

$$\dot{\hat{x}}(t) = \hat{A}\hat{x}(t), \quad t \neq i; \ \hat{x}(0) = 0 \tag{26}$$

$$\hat{x}(i) = \hat{x}(i^-) + S(i)C^T(i)V^{-1}[y(i) - C(i)\hat{x}(i^-)], \tag{27}$$

$$\hat{z}(t) = L(t)\hat{x}(t), \quad \forall t \in [0, T]. \tag{28}$$

Proof. Firstly, associated with the system (1)-(3) and the filter (26)-(28), we define $\tilde{x} = x - \hat{x}$. Since $x(i) = x(i^-)$, from (1)-(3) and (26)-(28), we have that

$$\dot{\tilde{x}}(t) = [A(t) + \triangle A_e]\tilde{x}(t) + [\triangle A(t) - \triangle A_e]x(t) + B(t)w(t), \quad t \neq i; \ \tilde{x}(0) = x_0$$

$$\tilde{x}(i) = A_d(i)\tilde{x}(i^-) + B_d(i)\triangle C(i)x(i^-) + B_d(i)D(i)v(i),$$

where

$$A_d(i) = I - S(i)C^T(i)V^{-1}C(i),$$

$$\triangle A_e = \gamma^{-2}\hat{B}\hat{B}^TP(t),$$

$$B_d(i) = -S(i)C^T(i)V^{-1}.$$

Hence, defining $\eta = [x^T \ \tilde{x}^T]^T$, the estimation error $z - \hat{z}$ is described by:

$$\dot{\eta}(t) = [A_c + H_cFE_c]\eta(t) + B_cw(t), \tag{29}$$

$$\eta(0) = [x_0^T \ \ \tilde{x}_0^T]^T$$

$$\eta(i) = [A_{dc} + H_{dc}F_dE_{dc}]\eta(i^-) + B_{dc}v(i), \tag{30}$$

$$z(t) - \hat{z}(t) = C_c\eta(t), \tag{31}$$

where

$$A_c = \begin{bmatrix} A(t) & 0 \\ -\triangle A_e & A(t) + \triangle A_e \end{bmatrix}, \quad A_{dc} = \begin{bmatrix} I & 0 \\ 0 & A_d(i) \end{bmatrix},$$

$$B_c = \begin{bmatrix} B(t) \\ B(t) \end{bmatrix}, \quad B_{dc} = \begin{bmatrix} 0 \\ B_d(i)D(i) \end{bmatrix}, \quad H_c = \begin{bmatrix} H \\ H \end{bmatrix},$$

$$H_{dc} = \begin{bmatrix} 0 \\ B_d(i)H_d \end{bmatrix}, \quad C_c = [0 \ \ L(t)],$$

$$E_c = [E \ \ 0], \quad E_{dc} = [E_d \ \ 0].$$

Here, from Theorem 3.1 of [9], condition (b) is necessary and sufficient for the solvability on the finite moving horizon $[0, T]$ of H_∞ FIR filtering problem for the linear system with sampled-data measurements

$$\dot{\xi}(t) = \hat{A}\xi(t) + \hat{B}\hat{w}(t), \quad t \in [0, T]; \quad \xi(0) = \xi_0 \tag{32}$$

$$y(i) = C(i)\xi(i) + \hat{D}\hat{v}(i), \quad \forall i \in (0, T) \tag{33}$$

$$z_e(t) = L(t)\xi(t), \tag{34}$$

where $\xi \in R^n$ is the state, ξ_0 is an unknown initial state, $\hat{w} \in R^{q+a}$ is the process noise, $y(i) \in R^m$ is the sampled measurement, $\hat{v}(i) \in R^{r+a}$ is the measurement noise, $z_e \in R^p$ is a linear combination of the state variables to be estimated, i is an integer. The filtering performance measure is given by

$$\sup\left\{\left[\frac{\|z_e - \hat{z}_e\|^2}{\|\hat{w}\|^2_{[t-T,t]} + \|\hat{v}\|^2_{(i-T,i)} + \xi_0^T[R - \gamma^{-2}P(0)]\xi_0}\right]^{1/2}\right\} < \gamma, \tag{35}$$

where $\hat{z}_e$ is the estimate of z_e. Also, observe that a suitable estimate $\hat{z}_e$ is given by

$$\hat{z}_e(t) = L(t)\hat{x}(t),$$

where $\hat{x}(t)$ is as in (26)-(27) and with $y(i)$ as defined in (33).

Now, letting $\tilde{\xi} = \xi - \hat{x}$

$$\dot{\tilde{\xi}}(t) = \hat{A}\tilde{\xi}(t) + \hat{B}\hat{w}(t), \quad t \neq i; \quad \tilde{\xi}(0) = \xi_0$$

$$\tilde{\xi}(i) = A_d(i)\tilde{\xi}(i^-) + B_d(i)\hat{D}\hat{v}(i), \quad \forall i \in (0, T)$$

$$z_e(t) - \hat{z}(t) = L(t)\tilde{\xi}(t).$$

Since the above system satisfies (35) by Lemma1, this implies that there exists a matrix $M(t)=M^T(t)\geq 0$, satisfying the following Riccati differential equation with jumps

$$\dot{M}(t)+\hat{A}^TM(t)+M(t)\hat{A}+\gamma^{-2}M(t)\hat{B}\hat{B}M(t) + L^T(t)L(t) = 0,\quad t\neq i;\ M(T)=0 \tag{36}$$

$$\gamma^2I-\hat{D}^TB_d^T(i)M(i^+)A_d(i) > 0,\ \forall i\in(0,T) \tag{37}$$

$$\begin{aligned}M(i) &= A_d(i)^TM(i^+)A_d(i)+A_d^T(i)M(i^+)B_d(i)D(i)\\ &\cdot[\gamma^2I-\hat{D}^TB_d(i)^TM(i^+)B_d(i)\hat{D}]^{-1}\\ &\cdot \hat{D}^TB_d(i)^TM(i^+)A_d(i),\end{aligned} \tag{38}$$

$$M(0) < \gamma^2R-P(0). \tag{39}$$

Next, let

$$X(t) = \begin{bmatrix}P(t) & 0\\ 0 & M(t)\end{bmatrix}$$

where $P(t)$ and $M(t)$ are the non-negative definite solution of (22)-(23) and (36)-(39), respectively. Note that since $M(0) < \gamma^2R-P(0)$, there exists a sufficiently small scalar $\delta > 0$ such that

$$X(0) < X_0 = \begin{bmatrix}P(0)+\delta I & 0\\ 0 & \gamma^2R-P(0)-\delta I\end{bmatrix}.$$

It is straightforward to verify that $X(t)$, $\forall t\in[0,T]$ satisfies the following Riccati differential equation with jumps:

$$\begin{aligned}&\dot{X}(t)+A_c^TX(t)+X(t)A_c+\gamma^{-2}X(t)B_cB_c^TX(t)\\ &+\frac{1}{\varepsilon^2}X(t)H_cH_c^TX(t)+C_c^TC_c+\varepsilon^2E_c^TE_c=0,\\ &t\neq i;\ X(T)=0\\ &\gamma^2I-\hat{B}^T_{dc}X(i^-)\hat{B}_{dc} > 0, \forall i\in(0,T)\\ &X(i)=A^T_{dc}X(i^-)A_{dc}+A^T_{dc}X(i^+)\hat{B}_{dc}\\ &\quad\cdot[\gamma^2I-\hat{B}_{dc}{}^TX(i^+)\hat{B}_{dc}]^{-1}\hat{B}_{dc}{}^TX(i^+)A_{dc}\\ &\quad+\nu^2E^T_{dc}E_{dc},\ \forall i\in(0,T)\\ &X(i-T) < X_0.\end{aligned}$$

where

$$\hat{B}_{dc} = \begin{bmatrix}0\\ B_d(i)\hat{D}\end{bmatrix}.$$

By Lemma 1, it follows that there exists a time- varying matrix $X_1(t)=X_1^T(t)>0$, $\forall t\in[0,T]$, such that

$$\dot{X}_1(t)+A_c^TX_1^T(t)+X_1(t)A_c+\gamma^{-1}X_1(T)B_cB_c^TX_1(t) +\frac{1}{\varepsilon^2}X_1(t)H_cH_c^TX_1(t)+C_c^TC_c+\varepsilon^2E_c^TE_c<0,\ t\neq i; \tag{40}$$

$$X_1(0) = 0 \tag{41}$$

$$\gamma^2I-\hat{B}_{dc}{}^TX_1(i^+)\hat{B}_{dc} > 0,\ \forall i\in(0,T) \tag{42}$$

$$\begin{aligned}X_1(i) > &A^T_{dc}X_1(i^+)A_{dc}+A^T_{dc}X_1(i^+)\hat{B}_{dc}\\ &\cdot[\gamma^2I-\hat{B}_{dc}{}^TX_1(i^+)\hat{B}_{dc}]^{-1}\hat{B}_{dc}{}^TX_1(i^+)A_{dc}\\ &+\nu^2E^T_{dc}E_{dc},\ \forall i\in(0,T)\end{aligned} \tag{43}$$

$$X_1(0) < X_0. \tag{44}$$

Recalling that for any non-zero scalar ε, any real matrices E, F and H of appropriate dimensions, with $F^TF < I$,

$$HFE+E^TF^TH^T \leq \frac{1}{\varepsilon^2}HH^T+\varepsilon^2E^TE,$$

it results from (40) that $X_1(t)$, $\forall t\in[0,T]$ satisfies

$$\dot{X}_1(t)+[A_c+H_cFE_c]^TX_1(t)+X_1(t)[A_c+H_cFE_c] +\gamma^{-2}X_1(t)B_cB_c^TX_1(t)+C_c^TC_c<0,\ t\neq i. \tag{45}$$

Next, applying the matrix inversion lemma to (43) implies that

$$X_1(i) > A^T_{dc}[Y^{-1}-\nu^{-2}H_{dc}H^T_{dc}]^{-1}A_{dc}+\nu^2E^T_{dc}E_{dc},$$

where $Y^{-1} = X_1^{-1}(i^+)-\gamma^{-2}B_{dc}B^T_{dc}$.. Now, using the matrix inversion lemma we derive that

$$\begin{aligned}X_1(i) > &A^T_{dc}YA_{dc}+A^T_{dc}YH_{dc}[I-H^T_{dc}YH_{dc}]^{-1}H^T_{dc}YA_{dc}\\ &+\nu^2E^T_{dc}E_{dc}, \forall i\in(0,T).\end{aligned}$$

Hence, by considering Lemma 2, it follows that

$$\begin{aligned}X_1(i) > &[A_{dc}+H_{dc}F_dE_{dc}]^T[X_1^{-1}(i^+)-\gamma^{-2}B_{dc}B^T_{dc}]^{-1}\\ &\cdot[A_{dc}+H_{dc}F_dE_{dc}],\ \forall i\in(0,T).\end{aligned}$$

Furthermore, we observe that $\gamma^{-2}\eta(0)^TX_0\eta(0)=x_0^TRx_0$. Finally, taking into account (41)-(42) and (44)-(46), Lemma 1 implies that the system (29)-(31) satisfies

$$\|z-\hat{z}\|^2<\gamma^2\{\|w\|^2_{[t-T,t]}+\|v\|^2_{(i-T,i)}+x_0^TRx_0\}$$

for all admissible uncertainties, which complete the proof. ■

5. Conclusion

This paper has presented a new methodology of robust H_∞ FIR filtering based on sampled measurements for a class of linear continuous time-varying systems subject to real norm-bounded parameter uncertainty and unknown initial state. Attention is focused on the simultaneous estimation of a continuous and discrete time-varying signal using a H_∞ performance measure which involves a mixed L_2/l_2 norm of the estimation error of the moving horizon $[t-T, t]$ and $(i-T, i)$ for the continuous and discrete time signal, respectively.

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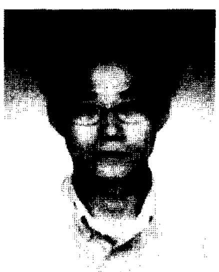

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