Robust Non-fragile Decentralized Controller Design for Uncertain Large-Scale Interconnected Systems

Ju H. Park

Abstract - In this brief, the design method of robust non-fragile decentralized controllers for uncertain large-scale interconnected systems is proposed. Based on Lyapunov second method, a sufficient condition for asymptotic stability is derived in terms of a linear matrix inequality (LMI), and the measure of non-fragility in controller is presented. The solutions of the LMI can be easily obtained using efficient convex optimization techniques. A numerical example is given to illustrate the proposed method.
Key words - Large-scale systems, non-fragile controller, LMI.

1. Introduction

With the enlargement of dimension of a control system, analysis and control for the system becomes very complicated. It is standard to divide such systems into a number of interconnected subsystems. In general, a large-scale interconnected dynamical system can be usually characterized by a large number of state variables, system parametric uncertainties, and a complex interaction between subsystems (Mahmoud *et al.*, [14], Siljak [16]). During the last decade, the problem of decentralized stabilization of large-scale systems has received considerable attention, because there are a large number of large scale interconnected dynamical systems in many practical control problems, e.g. transportation systems, power systems, communication systems, ecomonic systems, social systems, and so on (Chen *et al.* [2], Chen [3], Geromel and Yamakami [8], Ho *et al.* [9], Hu [10], Lee and Rodovic [13], Shi and Gao [15], Yan *et al.* [17]).

On the other hand, it is generally known that feedback systems designed for robustness with respect to plant parameters, may require very accurate controllers (Dorato [4], Keel and Bhattacharyya [11]). Therefore it is necessary that any controller should be able to tolerate some uncertainty in parameters. Since the controller fragility is basically the performance deterioration of a feedback control system due to inaccuracies in controller implementation, non-fragile control problem has been important issues (Dorato et al. [5], Famularo et al. [6], Kim and Park [12]). However, there are no papers considering non-fragile controller design methods of large-scale interconnected systems.

This paper is concerned with the design problem of robust non-fragile decentralized controller for the large-scale systems with parametric time-varying uncertainties. A sufficient condition for robust stability of the system is derived in terms of LMI using Lyapunov method. Moreover, the measure of non-fragility in controller can be calculated by solving the LMI. The LMI approach has been one of the hot spots in the control problem due to its computational advantage and simplicity in solving the addressed problems (Boyd et al. [1]). The controller parameters which satisfy the above LMIs can be easily found by various efficient convex optimization algorithms.

Notations: Throughout the paper, R^n denotes the n dimensional Euclidean space, and $R^{n\times m}$ is the set of all $n\times m$ real matrices. I denotes the identity matrix with appropriate dimensions. For symmetric matrices X and Y, the notation $X>Y$(respectively, $X\geq Y$) means that the matrix $X-Y$ is positive definite, (respectively, nonnegative).

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2. Problem Formulation

Consider a class of uncertain large-scale system composed of N interconnected subsystems described by

$$S_i: \dot{x}_i(t) = [A_i+\Delta A_i(t)]x_i(t)+\sum_{j\neq i}^{N}[A_{ij}+\Delta A_{ij}(t)]x_j(t) + B_i u_i(t), \quad i=1,2,\cdots,N \tag{1}$$

where $x_i(t)\in R^{n_i}$ is the state vector, and $u_i(t)\in R^{m_i}$ is the control vector. The system matrices A_i, B_i, and A_{ij} are of appropriate dimensions, and $\Delta A_i(t)$, and $\Delta A_{ij}(t)$ are real-valued matrices representing time-varying parameter uncertainties in the system.

Assume that the pair $(A_i, B_i), i = 1, \cdots, N,$ is stabilizable, and the time-varying uncertainties are of the form

$$\Delta A_i(t) = D_{ai}F_{ai}(t)E_{ai},\ \Delta A_{ij}(t) = D_{aij}F_{aij}(t)E_{aij}, \quad (2)$$

where D_{ai}, D_{aij}, E_{ai}, and E_{aij} are known constant real matrices with appropriate dimensions, and $F_{ai}(t)$, and $F_{aij}(t)$ are unknown matrix functions which are bounded as

$$F_{ai}^T(t)F_{ai}(t) \le I,\ F_{aij}^T(t)F_{aij}(t) \le I,\ \forall\ i,\ j \ge 0. \quad (3)$$

Now, although one finds the controller $u_i(t) = K_i x_i(t)$ for each subsystems, the actual controller implemented is

$$u_i(t) = -[I + \delta_i \Phi_i(t)]K_i x_i(t) \quad (4)$$

where K_i is the nominal controller gain, δ_i is the positive scalar, the term $\delta_i \Phi_i(t) K_i$ represents controller gain variations, and $\Phi_i(t)$ is assumed to be bounded as

$$\Phi_i^T(t)\Phi_i(t) \le I. \quad (5)$$

Here, the value of δ_i indicates the measure of non-fragility against controller gain variations.

With the control law (4), the resulting closed-loop subsystem becomes

$$\dot{x}_i(t) = [A_i + \Delta A_i(t) - B_i(I + \delta_i\Phi_i(t))K_i]x_i(t) + \sum_{j \ne i}^{N}[A_{ij} + \Delta A_{ij}(t)]x_j(t). \quad (6)$$

Then, the problem is to find the feedback gain matrix K_i of the control law (4) so that the closed-loop system (6) is asymptotically stabilized with non-fragility δ_i.

Before proceeding further, we will state well known lemma.

Lemma 1 (Boyd et al., [1]): The linear matrix inequality

$$\begin{bmatrix} Q(x) & S(x) \\ S(x)^T & R(x) \end{bmatrix} > 0$$

is equivalent to

$$R(x) > 0,\quad Q(x) - S(x)R(x)^{-1}S(x)^T > 0,$$

where $Q(x) = Q(x)^T$, $R(x) = R(x)^T$ and $S(x)$ depend affinely on x.

3. Robust Controller Design

In this section, we give a stability criterion for robust stability of the system (1) using Lyapunov method and LMI technique.

Now, we synthesize the gain matrices K_i as follows:

$$K_i = \gamma_i B_i^T P_i, \quad i = 1, 2, \cdots, N \quad (7)$$

where γ_i is positive scalar, and P_i is positive-definite matrix.

Note that the K_i given in (7) is a class of gain matrix of optimal control law, which is often utilized as controller structure in many control applications.

Here, for simplicity, we define

$$A_{di} = \Big(\sum_{j \ne i} A_{ij}A_{ij}^T\Big)^{1/2},\quad D_{di} = \Big(\sum_{j \ne i} D_{aij}D_{aij}^T\Big)^{1/2},$$
$$E_{di} = \Big(\sum_{j \ne i} E_{aji}^T E_{aji}\Big)^{1/2}. \quad (8)$$

Then, using Lyapunov method, we have following theorem for robust stability of the system (1).

Theorem 1: The closed-loop system (6) is asymptotically stable with non-fragility δ_i, if for $i = 1, 2, \cdots, N$, there exist positive definite matrix X_i, and positive scalars γ_i, ε_{0i}, α_{i1} and α_{i2}, which satisfy the following LMI:

$$\Omega_i(X_i, \gamma_i, \alpha_{i1}, \alpha_{i2}, \varepsilon_{0i}) = \begin{bmatrix} Q_i(X_i, \gamma_i) & \gamma_i B_i & B_i & X_i E_{di}^T & X_i & X_i E_{ai}^T \\ \gamma_i B_i^T & -\alpha_{i1} I & 0 & 0 & 0 & 0 \\ B_i^T & 0 & -\alpha_{i2} I & 0 & 0 & 0 \\ E_{di} X_i & 0 & 0 & -I & 0 & 0 \\ X_i & 0 & 0 & 0 & -(1/(N-1))I & 0 \\ E_{ai} X_i & 0 & 0 & 0 & 0 & -\varepsilon_{0i} I \end{bmatrix} < 0, \quad (9)$$

where

$$\begin{aligned} Q_i &= X_i A_i^T + A_i X_i + \varepsilon_{0i} D_{ai} D_{ai}^T - 2\gamma_i B_i B_i^T + A_{di} A_{di}^T + D_{di} D_{di}^T \\ \alpha_{i1} &= \varepsilon_i/\delta_i,\quad \alpha_{i2} = 1/(\varepsilon_i \delta_i),\quad X_i = P_i^{-1},\quad \varepsilon_i > 0. \end{aligned} \quad (10)$$

Proof: Consider a Lyapunov functional candidate

$$V = \sum_{i=1}^{N} V_i = \sum_{i=1}^{N} x_i^T(t) P_i x_i(t) \quad (11)$$

where P_i is the positive-definite matrix to be defined in (7).

The time derivative of V is given by

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{N} \dot{x}^T(t) P_i x_i(t) + x_i^T(t) P_i \dot{x}(t) \\ &= \sum_{i=1}^{N} 2x_i^T(t) P_i \dot{x}_i(t). \end{aligned} \quad (12)$$

Substituting (6) into (12), we have

$$\begin{aligned} \dot{V} = & \sum_{i=1}^{N} \{ x_i^T(t) [A_i^T P_i + P_i A_i + 2P_i D_{ai} F_{ai}(t) E_{ai} \\ & - 2\gamma_i P_i B_i B_i^T P_i - 2\gamma_i P_i B_i \delta_i \Phi_i(t) B_i^T P_i] \\ & x_i(t) + 2x_i^T(t) P_i \sum_{j=1, i \ne j}^{N} (A_{ij} + D_{aij} F_{aij}(t) E_{aij}) x_j(t) \Big\}. \end{aligned} \quad (13)$$

Using the known fact that

$$U\Delta V^T + V\Delta U^T \le \varepsilon UU^T + \varepsilon^{-1} VV^T, \quad \varepsilon > 0$$

for any matrices U, V and Δ with $\Delta^T\Delta \le I$, we can eliminate the unknown factor, $F_{ai}(t), F_{aij}(t)$ and $\Phi_i(t)$, of parameter uncertainties. Then the terms on right-hand side of (13) are bounded as

$$\begin{aligned}
&2x_i^T(t)P_iD_{ai}F_{ai}(t)E_{ai}x_i(t) \le \\
&\quad \varepsilon_0 x_i^T(t)P_iD_{ai}F_{ai}(t)F_{ai}^T(t)D_{ai}^TP_ix_i(t) \\
&\quad + \varepsilon_{0i}^{-1}x_i^T(t)E_{ai}^TE_{ai}x_i(t) \\
&\le \varepsilon_0 x_i^T(t)P_iD_{ai}D_{ai}^TP_ix_i(t) + \varepsilon_{0i}^{-1}x_i^T(t)E_{ai}^TE_{ai}x_i(t)
\end{aligned}$$

$$\begin{aligned}
&-\sum_{i=1}^N 2x_i^T(t)\gamma_iP_iB_i\delta_i\Phi_i(t)B_i^TP_ix_i(t) \le \\
&\quad \sum_{i=1}^N(\varepsilon_i^{-1}\gamma_i^2\delta x_i^T(t)P_iB_iB_i^TP_ix_i(t) \\
&\quad + \varepsilon_i\delta x_i^T(t)P_iB_i\Phi_i^T(t)\Phi_i(t)B_i^TP_ix_i(t)) \\
&\le \sum_{i=1}^N(\varepsilon_i^{-1}\delta_i\gamma_i^2x_i^T(t)P_iB_iB_i^TP_ix_i(t) \\
&\quad + \varepsilon_i\delta x_i^T(t)P_iB_iB_i^TP_ix_i(t))
\end{aligned}$$

$$\begin{aligned}
&\sum_{i=1}^N 2x_i^T(t)P_i\sum_{j\ne i}^N A_{ij}x_j(t) \le \sum_{i=1}^N \\
&\quad \left(x_i^T(t)P_i\sum_{j\ne i}^N A_{ij}A_{ij}^TP_ix_i(t) + \sum_{j\ne i}^N x_j^T(t)x_j(t)\right) \\
&= \sum_{i=1}^N(x_i^T(t)P_iA_{di}A_{di}^TP_ix_i(t) + (N-1)x_i^T(t)x_i(t))
\end{aligned}$$

$$\begin{aligned}
&\sum_{i=1}^N 2x_i^{T(t)}P_i\sum_{j\ne i}^N D_{aij}F_{aij}(t)E_{aij}x_j(t) \le \\
&\quad \sum_{i=1}^N\Big(x_i^T(t)P_i\sum_{j\ne i}^N D_{aij}F_{aij}(t)F_{aij}^T(t)D_{aij}^TP_ix_i(t) \\
&\qquad + \sum_{j\ne i}^N x_j^T(t)E_{aij}^TE_{aij}x_j(t)\Big) \\
&\le \sum_{i=1}^N\Big(x_i^T(t)P_i\sum_{j\ne i}^N D_{aij}D_{aij}^TP_ix_i(t) \\
&\qquad + \sum_{j\ne i}^N x_j^T(t)E_{aij}^TE_{aij}x_j(t)\Big) \\
&= \sum_{i=1}^N\Big(x_i^T(t)P_iD_{di}D_{di}^TP_ix_i(t) \\
&\qquad + \sum_{j\ne i}^N x_j^T(t)E_{aij}^TE_{aij}x_j(t)\Big)
\end{aligned}$$

where A_{di} and D_{di} are defined in (8), and ε_{0i} and ε_i are positive scalars to be chosen.

Using (14), we obtain a new bound of $\dot V$ as

$$\begin{aligned}
\dot V \le &\sum_{i=1}^N\{x_i^T(t)[A_i^TP_i + P_iA_i + \varepsilon_{0i}P_iD_{ai}D_{ai}^TP_i \\
&+ \varepsilon_{0i}^{-1}E_{ai}^TE_{ai} + \varepsilon_i^{-1}\delta_i\gamma_i^2P_iB_iB_i^TP_i - 2\gamma_iP_iB_iB_i^TP_i \\
&+ \varepsilon_i\delta_iP_iB_iB_i^TP_i + P_iA_{di}A_{di}^TP_i + P_iD_{di}D_{di}^TP_i \\
&+ (N-1)I]x_i(t) + \sum_{j=1, i\ne j}^N x_j^T(t)E_{aij}^TE_{aij}x_j(t)\Big\}.
\end{aligned} \tag{15}$$

Now, note that

$$\begin{aligned}
&\sum_{i=1}^N\sum_{j=1, i\ne j}^N x_j^T(t)E_{aij}^TE_{aij}x_j(t) \\
&= \sum_{i=1}^N x_i^T(t)\sum_{j=1, i\ne j}^N(E_{aji}^TE_{aji})x_i(t) \\
&= \sum_{i=1}^N x_i^T(t)E_{di}^TE_{di}x_i(t).
\end{aligned} \tag{16}$$

Then (15) is simplified as

$$\begin{aligned}
\dot V \le &\sum_{i=1}^N\{x_i^T(t)[A_i^TP_i + P_iA_i + \varepsilon_{0i}P_iD_{ai}D_{ai}^TP_i \\
&+ \varepsilon_{0i}^{-1}E_{ai}^TE_{ai} - 2\gamma_iP_iB_iB_i^TP_i + \varepsilon_i^{-1}\delta_i\gamma_i^2P_iB_iB_i^TP_i \\
&+ \varepsilon_i\delta_iP_iB_iB_i^TP_i + P_iA_{di}A_{di}^TP_i + P_iD_{di}D_{di}^TP_i \\
&+ (N-1)I + E_{di}^TE_{di}]x_i(t)\}.
\end{aligned} \tag{17}$$

Therefore, $\dot V$ is negative if the following condition holds:

$$\begin{aligned}
&A_i^TP_i + P_iA_i + \varepsilon_{0i}P_iD_{ai}D_{ai}^TP_i + \varepsilon_{0i}^{-1}E_{ai}^TE_{ai} \\
&\quad - 2\gamma_iP_iB_iB_i^TP_i + \varepsilon_i^{-1}\delta_i\gamma_i^2P_iB_iB_i^TP_i + \varepsilon_i\delta_iP_iB_iB_i^TP_i \\
&\quad + P_iA_{di}A_{di}^TP_i + P_iD_{di}D_{di}^TP_i + (N-1)I + E_{di}^TE_{di} < 0, \\
&\quad \text{for } i = 1,2,\cdots,N.
\end{aligned} \tag{18}$$

By premultiplying and postmultiplying X_i onto (18), we get

$$\begin{aligned}
&X_iA_i^T + A_iX_i + \varepsilon_{0i}D_{ai}D_{ai}^T + \varepsilon_{0i}^{-1}X_iE_{ai}^TE_{ai}X_i - 2\gamma_iB_iB_i^T \\
&\quad + \varepsilon_i^{-1}\delta_i\gamma_i^2B_iB_i^T + \varepsilon_i\delta_iB_iB_i^T + A_{di}A_{di}^T + D_{di}D_{di}^T \\
&\quad + (N-1)X_i^TX_i + X_iE_{di}^TE_{di}X_i < 0, \quad \text{for i} = 1,2,\cdots,\text{N}.
\end{aligned} \tag{19}$$

Then, Lemma 1, the equation (19) is equivalent to (9). This completes the proof.

Remark 1: From the solutions, α_{i1} and α_{i2}, of the LMI (9), the measure of non-fragility in each controller for subsystems, δ_i, can be calculated by $\delta_i = (\alpha_{i1}\alpha_{i2})^{-1/2}$.

Remark 2: In order to solve the LMI (9) given in Theorem 1, we can utilize Matlab's LMI Control Toolbox [7], which implements state-of-the-art interior-point algorithms, which is significantly faster than classical convex optimization algorithms [1].

Remark 3: The γ_i and P_i, which is the gain parameters of the decentralized controller (7) for subsystem i, is not unique by solving the LMI (9). So, one can choose the solution taking into account the design specification such as the maximum magnitude of the gain parameters, etc.

We now give an example to illustrate the proposed method.

Numerical Example: Consider a large-scale system which is composed of three subsystems

$$\dot{x}_1(t)=\begin{bmatrix}0 & 1\\ 1 & -2\end{bmatrix}x_1(t)+\begin{bmatrix}0.4 & -0.1\\ -0.1 & 0.4\end{bmatrix}x_2(t)+\begin{bmatrix}0.3 & 0\\ 0 & 0.3\end{bmatrix}x_3(t)$$
$$+\Delta A_1(t)x_1(t)+\Delta A_{12}(t)x_2(t)+\Delta A_{13}(t)x_3(t)+\begin{bmatrix}1\\ 0\end{bmatrix}u_1(t),$$

$$\dot{x}_2(t)=\begin{bmatrix}-1 & 1\\ 0 & 1\end{bmatrix}x_2(t)+\begin{bmatrix}0 & 0.3\\ 0.1 & 0.3\end{bmatrix}x_1(t)+\begin{bmatrix}0.2 & 0.1\\ 0.1 & 0.2\end{bmatrix}x_3(t)$$
$$+\Delta A_2(t)x_2(t)+\Delta A_{21}(t)x_1(t)+\Delta A_{23}(t)x_3(t)+\begin{bmatrix}0\\ 1\end{bmatrix}u_2(t),$$

$$\dot{x}_3(t)=\begin{bmatrix}0 & 1\\ -1 & -1\end{bmatrix}x_3(t)+\begin{bmatrix}0 & 0.5\\ 0.1 & 0.2\end{bmatrix}x_1(t)+\begin{bmatrix}0.3 & 0.2\\ 0 & 0.2\end{bmatrix}x_2(t)$$
$$+\Delta A_3(t)x_3(t)+\Delta A_{31}(t)x_1(t)+\Delta A_{32}(t)x_2(t)+\begin{bmatrix}2\\ 1\end{bmatrix}u_3(t),$$

where

$$\begin{aligned}
\Delta A_1(t)&=\begin{bmatrix}0 & 0\\ 0.1 & 0.2\end{bmatrix}\begin{bmatrix}\sin(t) & 0\\ 0 & \sin(2t)\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\\
\Delta A_{12}(t)&=\begin{bmatrix}0\\ 0.1\end{bmatrix}\sin(2t)[0\ \ 1],\\
\Delta A_{13}(t)&=\begin{bmatrix}0\\ 0.1\end{bmatrix}\cos(t)[1\ \ 1],\\
\Delta A_2(t)&=\begin{bmatrix}0 & 0\\ 0.1 & 0.1\end{bmatrix}\begin{bmatrix}\sin(2t) & 0\\ 0 & \sin(t)\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\\
\Delta A_{21}(t)&=\begin{bmatrix}0\\ 0.1\end{bmatrix}\sin(t)[1\ \ 1],\\
\Delta A_{23}(t)&=\begin{bmatrix}0\\ 0.1\end{bmatrix}\cos(t)[0\ \ 1]\\
\Delta A_3(t)&=\begin{bmatrix}0.1 & 0\\ 0 & 0.2\end{bmatrix}\begin{bmatrix}\sin(t) & 0\\ 0 & \sin(t)\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\\
\Delta A_{31}(t)&=\begin{bmatrix}0\\ 0.1\end{bmatrix}\sin(t)[1\ \ 0]\\
\Delta A_{32}(t)&=\begin{bmatrix}0\\ 0.1\end{bmatrix}\sin(2t)[1\ \ 1].
\end{aligned}$$

First, we find the positive solutions of the LMI (9) for the subsystem 1 as

$$X_1=\begin{bmatrix}0.1081 & -0.0838\\ -0.0838 & 0.3160\end{bmatrix},\quad \gamma_1=0.929$$
$$\alpha_{11}=1.4339,\quad \alpha_{12}=1.5136,\quad \varepsilon_{01}=1.2435.$$

Similarly, the other solutions of the LMI (9) for the subsystem 2 and 3 are as follows:

$$X_2=\begin{bmatrix}0.4476 & -0.2215\\ -0.2215 & 0.3491\end{bmatrix},\quad \gamma_2=1.7538$$
$$\alpha_{21}=3.9388,\quad \alpha_{22}=2.4683,\quad \varepsilon_{02}=3.0038$$
$$X_3=\begin{bmatrix}0.4032 & 0.1508\\ 0.1508 & 0.5870\end{bmatrix},\quad \gamma_3=0.6245$$
$$\alpha_{31}=3.9962,\quad \alpha_{32}=2.7687,\quad \varepsilon_{03}=3.6907.$$

Therefore, the gain matrices, K_i, of the stabilizing controller, u_i, for three subsystems are

$$\begin{aligned}
K_1&=\gamma_1 B_1^T P_1=[10.8157\ \ 2.8695],\\
K_2&=\gamma_2 B_2^T P_2=[3.6246\ \ 7.3240],\\
K_3&=\gamma_3 B_3^T P_3=[2.9868\ \ 0.2968],
\end{aligned}$$

and the value of non-fragility in controller are as follows:

$$\begin{aligned}
\delta_1&=(\alpha_{11}\alpha_{12})^{-1/2}=0.6788,\\
\delta_2&=(\alpha_{21}\alpha_{22})^{-1/2}=0.3207,\\
\delta_3&=(\alpha_{31}\alpha_{32})^{-1/2}=0.3006.
\end{aligned}$$

From the above value of non-fragility of each subsystem, one can see that the obtained robust decentralized controller guarantees the asymptotic stability of the closed-loop system in spite of each controller gain variations of the subsystem 1,2 and 3 within 67.88%, 32.07%, and 30.06%, respectively.

For computer simulation, the following control laws and initial conditions are employed:

$$\begin{aligned}
u_1(t)&=-(1+0.6788\sin(t))K_1x_1(t)\\
u_2(t)&=-(1+0.3207\sin(t))K_2x_2(t)\\
u_3(t)&=-(1+0.3006\sin(t))K_3x_3(t)\\
x_1(0)&=[1\ \ -0.5]^T\\
x_2(0)&=[1.5\ \ -1.5]^T\\
x_3(0)&=[-1\ \ 0.5]^T.
\end{aligned}$$

Note that it is assumed that the controllers for the subsystem 1, 2, and 3 are subjected to 67.88%, 32.07%, and 30.06% gain variations, respectively. The simulation results are given in Figs. 1-4. Figure 1, 2, and 3 show the

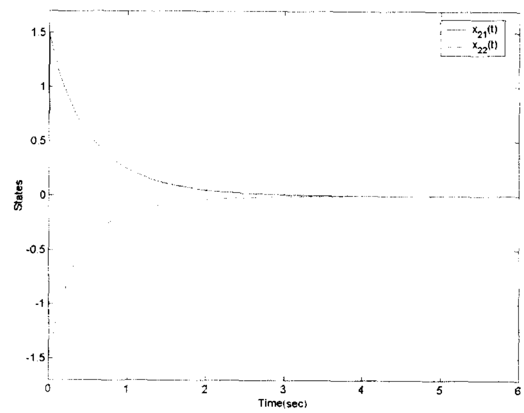

Fig. 1 State responses of subsystem 1

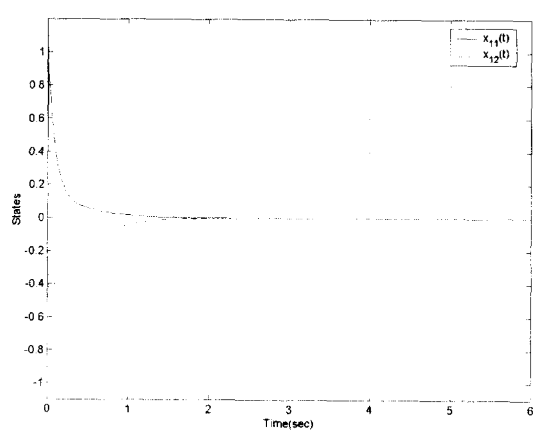

Fig. 2 State responses of subsystem 2

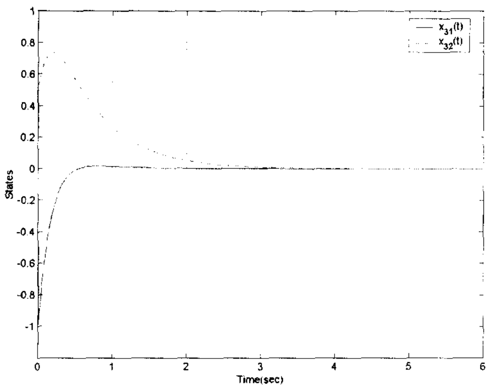

Fig. 3 State responses of subsystem 3

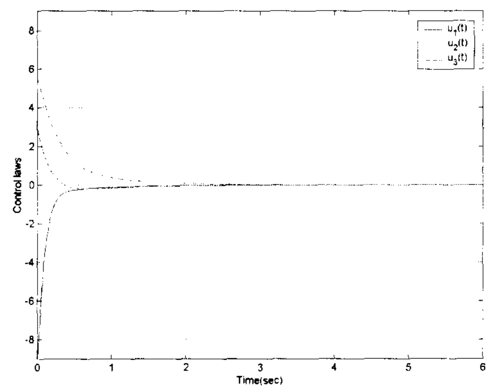

Fig. 4 Control inputs for subsystem 1, 2, and 3

state responses of the subsystems 1, 2, and 3, respectively, Figure 4 shows the control laws of each subsystem. From the figures, one can see that the system is indeed well stabilized irrespective of uncertainties and controller gain variations.

4. Conclusion

In this brief, a robust non-fragile controller design method for uncertain large-scale interconnected systems, is presented. Using Lyapunov method, a sufficient condition for asymptotic stability of the system is derived in terms of LMI. Finally, a numerical example is given for illustration of controller design, and simulation result shows that the system is well stabilized in spite of controller gain variations and uncertainties.

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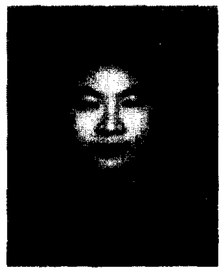

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