Analytical Approach to Evaluate the Inelastic Behaviors of Reinforced Concrete Structures under Seismic Loads

지진하중을 받는 철근콘크리트 구조물의 해석적 방법에 의한 비탄성 거동 평가

김태훈*　　신현목**
Kim, Tae Hoon　　Shin, Hyun Mock

국문요약

이 연구는 철근콘크리트 구조물의 비탄성 거동을 파악하고 합리적이면서 경제적인 내진설계기준의 개발을 위한 자료를 제공하는데 그 목적이 있다. 정확하고 올바른 내진성능의 파악을 위하여 비탄성 해석프로그램을 사용하였다. 사용된 프로그램은 철근콘크리트 구조물의 해석을 위해서 유한요소법을 이용하여 개발된 RCAHEST 이다. 재료적 비선형성에 대해서는 균열콘크리트에 대한 인장, 압축, 전단모델과 콘크리트 속에 있는 철근모델을 조합하여 고려하였다. 이에 대한 콘크리트의 균열모델로서는 분산균열모델을 사용하였다. 또한, 횡방향 구속철근으로 인한 강도의 증가 효과를 고려하였다.

주요어 : 비탄성 거동, 내진설계기준, 비탄성 해석프로그램, 분산균열

ABSTRACT

The purpose of this study is to investigate inelastic behavior of reinforced concrete structures and to provide result for developing improved seismic design criteria. The accuracy and objectivity of the assessment process may be enhanced by the use of sophisticated inelastic analysis program. A computer program, named RCAHEST (reinforced concrete analysis in higher evaluation system technology), for the analysis of reinforced concrete structures was developed using the finite element method. Material nonlinearity is taken into account by comprising tensile, compressive and shear models of cracked concrete and a model of reinforcing steel. The smeared crack approach is incorporated. The strength increase of concrete due to the lateral confining reinforcement has been also taken into account to model the confined concrete.

Key words : inelastic behavior, seismic design criteria, inelastic analysis program, smeared crack

1. Introduction

In the modern construction industry, composite steel/concrete structural systems are extensively used. Existing advantages of this form of construction are due to the complementary properties of both steel and concrete. RC structural systems can be considered an extension of composite structures. The design of these structures must satisfy the requirements of safety and serviceability. While this can be accomplished in most cases by following approximate or empirical procedures prescribed in codes or recommended practices, it is desirable to have refined analytical models and methods available which can trace the structural response of these structures throughout their service load history and under increasing loads through their elastic, cracking, inelastic, and ultimate ranges. Because of the difficulties in developing a rational analysis procedure, empirical approaches have been used for ordinary design. However, the application of the FEM to RC structures has also been underway for last 30 years, during which time it has proven to be a very powerful tool in engineering analysis.(1) It is capable of providing analytic solutions to problems involving columns, beams, plates, shells, and even three-dimensional solids.

Applications of the method to problems of the behavior of reinforced concrete structure including such aspects as tensile cracking, crushing, tension stiffening, multiaxial non-linear concrete properties, creep, shrinkage, concrete-steel interaction, etc. have been undertaken by a number of investigators.(1),(2) At present, material nonlinearities are taken into account by comprising the tension, compression and shear model of cracked concrete and a model for reinforcement in the concrete; and also a so-called smeared crack model is incorporated. These constitutive models comprise the model of cracked concrete in compression, incorporating compression-softening effects due to transverse cracking, the model of cracked concrete in tension, reflecting tension-stiffening effects due to bond interactions with reinforcement the model of cracked concrete in shear, reflecting the aggregate interlocking, and the model of reinforcement

---

* 학생회원 · 성균관대학교 토목환경공학과, 박사과정 수료
(대표저자 : kth7love@mail.skku.ac.kr)
** 정회원 · 성균관대학교 토목환경공학과, 교수

본 논문에 대한 토의를 2001년 6월 30일까지 학회로 보내 주시면 그 결과를 게재하겠습니다.

in reinforced concrete. The models cover loading, unloading and reloading paths. A finite element program incorporating these constitutive models has been used to predict the response of concrete panel under both in-plane monotonic and cyclic loads successfully.$^{(3),(4)}$ In fact, this reinforced concrete model is only one code which can deal with two-way cracking accompanying crack opening and closing under the reversed cyclic loads. In the model presented in this paper, the emphasis is on its ability to model cyclic behavior by proper theoretical representation or the material parameters.

The proposed structural element library RCAHEST(reinforced concrete analysis in higher evaluation system technology) is built around the finite element analysis program FEAP developed by Taylor.$^{(5)}$

This paper presents some salient features of the structural element library RCAHEST. Some of RCAHEST's capabilities are discussed in the examples. To examine the above correlation between experiment and analysis, two different types of examples have been chosen which are introduced as follows together with an explanation of their results. (1) three panels tested at the University of Toronto, (2) pseudo-dynamic tested by Chung-Ang University.

The goal of this study is to make seismic assessment by analytic method be possible. To this end an integrated dynamic finite element analysis method is developed that is based on the improved modeling of dynamic nonlinear behavior of reinforced concrete. It is believed that accurate and rational seismic assessment can be achieved very efficiently economical by this method.

2. Finite element analysis of reinforced concrete structures

In engineering science, experimentation and analysis are two complementary processes for developing and understanding phenomena. This is because experimental investigation is expensive and time consuming and only simple problems can be experimentally investigated. It is also obvious that analytical investigation cannot be performed without reliable experimental results.

The vast use of RC structures, necessitates the intensive investigation of these structures experimentally and analytically. Because of the complexity associated with the development of rational analytical procedures, empirical formulations which are based on a large number of experimental data have been applied in design. The use of such approaches was compulsory in the past and may continue to be most convenient in ordinary design. To improve the accuracy of analysis in the design of complex structures, such as offshore oil platforms and nuclear containment structures, the use of more accurate methods is required. Development of FEA and its application to RC structures has given an opportunity for researchers to take all ranges of problems as well as achieving improvement in the accuracy of results.$^{(1),(2)}$

For any type of structure, the more complicated its structural geometric configuration is, the more a computer-based numerical solution becomes necessary. It has also been shown that experimental investigations are time consuming, capital intensive and even often impractical. The FEM is now firmly accepted as a very powerful general technique for the numerical solution of a variety of problems encountered in engineering.$^{(1)}$

The nonlinear constitutive model is the most important factor that governs the accuracy of the solution in the analysis of reinforced concrete structures. Reinforced concrete is a composite material consisting of concrete and steel. Under the applied load, crack will be developed in the concrete. After cracking the mechanical behavior of concrete and bonding between steel and concrete will become very complex. The accuracy of the analysis is practically dependent on the quality of the constitutive model.

3. Nonlinear material modeling of reinforced concrete

Reinforced concrete exhibits highly nonlinear material behavior. Realistic constitutive models for the concrete and reinforcement must be incorporated in the nonlinear finite element model. The reinforced concrete can efficiently be expressed as the superposition of concrete and reinforcing bar. However, the reinforced concrete is not of simply adding each of these analytical models but of properly combining each of them, recognizing that there exists a bond effect between the concrete and the reinforcement.

3.1 Model for uncracked concrete

3.1.1 Elasto-plastic fracture model for biaxial state of stress

The Elasto-Plastic and Fracture model for the biaxial state of stress proposed by Maekawa and Okamura was used as a constitutive equation of uncracked concrete. In this model, the nonlinearity and anisotropy of the concrete are expressed regardless of the loading history including strain-softening domain. The equivalent stress-strain relation with the initial elastic modulus, fracture parameter and equivalent plastic strain.$^{(6),(7)}$

3.1.2 Crack representation

An analytical model for reinforced concrete element can be expressed through the average stress-strain relation based on the smeared crack concept that regards the tributary area with multiple crack and reinforcement as a finite continuum element.(8)

The smeared crack approach can be divided into the fixed and rotating crack approach.(9) The rotating crack approach assumes that each direction of the first and second crack is continuously varied to coincide with the direction of the principal strain, providing a reasonable result for the localized crack at arbitrary loading level. Since it is necessary to restore the load history at the crack plane when subjected to cyclic loads, the application of rotating crack approach becomes impractical.(8) The fixed crack model is divided into the orthogonal and non-orthogonal fixed crack concept. The former may overestimate a concrete stiffness and consequently, produces a stiffer response when the principle tensile stress exceeds a cracking stress prior to crack detection. The reason is that the direction of new crack is limited to be perpendicular to the first crack.(10)-(12)

In this study, a non-orthogonal crack approach is adopted to define the crack initiation in reinforced concrete element with respect to real direction of principal stress, while the localized discontinuous deformations in the boundary plane were considered by applying discrete crack approach.(6),(7)

The crack initiation as a transient point of the concrete nonlinearity is greatly influenced by the several factors includes uniaxial strength of concrete, biaxial state of stress and so on. It is assumed in this study that the crack occurs when the principle tensile strain reaches the stress of concrete reaches the fracture envelope.(6),(7)

3.2 Model for cracked concrete

Once the crack takes place in concrete, an anisotropy becomes significant so that the stress-strain relationship takes on an orthogonal anisotropy in the direction normal and parallel to crack. This means that the stress-strain relations have to be modeled respectively in the direction parallel as well as normal to crack and in the shear direction. Hence, each constitutive model for cracked reinforced concrete element in this study is formulated in the direction of orthogonal anisotropy.

3.2.1 Model for concrete normal to crack

The cracked concrete still carries a certain amount of the tensile stress normal to the cracked plane due to the bond effect of concrete to the reinforcing bar.

Okamura et al.(13) have developed an average stress vs. average strain relationship for concrete normal to crack. However they did not considered the fact that the tensile stress resisted by the concrete results from the bond effect between the concrete and the reinforcing bar. Therefore, it may cause an inaccuracy of the analysis results under the complicated stress distribution and overestimate the tensile stiffness normal to crack of reinforced concrete element, especially when the reinforcing ratios in orthogonal directions show a remarkable discrepancy or when the reinforcing bars are distributed only in one direction.

In this study, the bond model was applied in each direction of the reinforcing bar as shown in Fig. 1. Hence, more refined approach could be made to the tension stiffness model, by transforming the tensile stresses of concrete to those in the direction normal to the crack and the accuracy of result is expected to be improved.(6),(7)

As a tension stiffness model for unloading and reloading, the model proposed by Shima et al.(14) was basically used.(15) The unloading process brings about closing of crack that has taken place. But it should be noted that the two surfaces of crack start contacting each other even before the average strain ε_{xo} of concrete becomes zero. Hence, 150×10^{-6} has been given as the strain when the crack surfaces start contacting as shown in Fig. 2.(6),(7)

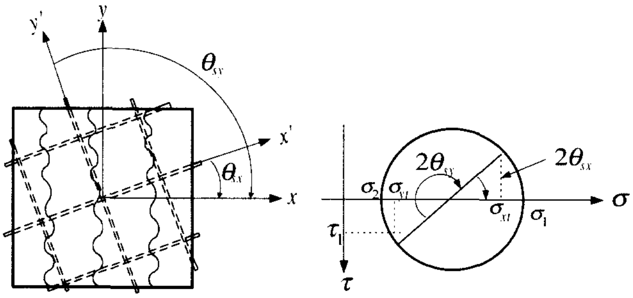


Fig. 1 Tensile stress evaluation of concrete normal to crack

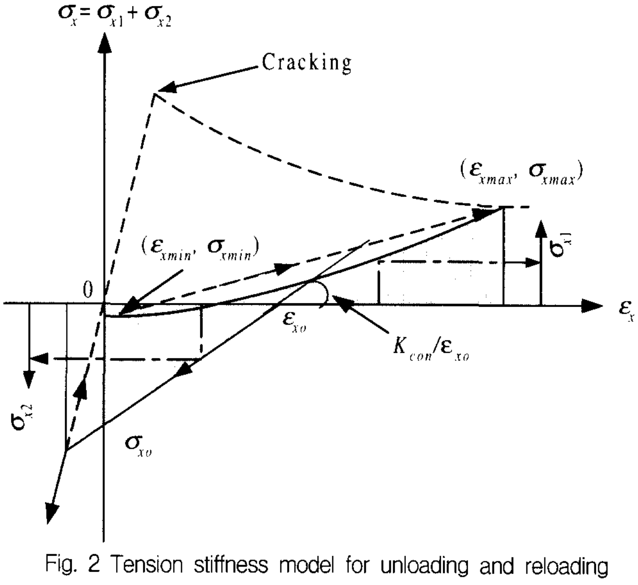


Fig. 2 Tension stiffness model for unloading and reloading

3.2.2 Model for concrete parallel to crack

The modified elasto-plastic fracture model[16] was used in this study as a model for concrete parallel to crack. This model describes the degradation in compressive stiffness by modifying the fracture parameter in terms of the strain perpendicular to the crack plane. The main reason for the degradation in the stiffness of cracked concrete may be attributed to the fact that the ability of concrete carrying a compressive stress is degraded in the vicinity of crack on account of the roughness of crack surfaces.[17]

It is necessary to model the damage of inner concrete under the cyclic load in order to consider the energy consumption at unloading and reloading process. But this study assumed that the energy consumption could be considered by modifying the stress-strain curve at unloading process to an a quadratic curve. The tangential stiffness of the curve at the beginning of unloading becomes infinite and passing through the point of residual strain as shown in Fig. 3.[6],[7]

3.2.3 Model for concrete in shear direction

In order to consider the effect of shear stress transfer due to the aggregate interlock at crack surface, the shear transfer model based on the contact surface density function[18] was basically used. Since this model defines the form of crack surface in terms of three parameters and assumes the contact surface to respond elasto-plastically, it is applicable to the arbitrary loading history as shown in Fig. 4.

The shear stiffness shows sudden increase on closing of crack under the reversed cyclic loading. This effect should be modeled not only to ensure the continuity of shear stiffness evaluation through the opening and closing of the crack during the reversed loading but also to obtain the equilibrium solution by the numerical analysis.

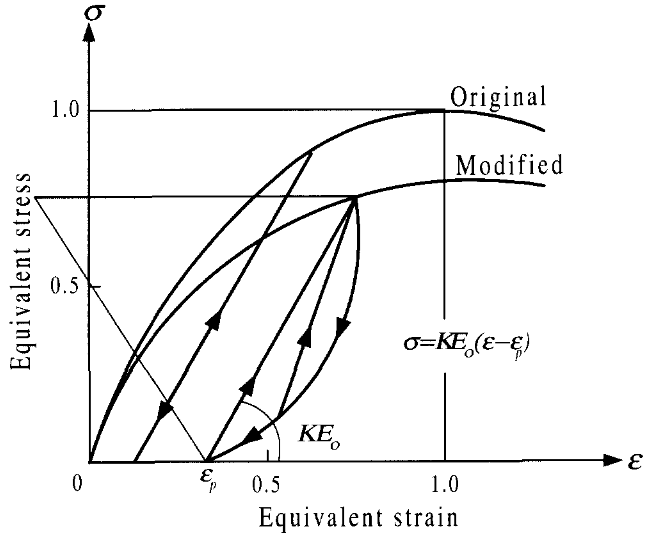


Fig. 3 Equivalent stress-equivalent strain relationship at unloading and reloading

Introducing the average strain concept of the smeared crack model just as in the monotonic loading, the shear transfer model for unloading and reloading is expressed as shown in Fig. 5.[6],[7]

3.3 Model for reinforcing bar in concrete

The stress acting on the reinforcing bar embedded in concrete is not uniform and takes the maximum values where the bar is exposed to a crack plane. The constitutive model for the bare bar can be used if the stress-strain relation belongs to an elastic range. However, the elastic relationship between the average stress and strain of reinforcing bar is lost as soon as the bar yields at crack plane, even if it remains unyielded elsewhere. And the average steel stress at that instant is lower than its yield strength.[19]

Since the bar stress remains lower than the yield strength elsewhere except for the vicinity of crack plane, the average strain of reinforcing bar does not exhibit an yield shelf

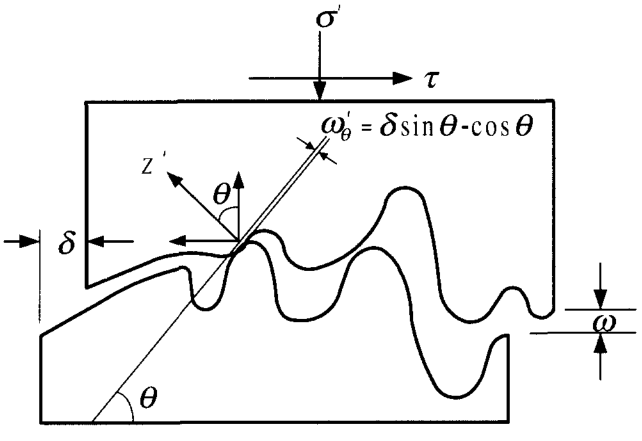


Fig. 4 Shear transfer mechanism at crack surface

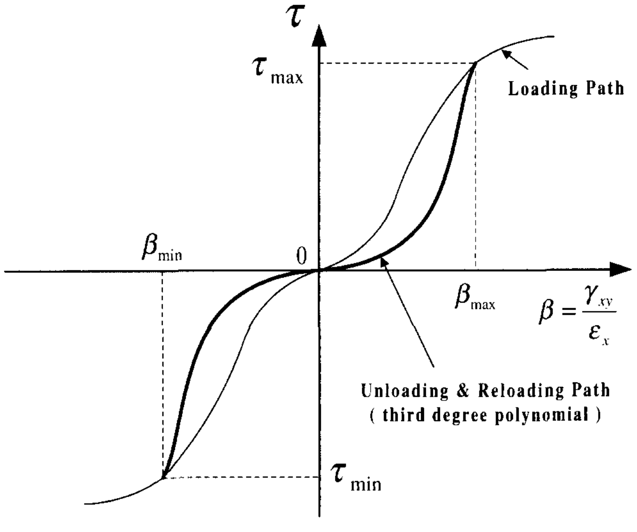


Fig. 5 Shear transfer model for unloading and reloading

just as for a bare bar, entering into the strain hardening domain immediately after yielded as shown in Fig. 6. In modeling the post-yield constitutive law for the reinforcing bar in concrete, the bond characteristic should be considered.

The average stress-average strain relationship of reinforcing bar for unloading or reloading can be determined once the stress distribution of bar between the cracks and the stress-strain relation for the bare bar has been determined for that mode of loading. Kato's model(20) for the bare bar under the reversed cyclic loading and the assumption of stress distribution denoted by a cosine curve was used in deriving the mechanical behaviors of reinforcing bars in concrete under the reversed cyclic loading. By taking the average stress and strain for Kato's bar stress and strain respectively, the modified Kato's model was developed for the post-yield steel model for unloading and reloading branch as shown in Fig. 7.(6),(7)

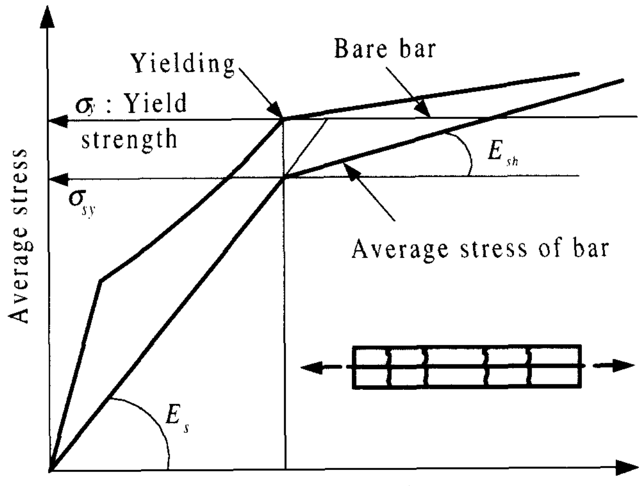


Fig. 6 Yield condition of reinforced concrete

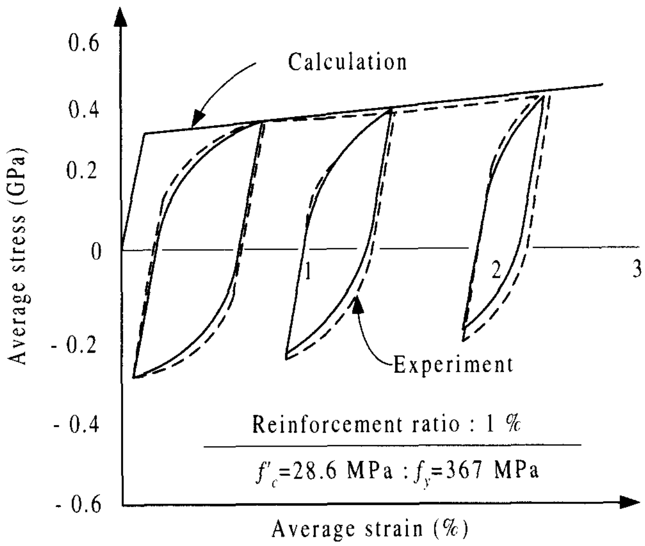


Fig. 7 Reinforcement model under reversed cyclic loading

3.4 Material modeling of interface element

This study introduced an interface element based on discrete crack concept, which describes the stress versus localized deformation relationships at the joint plane connecting two reinforced concrete elements with different sections, as shown in Fig. 8. This interface element is a one-dimensional one with zero thickness, and describes the relations between the normal force versus normal displacement and the shear force versus shear displacement.

3.4.1 Model for pulling-out of reinforcing bar

The strain-slip relation of reinforcing bar proposed by Shima et al.(14) was used as an analytical model on pulling-out of reinforcing bar from the base caused by the steel tension. This model describes a relation between the bar strain and loaded end slip or relative displacement of the bar to concrete, which is applicable to both elastic and plastic stress states for arbitrary loading. This model gives a unique strain-slip relation for the reinforcing bar with long embedded length and no slip at the free-end.

Shima et al.(14) modeled the relation between the strain and the slip as simple equations for the monotonic loading based on the bond stress-strain-slip relations, assuming that the slip and bond stress at free-end of the bar was equal to zero. Shin(3) defines the range of decrease in bond stress by the five times of diameter of the bar in length from the loaded end. Authors also assumed that the bond stress becomes zero at the joint plane but remain its standard value at the position apart from that plane by five times the bar diameter and changes linearly between them.(21),(22)

In this study the strain-slip relationship based on these assumptions was used to express the effect of decrease in bond capacity in the vicinity of the loaded end.

Authors have adopted for the present model a simple quadratic curve for the unloading and reloading as shown in Fig. 9.(21),(22)

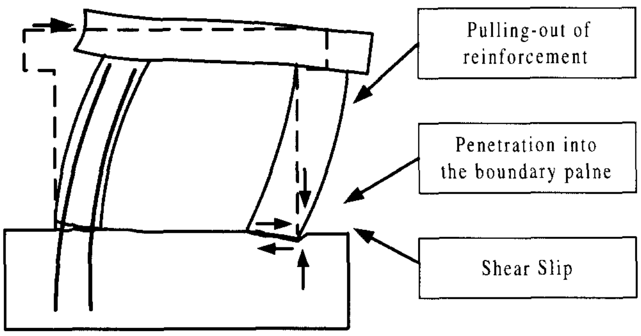


Fig. 8 Localized discontinuous deformation in boundary plane

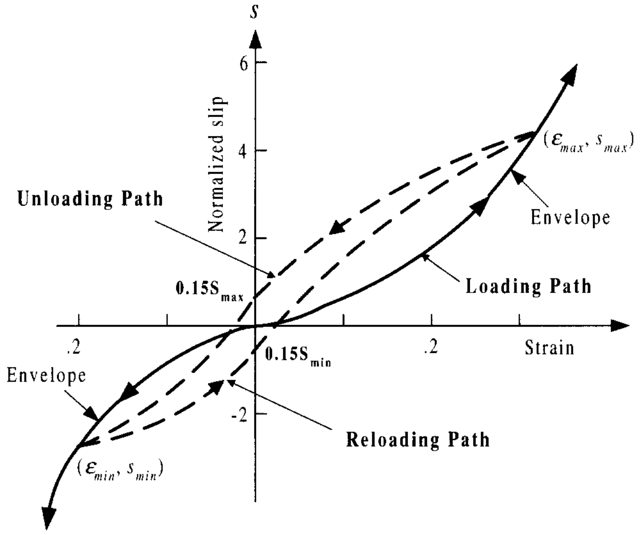


Fig. 9 Stain-slip relationship of bar at unloading and reloading

3.4.2 Model for closure at joint plane

Two-dimensional analysis necessarily assumes the stress distribution to be uniform in the direction of the element thickness. However, the stress distribution in the vicinity of joint plane made by two elements with different thickness is of three-dimensional in nature. Thus, the assumption of uniformity does not hold true. The predicted deformation tends to be smaller than the actual response. Hence it must be included in total deformation for the reliable estimation of structural behavior.

This study took into account the effect of the localized stress distribution, using the model for closure in boundary plane.(3)

This model assumes that the joint plane with virtual height h_i deforms to resist a compressive stress as shown in Fig. 10. The reinforced concrete element model in the previous section was also applied to the compressive model of concrete for interface element with virtual volume under loading, unloading and reloading.

The compressive stress(c) - displacement(ω) relation for the joint plane can now be given by replacing the compressive strain in the elasto-plastic fracture model with the compressive displacement at the joint plane.(21),(22)

3.4.3 Model for shear slip at joint plane

The shear stress(τ_c) - shear displacement(δ) relation of the interface element can be obtained from Li and Maekawa model(18) for the reinforced concrete plane stress element for monotonic loading. And for unloading and reloading branches, the relationships for the reinforced concrete element were also used with the shear strain γ_{xy} replaced by the shear displacement δ and the tensile strain ε_x by the tensile displacement ω.(21),(22)

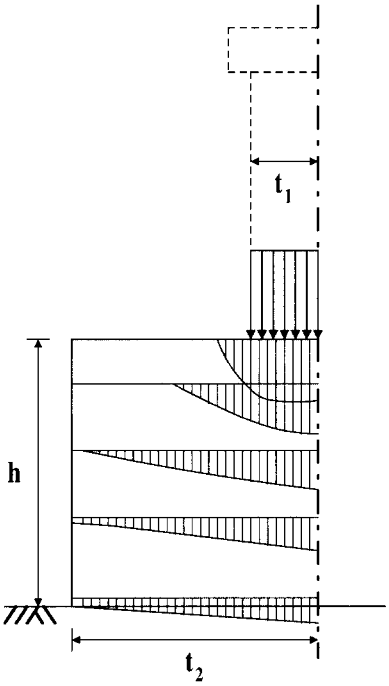


Fig. 10 3-D effect of interface for compression

3.5 Effects of confining reinforcements

A transverse reinforcement is provided to confine the compressed concrete within the core region and to prevent buckling of the longitudinal reinforcement. The reinforced concrete column confined by the transverse reinforcement shows an improved ultimate strength and strain capacity.(23),(24) It also has a superior ductile capacity to the unconfined concrete. This study takes the confining effects of the transverse reinforcement into account by modifying the compressive stress-strain model of the unconfined concrete as shown in Fig 11.(7),(21)

Many different stress-strain relationships have been proposed for the confined concrete.(23),(24) This study used a model proposed by Mander et al.(24) It considers the amount of the longitudinal and transverse reinforcement, and the yield strength and distribution type of the transverse reinforcement.

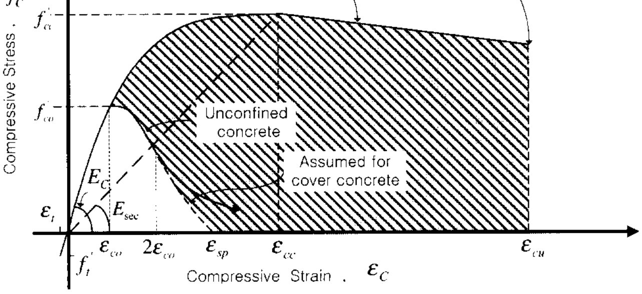


Fig. 11 Stress-strain model proposed for confined and unconfined concrete

4. Analysis program by finite element method

One of the current trends in the development of finite element engineering application program is to implement new material models and new numerical methods into existing general purpose finite element codes. New material models and new numerical methods reported in the open literature will certainly make the current analysis programs more reliable and more efficient. However, because of a lack of good organization for most well established engineering software systems, they are generally not suitable for a direct updating to include these new features as required for such developments. New architectures should be used in the design of such software systems. The overall goal of this work is to improve the quality and productivity in structural engineering research and instructional software development through creation of a domain-specific programming environment consisting of reusable software component and computer-aided software engineering tools which support reuse.

FEAP is characterized by modular architecture and by the facility of introducing any type of custom elements, input utilities and custom strategies and procedures. The FEAP will help alleviate many of the difficulties commonly encountered in maintaining the integrity of existing software components during the development of a new research capabilities. The FEAP system includes a general element library. Elements are available to model one, two and three dimensional problems in linear and nonlinear structural and solid mechanics and for heat conduction problems. FEAP permits users to add their own element modules to the program.(5)

Accompanying with the present study, we will attempt to implement such a constitutive model for reinforced concrete and structural element library RCAHEST as shown in Table 1.

Table 1 Element library RCAHEST

Element library	Element number	Description
RCAHEST (reinforced concrete analysis in higher evaluation system technology)	Elmt20	2D or 3D Flexibility-based fiber beam-column element
	Elmt21	2D or 3D spring element
	Elmt22	4 nodes elastic shell element
	Elmt23	4 nodes RC shell element
	Elmt24	2D elasto-plastic element
	Elmt25	RC plane stress element
	Elmt26	Joint element
	Elmt27	Reinforcing bar element
	Elmt28	Interface element

The RCAHEST computer program is organized into three parts: input, analysis and output, as shown in Fig. 12.

5. Applications

Using the developed RC plane stress element, element level and structural level specimens are analyzed. From the numerical analysis of the experimental specimens, it is concluded that the proposed technique is suitable for analyzing RC structures.

5.1 Elements subjected to in-plane reversed cyclic shear

The capability of finite element analysis was tested to predict the behavior of concrete panels subjected to in-plane reversed cyclic shear. The experimental works investigated in this section were done by Stevens et al.(25)

The specimens were all of the same overall size 1625 × 1625mm with effective test dimensions of 1524 × 1524mm. The thickness of all specimens was 285mm. Reinforcement,

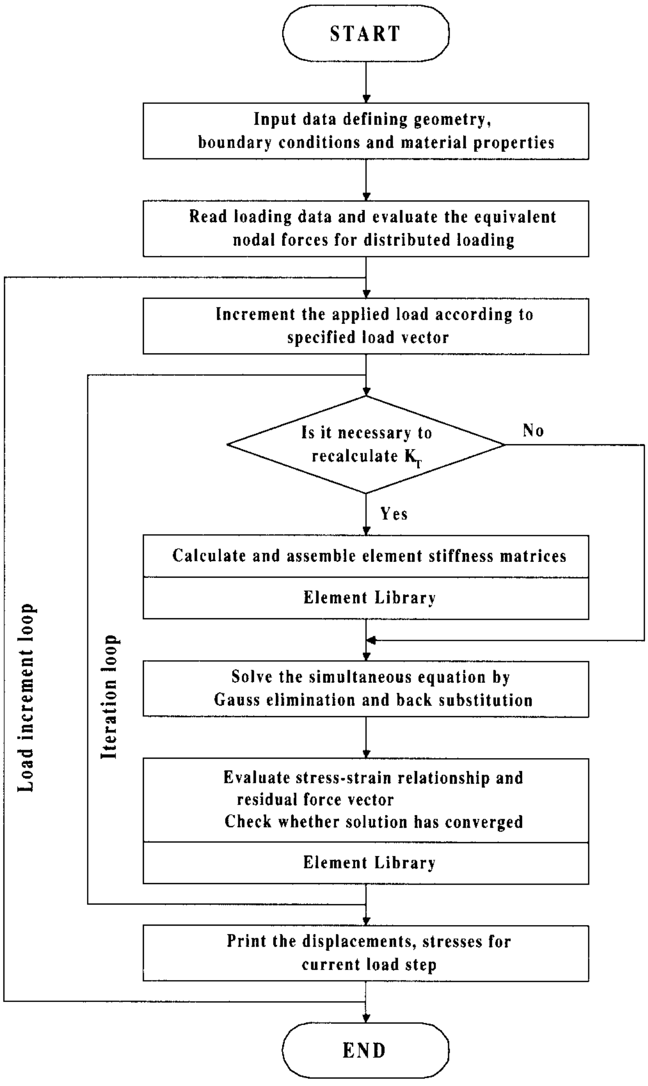


Fig. 12 Flowchart of program procedures

in all specimens, was placed in two layers in each of two orthogonal directions. One of the specimens has isotropic arrangement of reinforcement(SE9) and the others have anisotropic arrangement(SE8, SE10). The reinforcement layout and the application of stresses can be found in Fig. 13. Reversed cyclic shear loads were applied along the reinforcement directions, by applying equal tension and compression in the two orthogonal direction at 45° to the reinforcement. Details of material properties can be found in Table 2.

Since the force distribution is uniform across the element, only one finite element was used to predict the response of the specimen. The comparison between analytical and experimental results are shown in Fig. 14, Fig. 15, and Fig. 16. The analytical results show a good agreement with what was observed in the experiments not only loading condition, but also in unloading and reloading conditions.

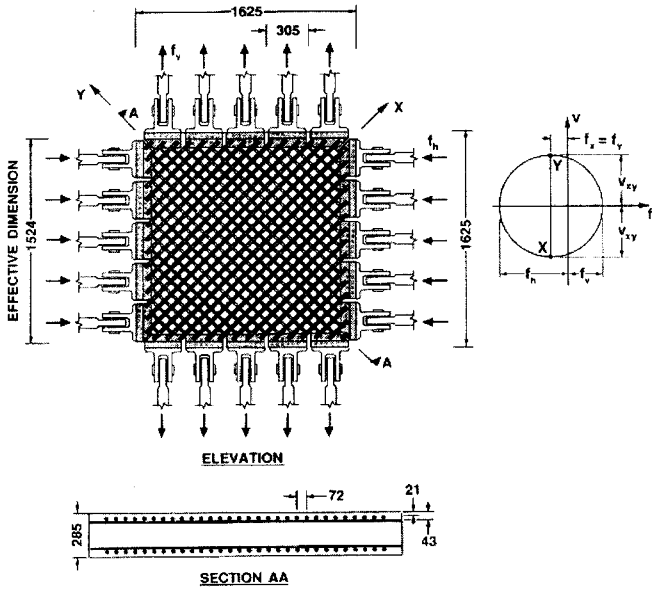


Fig. 13 Reinforcement layout and application of stresses of specimens

Table 2 Material properties of SE-specimens

Specimen	Concrete	Reinforcement				
	f_c' (MPa)	θ (degrees)	ρ_x (%)a	f_{yx} (MPa)	ρ_y (%)^a	f_{yy} (MPa)
SE8	37.0	0	1.465	492	0.49	479
SE9	44.2	0	1.465	422	1.465	422
SE10	34.0	0	1.465	422	0.49	479

f_c' : compressive strength of concrete

θ : angle of the orientation of reinforcing bars to x direction of specimen

ρ_x, ρ_y : reinforcement ratio in x and y directions

f_{yx}, f_{yy} : yield strength of steel in x and y directions : ^{a} Per layer

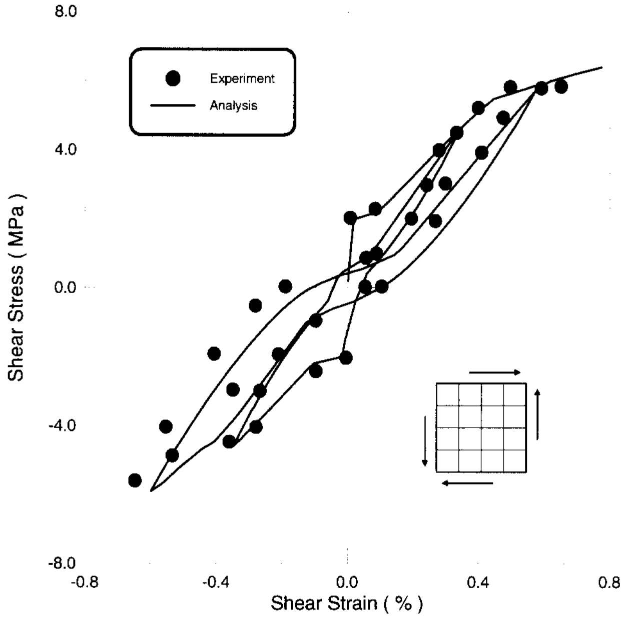


Fig. 14 Response of specimen SE8

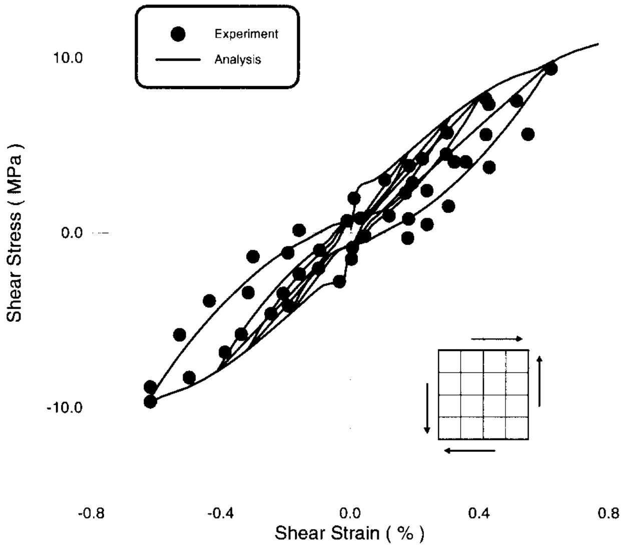


Fig. 15 Response of specimen SE9

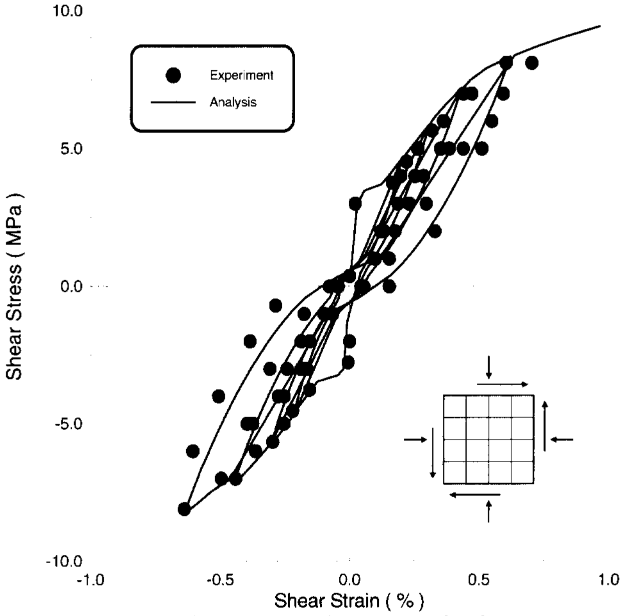
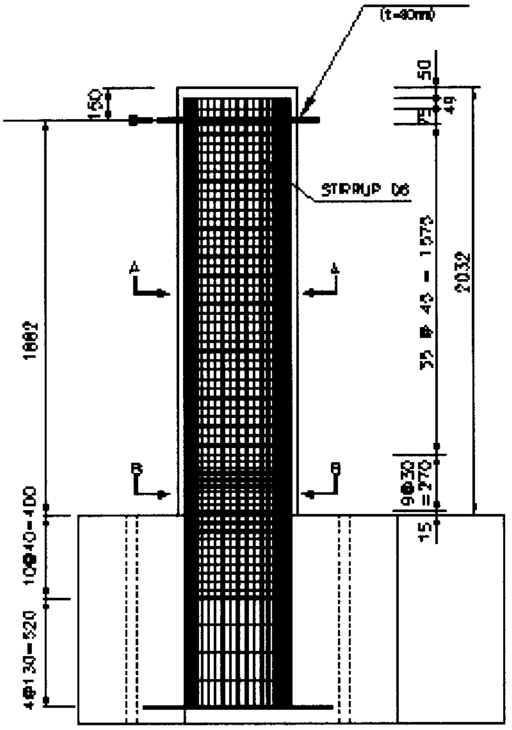


Fig. 16 Response of specimen SE10

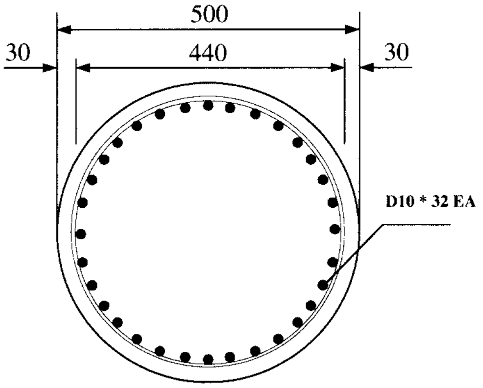
5.2 Pseudo-dynamic test

Pseudo-dynamic tests have been carried out on RC column specimens to investigate their hysteretic behavior under earthquake loading.

Test specimens had been made in circular RC column, being 1 to 3.4 scaled section of RC circular pier of Hagal bridge which had been seismically designed to specifications of the ministry of construction and transportation in Korea.(26) The section and the dimension of the pier are shown in Fig. 17. The details of the specimen are shown in Table 3.



(a) Dimensions of specimen



(b) Cross-section

Fig. 17 Detail of specimen (unit : mm)

Fig. 18 shows the set-up in which the specimen was tested. The input ground motion used in pseudo-dynamic test is specified in Fig. 19.

The circular section is divided into rectangular strips for the purpose of using plane stress element(Fig. 20). For rectangular sections, equivalent strips are calculated. Once the internal forces are calculated, the equilibrium is checked, while also considering any externally applied force that may be present.(7)

The analytical load-displacement relations are compared with test results as shown in Fig. 21. The agreements show the capability of this proposed model.

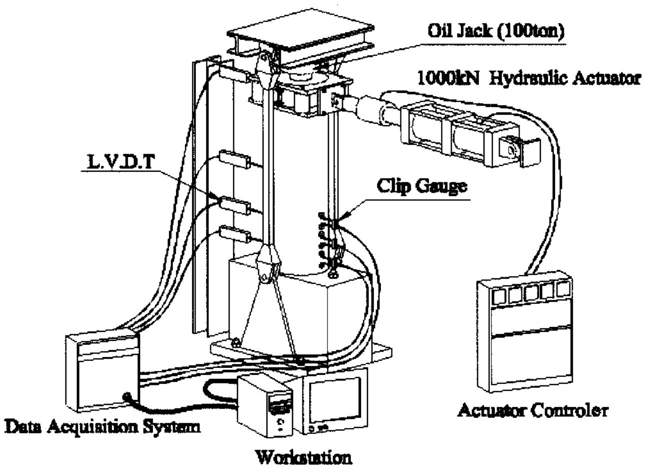


Fig. 18 Test schematic diagram

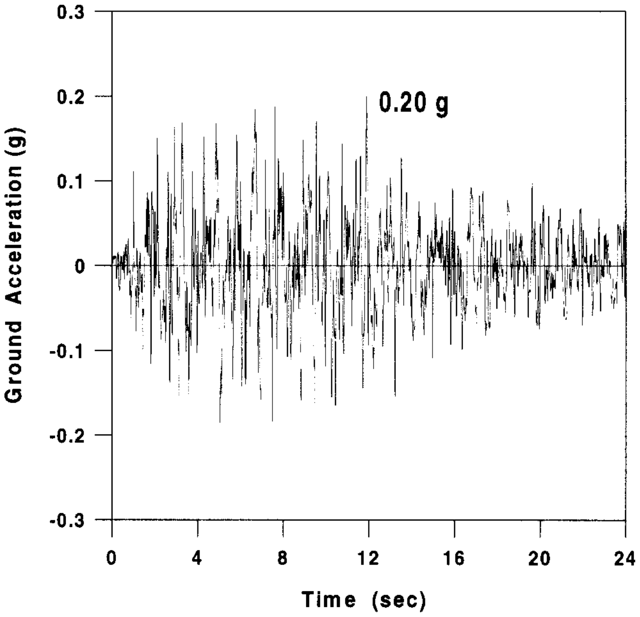


Fig. 19 T2 simulated earthquake record(26)

Table 3 Column unit details and material strengths

Aspect ratio, M/VD	$\frac{P}{f_c' A_g}$	f_c' (kgf/cm^2)	Longitudinal reinforcement		Transverse reinforcement		
			Quantity	f_{yl} (kgf/cm^2)	d_b (mm)	s (mm)	f_{yh} (kgf/cm^2)
3.8	0.1	261.0	32-D10	4,700	6	Plastic hinge zone : 30 Another zone : 45	4,400

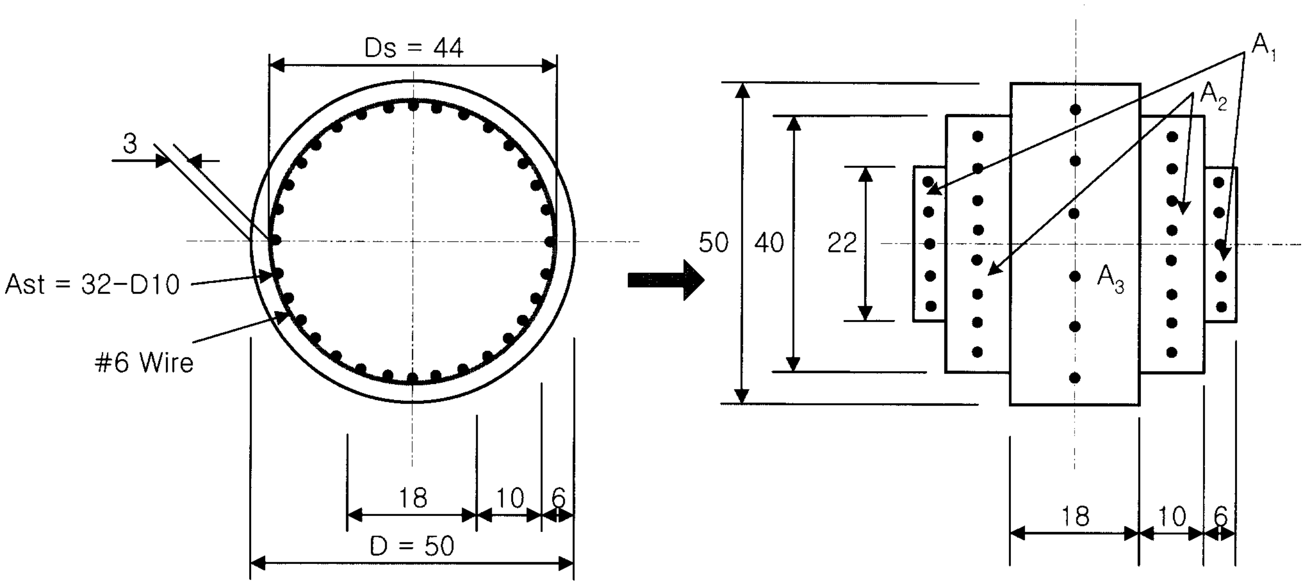


Fig. 20 Transform the circular section to an idealized equivalent rectangular section

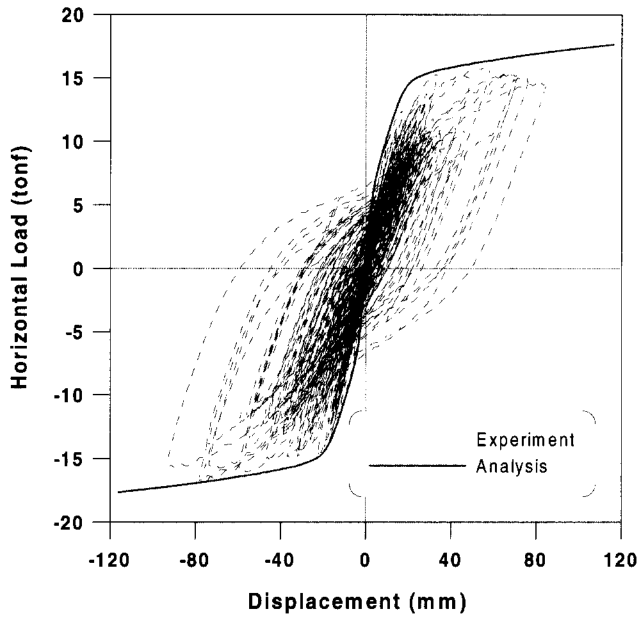


Fig. 21 Load-displacement curve

The test was continued up to the failure by increasing the amplitude of the input motion, as shown in Fig. 22 through 27. It can be seen that the results by RCAHEST trace very accurately the test results.

6. Conclusions

This integrated finite element program has been developed and it is named as RCAHEST. It is capable of static, dynamic and seismic analysis of reinforced concrete structures.

In comparison with test data, good predictions were obtained in regards to load capacities, failure modes, crack patterns, load-deformation responses of reinforced concrete structures under varied loadings.

From the results of the numerical examples and the above

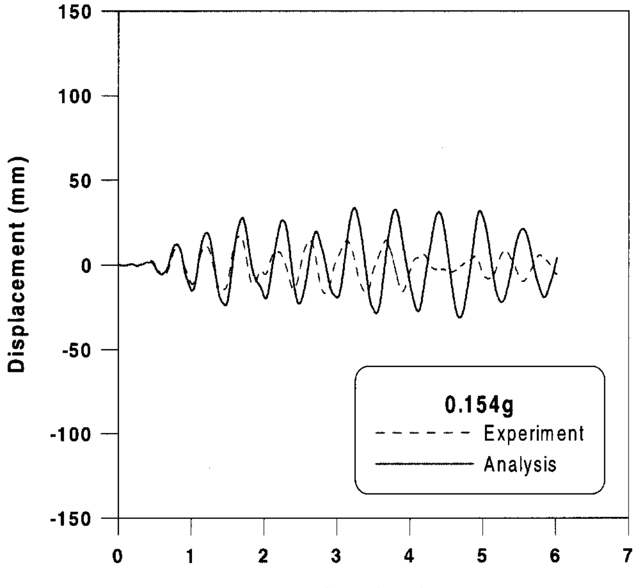


Fig. 22 Displacement response(PGA=0.154g)

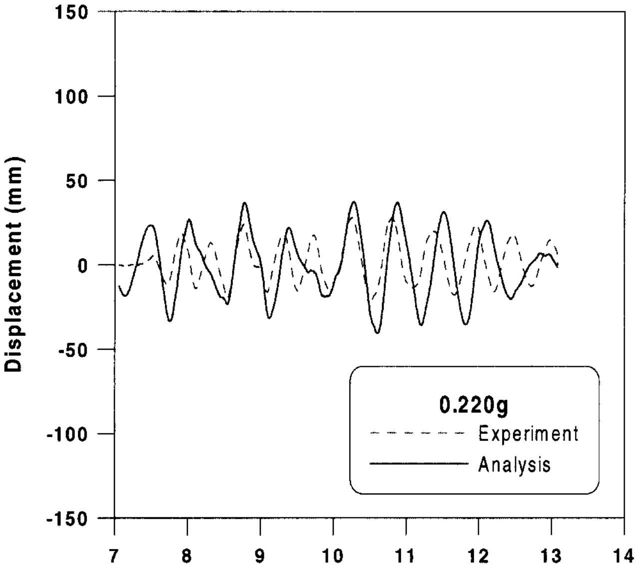


Fig. 23 Displacement response(PGA=0.220g)

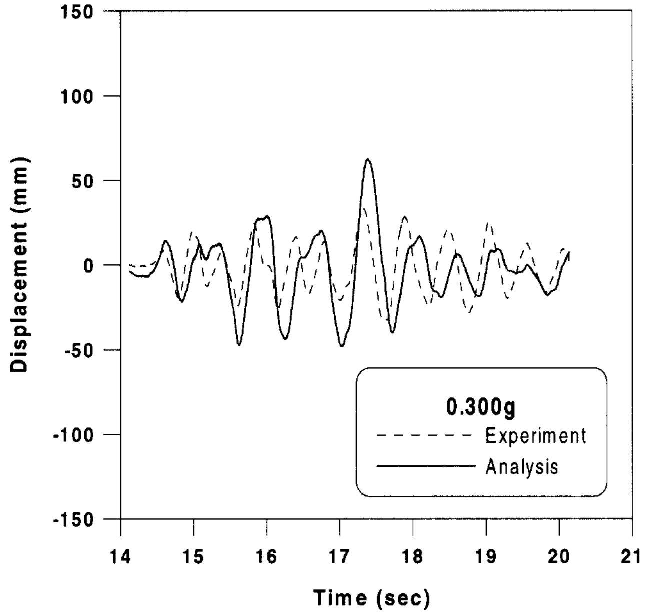


Fig. 24 Displacement response(PGA=0.300g)

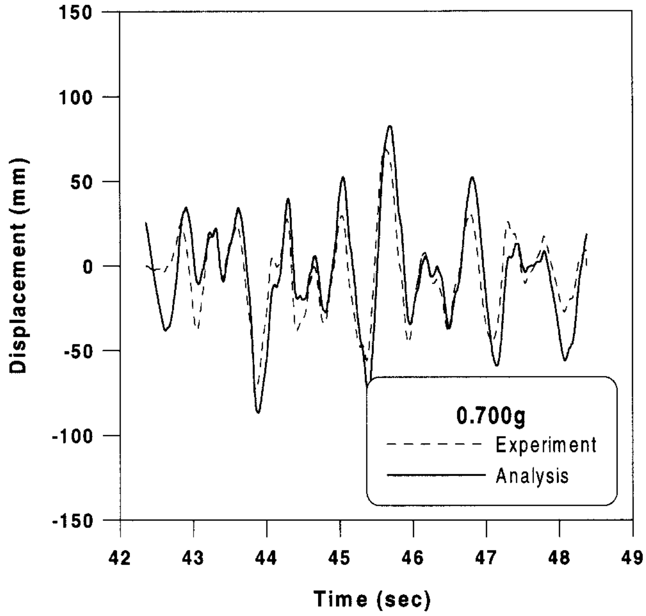


Fig. 26 Displacement response(PGA=0.700g)

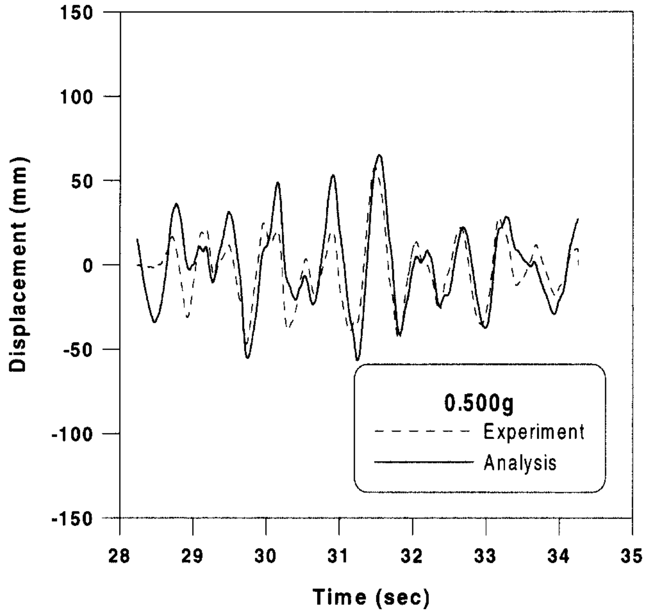


Fig. 25 Displacement response(PGA=0.500g)

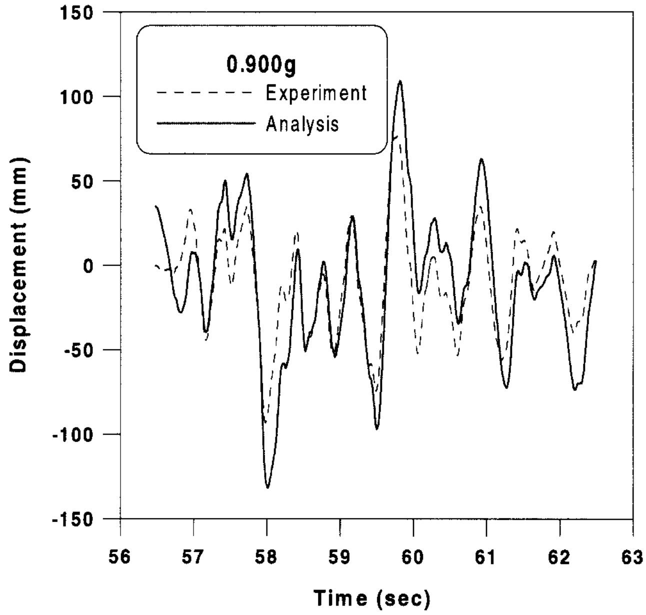


Fig. 27 Displacement response(PGA=0.900g)

discussion, the following summary of conclusions may be inferred.

1) An analytical approach of this paper could describe the overall behavior with high accuracy. The proposed formulation can be successfully used for the analysis of different types of reinforced concrete structures subjected to a variety of types of loading.
2) Nonlinear finite element analysis can be a useful tool in investigating design details or the load-deflection response of reinforced concrete structures.
3) Considering future work on the element, it is desirable that possibilities of improving the element performance in the general analysis of structures with slab, shear wall and beam-column.
4) Future work will extend the capacity of RCAHEST. Application to the prestressed concrete, three-dimensional material modeling and application to the seismic assessment of existing structures are planned as the research work to be done.

Acknowledgements

This study was supported by the Korea Science and Engineering Foundation(KOSEF) through the Korea Earthquake Research Center(KEERC), and funded by Grant No. 2000G0203. The authors wish to express their gratitude for the support received.

References

1. ASCE, "Finite element analysis of reinforced concrete structures," *Proceedings of the International Workshop*, 1993.
2. Scordelis, A. C., "Computer models for nonlinear analysis of reinforced and prestressed concrete structures," *PCI Journal*, 1984. 11-12, pp. 116-135.
3. Shin, H. M., "Finite Element Analysis of Reinforced Concrete Members Subjected to Load Reversals," Ph. D. Thesis, University of Tokyo, 1988.
4. Okamura, H. and Maekawa, K., *Nonlinear Analysis and Constitutive Models of Reinforced Concrete*, Gihodo-Shuppan, Tokyo, 1990.
5. Taylor, R. L., *FEAP - A Finite Element Analysis Program, Version 7.2 Users Manual, Volume 1 and Volume 2*, 2000.
6. Kim, T. H., Yoo, Y. H., and Shin, H. M., "Nonlinear analysis of reinforced concrete shells subjected to cyclic load," *Journal of the Earthquake Engineering Society of Korea*, EESK, Vol. 4, No. 1, 2000, pp. 35-49.
7. Kim, T. H., Yoo, Y. H., and Shin, H. M., "Inelastic analysis of circular R. C. bridge columns using equivalent transformed section," *Journal of the Korean Society of Civil Engineers*, KSCE, Vol. 20, No. 5-A, 2000, pp. 755-763.
8. Comite Euro-International Du Beton., "RC elements under cyclic loading," *State of The Art Report*, CEB, 1996.
9. Gupta, A. K. and Akbar, H., "Cracking in reinforced concrete analysis," *Journal of Structural Engineering*, ASCE, Vol. 110, No. 8, 1984, pp. 1735-1746.
10. Cervenka, V., "Constitutive model for cracked concrete," *ACI Structural Journal*, Vol. 82, No. 6, 1985, pp. 877-882.
11. Barzegar, F., "Analysis of RC membrane element with anisotropic reinforcement," *Journal of Structural Engineering*, ASCE, Vol. 115, No. 3, 1989, pp. 647-665.
12. Crisfield, M. A. and Wills, J., "Analysis of R/C panels using different concrete models," *Journal of Engineering Mechanics*, ASCE, Vol. 115, No. 3, 1989, pp. 578-597.
13. Okamura, H., Maekawa, K., and Sivasubramaniyam, S., "Verification of modeling for reinforced concrete finite element analysis of reinforced concrete structures," *ASCE*, 1985, pp. 528-543.
14. Shima, H., Chou, L., and Okamura, H., "Micro and macro models for bond behavior in reinforced concrete," *Journal of the Faculty of Engineering*, University of Tokyo (B), Vol. 39, N0. 2, 1987, pp. 133-194.
15. Shima, H. and Tamai, S., "Tension stiffness model under reversed loading including post yield range," IABSE Colloquium Delft, 1987, pp. 547-556.
16. Okamura, H., Maekawa, K., and Izumo, J., "RC plate element subjected to cyclic loading," IABSE Colloquium Delft, Vol. 54, 1987, pp. 575-590.
17. Vecchio, F. J. and Collins, M. P., "The modified compression-field theory for reinforced concrete element subjected to shear," ACI *Structural Journal*, 1986, pp. 219-231.
18. Li, B. and Maekawa, K., "Contact density model for stress transfer across cracks in concrete," *Concrete Engineering*, JCI, Vol. 26, No. 1, 1988.
19. Tamai, S., Shima, H., Izumo, J., and Okamura, H., "Average stress-strain relationship in post yield range of steel bar in concrete," *Proc. JSCE*, 378(V-6), 1987, pp. 117-129.
20. Kato, B., "Mechanical properties of steel under load cycles idealizing seismic action," *CEB Bulletin D'Information*, No. 131, 1979, pp. 7-27.
21. Kim, T. H., Yoo, Y. H., and Shin, H. M., "Analytical study on inelastic behavior and ductility capacity of reinforced concrete bridge columns subjected to seismic load," *Journal of the Earthquake Engineering Society of Korea*, EESK, Vol. 4, No. 4, 2000, pp. 37-51.
22. Kim, T. H. and Shin, H. M., "Analytical study on discontinuous displacement in reinforced concrete column-footing joint under earthquake," *Journal of the Korea Concrete Institute*, KCI, Vol. 12, No. 6, 2000, pp. 83-90.
23. Sheikh, S. A. and Uzumeri, S. M., "Strength and ductility of tied concrete columns," *Journal of Structural Division*, ASCE, Vol. 106, No. 5, 1980, pp. 1079-1102.
24. Mander, J. B., Priestley, M. J. N., and Park, R., "Theoretical stress-strain model for confined concrete," *Journal of Structural Engineering*, ASCE, Vol. 114, No. 8, 1988, pp. 1804-1826.
25. Steven, N. J., Uzumeri, S. M., and Collins, M. P., "Reinforced concrete subjected to reversed cyclic shear - experiments and constitutive model," ACI *Structural Journal*, 1991, pp. 135-146.
26. Cho, C. B., So, J. H., Park, J. H., and Chung, Y. S., "Seismic performance of circular reinforced concrete bridge piers by pseudo-dynamic test," *Proceedings of the Korea Concrete Institute*, KCI, Vol. 12, No. 1, 2000, pp. 464-469.