Evaluation of Proper Supplemental Damping for a Multi-Story Steel Frame Using Capacity Spectrum Method

능력스펙트럼법을 이용한 다층 철골조 건물의 적정 감쇠기 산정

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국문요약

본 연구에서는 능력스펙트럼법을 이용하여 성능목표를 만족하기 위하여 필요한 점성 감쇠기의 양을 간단하고 직접적인 방법으로 산정하는 방법에 관하여 연구하였다. 먼저 능력스펙트럼법을 이용하여 구조물의 비탄성 응답을 구하고 구조물의 응답과 목표변위의 차이를 이용하여 필요한 유효감쇠비를 구하였다. 그리고 이러한 유효감쇠비를 이용하여 필요한 점성감쇠기의 양을 산정하였다. 본 연구에서 제안한 방법의 타당성을 검증하기 위해 10층의 철골조 건물에 세 가지 유형의 층지진하중을 가하고 제안된 절차에 따라 필요한 감쇠기의 양을 구하였다. 해석결과에 따르면 제안된 방법에 의하여 설계된 점성 감쇠기를 해석모델에 설치하고 시간이력 해석을 수행한 결과 최대응답은 목표변위와 잘 일치함을 발견하였다.

주요어 : 점성 감쇠기, 능력스펙트럼법, 유효감쇠비, 성능에 기초한 내진설계

ABSTRACT

In this study, a simple and straightforward procedure is developed using capacity spectrum method to obtain the required amount of viscous dampers in order to meet given performance objectives. To this end nonlinear static response of a structure was obtained using capacity spectrum method first, and the required effective damping ratio was computed from the difference between the analysis result and the target displacement. Then, the amount of required viscous damping was obtained using the effective damping ratio. Three different types of seismic story force were considered. The procedure was applied to a 10-story steel frame with added viscous dampers. According to the earthquake time history analysis results, the maximum displacement of the model structure with the added viscous damping determined from the proposed method corresponds well with the target displacements.

Key words : viscous dampers, capacity spectrum method, equivalent viscous damping, performance based seismic design

1. Introduction

Performance-based seismic design provides insight about the actual performance of structures during earthquakes, and requires engineers to design a structure to meet performance objectives for multi-level seismic loads. For performance-based seismic design the capacity of a structure to resist seismic loads needs to be evaluated first. This can efficiently be carried out by the capacity spectrum method(CSM). It is basically an approximate method with the following significant simplifications: The first is the transformation of the response of a multi-degree-of-freedom(MDOF) structure to that of an equivalent single-degree-of-freedom(SDOF) system. The second is the calculation of the response of the inelastic SDOF system based on information extracted from elastic response spectrum.

To enhance seismic performance of a structure ATC-40[1] and FEMA-273[2] propose technical strategies which include such approaches as increasing strength, altering stiffness, and reducing demand by employing base isolation and energy dissipation devices. Specifically the energy dissipation devices directly increase the ability of the structure to dampen earthquake response. They are especially effective in the velocity sensitive region of a response spectrum corresponding to the natural period between about 0.5 to 2 seconds, in which maximum acceleration of a structure decreases significantly as the damping in the structure increases.

There are a number of different supplemental passive energy dissipation devices developed, each of which has different force-displacement and force-velocity relationships. Consequently, the selection and design of supplemental dampers should be based on the dynamic characteristics of the structure, including its mass and stiffness, the effective damping desired to satisfy the performance levels of the structure, etc. With these informations, the preliminary determination of the required damper size and the damping force is possible. Once the size of the dampers is estimated, the adequacy of the structure for the given seismic loads can be checked by dynamic analysis. However the nonlinear

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본 논문에 대한 토의를 2001년 6월 30일까지 학회로 보내 주시면 그 결과를 게재하겠습니다.

dynamic analysis of structural systems with supplemental dampers requires a lot of computation time, and a series of iterative process should be employed to find out an appropriate number of dampers to meet the required performance objectives.

Recently a simple procedure based on CSM was developed by the authors in a single-degree-of-freedom system to obtain the amount of supplemental viscous dampers required to satisfy the given performance objectives without iterative process.[3] In their study a parametric study was carried out for various natural periods, strain hardening ratios, ratio of elastic strength demand and yield strength. The results of the parametric study indicated that with the addition of the supplemental damping evaluated by the proposed method, the performance of the model structures were well restrained within the target points for most of the cases considered, and that the proposed process could be a potential alternative for performance-based design of structures with supplemental dampers in a single-degree-of-freedom system.

In this study the direct procedure is further extended to multi-degree-of-freedom system to verify the applicability of the method. Three different types of seismic story forces were applied in the pushover process. After the required supplemental damping was estimated in an equivalent SDOF system, it was redistributed to each story of the MDOF structure. Finally nonlinear dynamic time history analysis were carried out for the structure with the added dampers to evaluate the preciseness of the proposed method. For simplicity, the viscous dampers were modeled by a linear dashpot(Fig. 1).

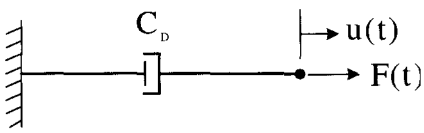


Fig. 1 Modeling of viscous dampers

2. Seismic performance evaluation by capacity spectrum method

2.1 Formation of a pushover curve

The relationship between the base shear and the top story displacement, which is generally called pushover curve or capacity curve, is obtained by gradually increasing the lateral loads appropriately distributed over the stories. There are many alternatives for the distribution of the lateral load. In this study the capacity curve was obtained by applying three different types of lateral seismic loads:

1) Forces are applied in proportion to the product of story masses and the fundamental mode shape of the elastic model structure.[1]

$$F_i = \frac{m_i \phi_{i1}}{\sum_{i=1}^{N} m_i \phi_{i1}} V \tag{1}$$

where F_i is the seismic story force in the i th floor, m_i is the mass of the i th floor, ϕ_{i1} is the i th component of the mode shape vector for the fundamental mode, V is the base shear, and N is the number of floors.

2) The effect of the higher modes are considered by means of combination of the story force of all modes using the SRSS method.[4]

$$F_i = \sqrt{\sum_{j=1}^{N} \left(\left[\frac{\sum_{i=1}^{N} m_i \phi_{ij}}{\sum_{i=1}^{N} m_i \phi_{ij}^2} \right] \phi_{ij} Sa_j m_i \right)^2} \tag{2}$$

where ϕ_{ij} is the i th component of the j th mode shape vector and S_{aj} is the spectral acceleration corresponding to the j th mode.

3) Equivalent mode shape considering modal participation factors is utilized.[5]

$$F_i = \frac{m_i \overline{\phi_i}}{\sum_{i=1}^{N} m_i \overline{\phi_i}} V \tag{3}$$

$$\Gamma_j = \frac{[\sum_{i=1}^{N} m_i \phi_{ij}]}{[\sum_{i=1}^{N} m_i \phi_{ij}^2]} \qquad \overline{\phi_i} = \sqrt{\sum_{i=1}^{N} (\phi_{ij} \Gamma_j)^2} \tag{4}$$

The first method can be properly applied to a structure in which the first mode dominates the dynamic motion. The second and third methods have advantage that higher mode effect can be included in the estimation of seismic lateral load.

2.2 Conversion to ADRS spectra

Application of the capacity spectrum technique requires that both the demand spectra and structural capacity curve be plotted in the spectral acceleration vs. spectral displacement domain, which is known as acceleration-displacement response spectra(ADRS). To convert a response spectrum from the standard S_a vs. T format to ADRS format, it is necessary to determine the value of S_d for each point on the curve, S_a and T. This can be done with the equation.

$$S_d = (T^2/4\pi^2) S_a g \tag{5}$$

In order to develop the capacity spectrum from the capacity curve, it is necessary to do a point by point conversion to the first mode spectral coordinates. Any point V, Δ_R on the capacity curve is converted to the corresponding point S_a and S_d on the capacity spectrum using the equations.

$$S_a = \frac{V}{M_1^*} \qquad S_d = \frac{\Delta_R}{\Gamma_1 \phi_{R1}} \tag{6}$$

where Γ_1 is the modal participation factor for the first natural mode of the structure and ϕ_{R1} is the roof level amplitude of the first mode. The mathematical expressions for the modal mass coefficient M_1^* is as follows.

$$M_1^* = \frac{\left(\sum_{j=1}^{N} m_j \phi_{j1}\right)^2}{\sum_{j=1}^{N} m_j \phi_{j1}^2} \tag{7}$$

The general process of obtaining structural responses using CSM is described in Fig. 2.

2.3 Estimation of equivalent viscous damping ratio

A bilinear representation of the capacity spectrum, as described in Fig. 3, is needed to estimate the effective

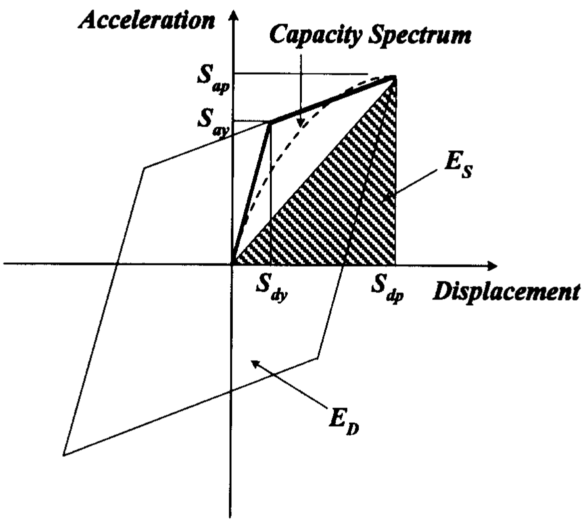
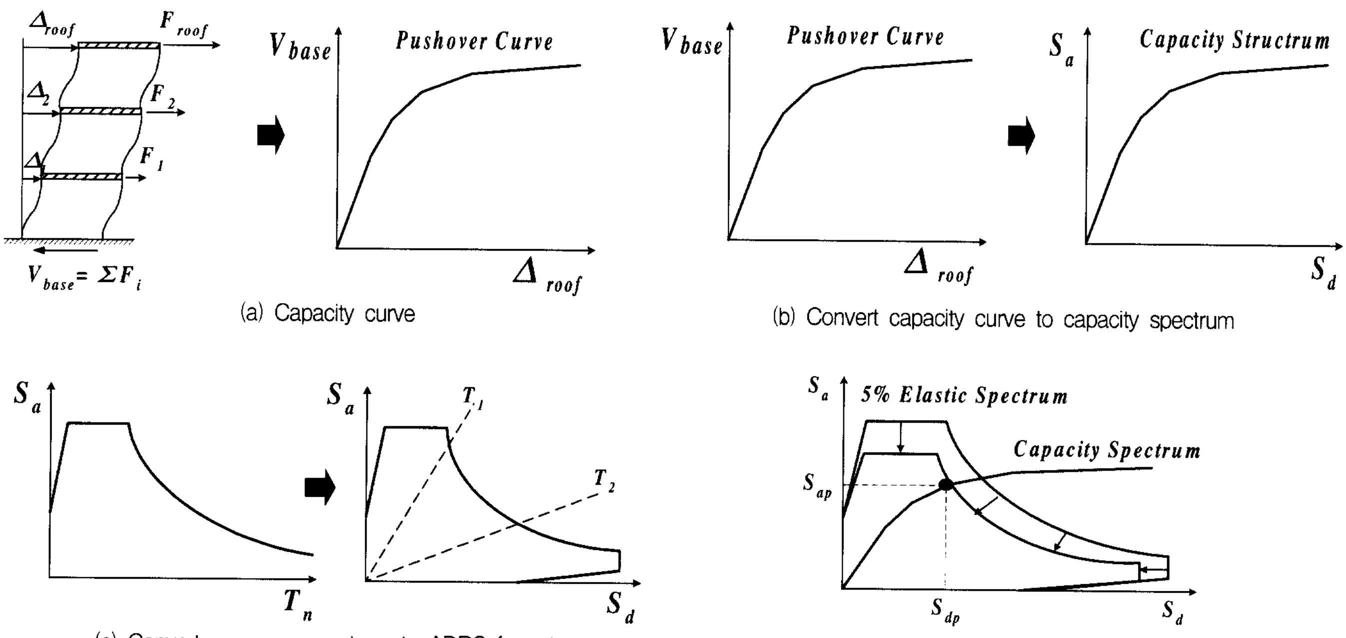


Fig. 3 Estimation of equivalent damping ratio

damping and appropriate reduction of spectral demand. ATC-40 recommends that the area under the original capacity curve and the equivalent bilinear curve be equal so that the energy associated with each curve is the same. When the bilinear system undergoes inelastic action at displacement S_{dp} with corresponding acceleration equal to S_{ap}, the effective period, T_{eff}, is determined from the secant (or effective) stiffness at maximum displacement.

$$T_{eff} = 2\pi \sqrt{\frac{S_{dp}}{S_{ap}}} \tag{8}$$

The equivalent viscous damping ratio for the yielding structure is determined from the energy dissipated by the



(a) Capacity curve

(b) Convert capacity curve to capacity spectrum

(c) Convert response spectrum to ADRS format

(d) Performance point

Fig. 2 Process of seismic capacity evaluation using capacity spectrum method hysteretic behavior, E_D, which is the area enclosed by the hysteresis loop at maximum displacement, and the stored potential energy corresponding to the area of the shaded triangle, E_S.[(1),(2),(6)]

$$\beta_{eq}=\frac{1}{4\pi}\frac{E_D}{E_S}=\frac{2(S_{ay}S_{dp}-S_{dy}S_{ap})}{\pi S_{ap}S_{dp}} \tag{9}$$

If the inherent viscous damping of the structure is assumed to be β, then the effective damping ratio of the system can be obtained as[(1),(2),(6)]

$$\beta_{eff}=\beta+\kappa\,\beta_{eq} \tag{10}$$

where κ is called the efficiency factor or damping modification factor which is equal to the actual area enclosed by the hysteresis loop divided by the loop area of the corresponding perfect bilinear hysteretic system. The factor is selected on the basis of experience with particular structural systems. In this study the factor is taken to be 1.0 because perfect bilinear system is assumed in the analysis.

When additional energy dissipation devices are added, the effective viscous damping ratio becomes[(2),(6)]

$$\beta_{eff}=\frac{1}{4\pi}\frac{E_{DS}+E_{DE}}{E_S}+\beta$$
$$=\frac{E_{DE}}{2\pi m S_{ap}S_{dp}}+\frac{2\kappa(S_{ay}S_{dp}-S_{dy}S_{ap})}{\pi S_{ap}S_{dp}}+\beta \tag{11}$$

where E_{DS} is the energy dissipated by the dampers, and the first term in the right hand side is the additional damping ratio contributed from the added dampers. The energy dissipated from the added viscous dampers for a cycle of harmonic motion can be computed as follows

$$E_{DE}=\int f_D\,du=\int_0^{2\pi/\omega}(c\,\dot{u})\,\dot{u}\,dt$$
$$=\pi c\omega\,u_o^2=2\pi\beta_v\frac{\omega}{\omega_n}k\,u_o^2 \tag{12}$$

where

$$\beta_v=\frac{c\,T_e}{4\pi m} \tag{13}$$

If Eq. (12) is substituted to Eq. (11) and solved for β_v, then the damping contributed from the devices can be obtained as follows after some manipulations,

$$\beta_v=\left(\beta_{eff}-\beta-\frac{2\kappa(S_{ay}S_{dp}-S_{dy}S_{ap})}{\pi S_{ap}S_{dp}}\right)\frac{T_e}{T_{eff}} \tag{14}$$

where T_e is the initial elastic stiffness.

3. Distribution of supplemental damping in MDOF system

In a SDOF system the additional damping required to achieve the target displacement can simply be added to the system. In a multi-story structure, however, the damping should properly be distributed over the stories. To this end the following relation between the viscous damping ratio and the dissipated and stored energy is used[(2),(7)]

$$\beta_{device}=\frac{E_{DE}}{4\pi E_S} \tag{15}$$

where E_{DE} is the energy dissipated by the dampers and E_S is the maximum strain energy stored in the structure. If the dampers are placed as diagonal members with the inclination of θ as shown in Fig. 4, then the energy dissipated by the dampers are formulated as follows

$$E_{DE}=\frac{2\pi^2}{T}\sum_j C_j\cos^2\theta_j\,\Delta_{ij}^2 \tag{16}$$

where, T is the fundamental natural period, C_j is the damping coefficient of the j th story, and Δ_{ij} is the relative displacement between i th and j th story. By substituting Eq. (16) into Eq. (15) and using the modal displacement instead of the displacement response, the required damping of the dampers can be obtained as

$$\beta_{device}=\frac{T\sum_i C_i\cos^2\theta_i(\phi_i-\phi_{i-1})^2}{4\pi\sum_i m_i\phi_i^2} \tag{17}$$

where m_i is the mass of the i th story and ϕ_i is the modal displacement at floor level i. If it is assumed that the same amount of dampers are used in every story, the damping coefficient of the damper in the i th story, C_i, can be easily computed from Eq. (17).

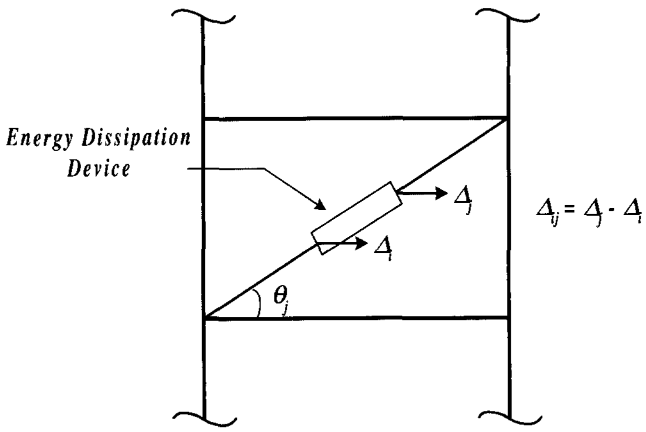


Fig. 4 Relative displacement of the damper

4. Application to a multi-story structure

4.1 Model structure

The model structure for analysis, shown in Fig. 5, is a 10-story steel framed structure designed to resist gravity and wind loads. For gravity load uniform dead load of 540kgf/cm^2 and live load of 250kgf/cm^2 were applied throughout the stories. Basic wind speed of 35m/sec was used for lateral static wind load. The yield stress of the structural steel is 2.4tonf/cm^2 and 3.3tonf/cm^2 for beams and columns, respectively. The structural analysis and design was carried out using the program MIDAS GEN.[8] The size of each selected structural member is tabulated in Table 1, and the dynamic modal characteristics are shown in Table 2.

Fig. 6 shows the mode shapes obtained for the three methods for lateral load distribution mentioned above, where it can be found that the mode shape obtained from SRSS and the equivalent mode shape is almost identical. However Fig. 7 indicates that the story forces resulting from the SRSS method are higher than those from other methods in the lower stories but smaller in the higher stories.

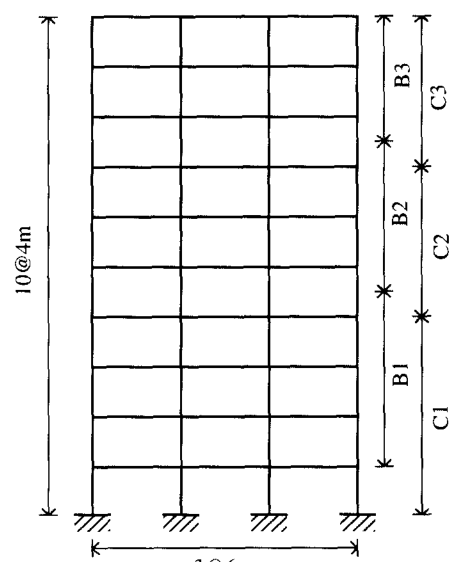


Fig. 5 10-story model structure

Table 1 Sectional properties of members

Columns (Beams)	Columns		Beams
	Interior columns	Exterior columns	
C1(B1)	H400×400×13×21	H344×348×10×16	H400×200×8×13
C2(B2)	H350×350×12×19	H300×300×10×15	H396×199×7×11
C3(B3)	H344×354×16×16	H298×299× 9×14	H350×175×7×11

Table 2 Dynamic characteristics of the model structure

Mode	1	2	3	4	5
Period(sec)	1.41	0.49	0.28	0.19	0.14
Period ratio(T_1/T_m)	1.00	2.87	4.98	7.49	10.14
Modal participation factor	1.34	0.51	0.30	0.22	-0.16
Effective mass(%)	77.54	11.68	4.28	2.23	1.53

4.2 Design spectrum and earthquake time history record

Site-specific elastic design response spectrum was constructed as shown in Fig. 8 in accordance with Fig. 16-3 of UBC-97 with the values C_a and C_v as suggested in the Seismic Design Guidelines II[9] for earthquakes with recurrence period of 2,400 years and soil type of S_E. Based on the design spectrum time history record shown in Fig. 10 was generated using the program SIMQKE[10] for comparison of the static procedure with the nonlinear dynamic time history analysis. The response spectrum constructed from the time history record generated from the design spectrum are plotted in Fig. 9 along with the design spectrum.

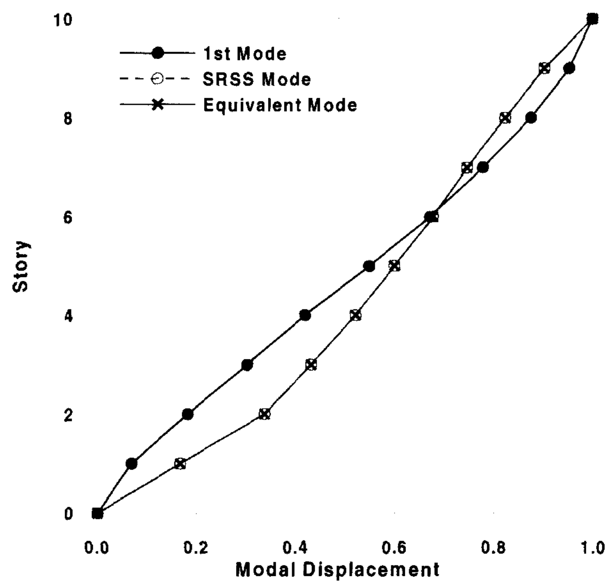


Fig. 6 Mode shapes obtained from the three different methods

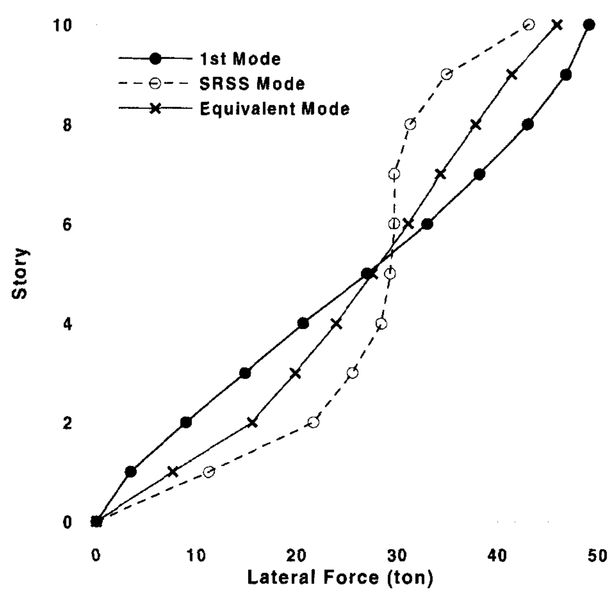


Fig. 7 Seismic story forces

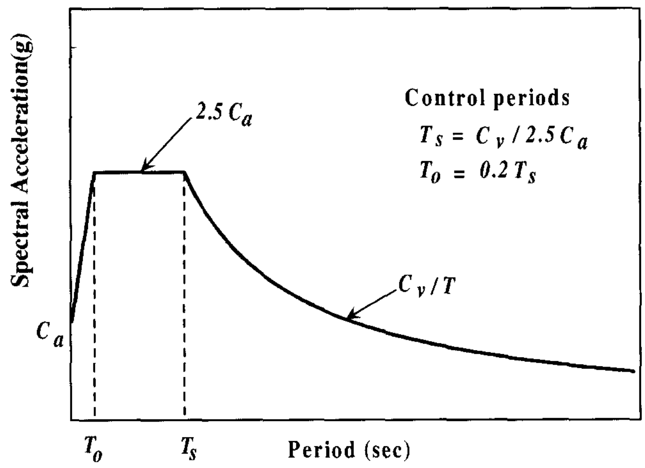


Fig. 8 Design spectra

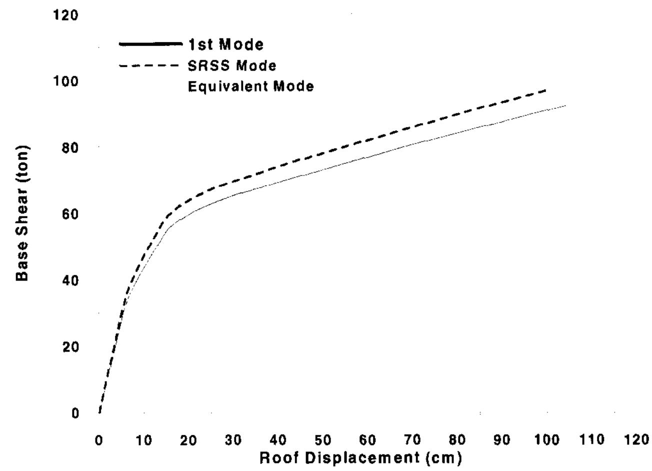


Fig. 11 Pushover curves

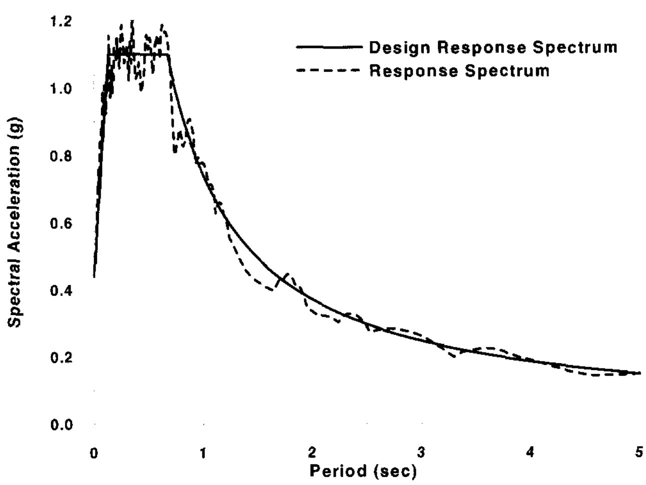


Fig. 9 Response spectrum of artificial ground motion

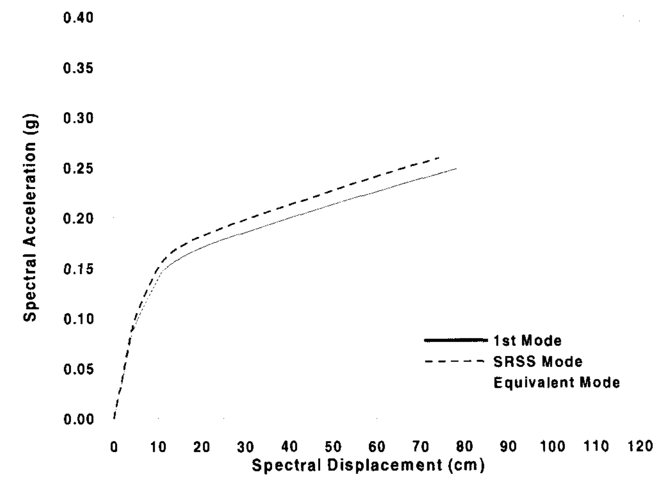


Fig. 12 Capacity spectra

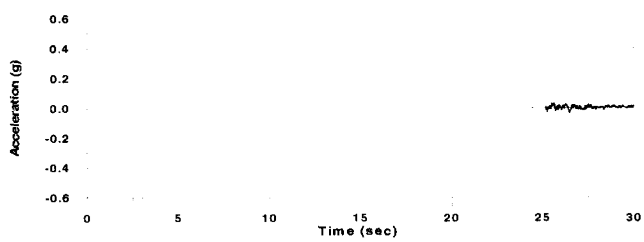


Fig. 10 Artificial ground excitation generated from the design spectrum

4.3 Capacity spectrum

The base shear-top story displacement relationship shown in Fig. 11 was obtained with the lateral static load gradually increased until the top story displacement reached 4% of the total structure height. In this study, pushover analysis was carried out using DRAIN-2D+[11] program. Then this force-displacement relationship was transformed into the capacity spectrum of an equivalent SDOF system in ADRS format using the procedure mentioned previously. The results are plotted in Fig. 12.

4.4 Estimation of performance point

To estimate the performance points the capacity spectrum and the demand spectra with various damping ratios are plotted simultaneously in ADRS format. Fig. 13 to 15 and Table 3 to 5 present the process of estimation of the performance points for each lateral story force distribution method.

Table 6 shows the displacement and acceleration obtained for the equivalent SDOF system and the displacement and base shear for the original MDOF structure induced from the results of the SDOF system using Eq. (6). The results

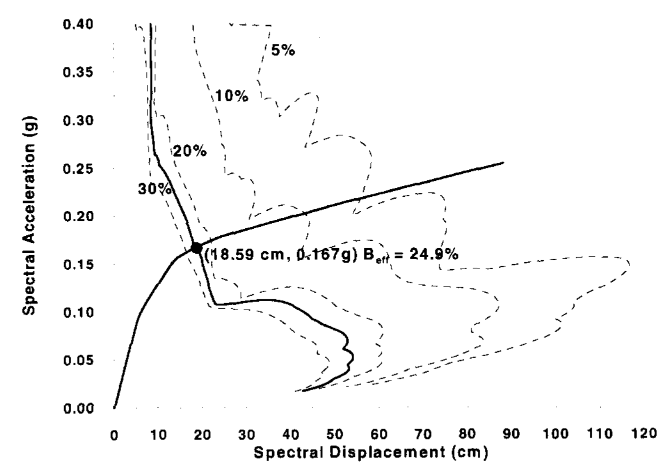


Fig. 13 Estimation of performance point(only first mode considered)

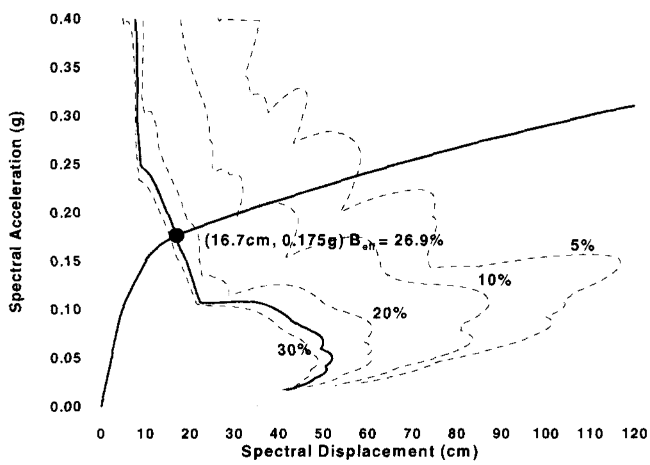


Fig. 14 Estimation of performance point(SRSS mode combination)

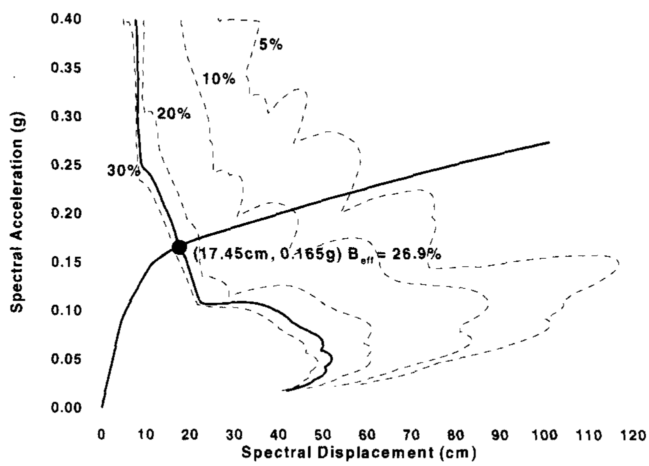


Fig. 15 Estimation of performance point(equivalent mode considered)

Table 3 Evaluation of the performance points considering the 1st mode (unit : cm)

Damping ratio	S_{di}	S_{ai}	β_{eff}
5	55.80	0.217	37.4
10	43.25	0.202	37.0
20	21.90	0.174	28.6
30	16.30	0.162	21.3
:	:	:	:
24.9	**18.59**	**0.167**	**24.9**

Table 4 Evaluation of the performance points considering SRSS mode combination (unit : cm)

Damping ratio	S_{di}	S_{ai}	β_{eff}
5	57.45	0.238	37.3
10	33.80	0.203	37.2
20	21.05	0.183	31.8
30	15.62	0.173	25.0
:	:	:	:
26.9	**16.72**	**0.175**	**26.9**

Table 5 Evaluation of the performance points considering equivalent mode (unit : cm)

Damping ratio	S_{di}	S_{ai}	β_{eff}
5	56.15	0.221	36.8
10	43.25	0.202	37.6
20	21.91	0.174	31.3
30	16.32	0.162	25.6
:	:	:	:
26.9	**17.45**	**0.165**	**26.9**

Table 6 Structural responses for each method

Method	Performance points		MDOF system		Effective damping (β_{eff})
	Displacement (cm)	Acceleration (g)	Displacement (cm)	Base shear (tonf)	
1st mode	18.59	0.167	24.85	56.70	24.9
SRSS combination	16.72	0.175	23.19	66.07	26.9
Equivalent mode	17.45	0.165	24.23	62.29	26.9

from the three different lateral story forces were also compared in the table, where it can be seen that the top story displacement obtained from the lateral load by SRSS combination of the mode shapes is the lowest while the base shear is the largest. The opposite is true for the lateral loads constructed from the fundamental mode shape of the structure. The results from the equivalent mode are placed in the middle.

4.5 Comparison with the results from time history analysis

To perform nonlinear time history analysis, the behavior of model structure was simulated by using the Beam-Column element of the DRAIN-2D+ program. It was assumed that hinge rotation takes place at the point-plastic hinge at element ends. Fig. 16 and 17 represent the maximum inter-story drift and the maximum story displacements of the model structure obtained from the nonlinear static analysis and the nonlinear time history analysis. According to the results the displacements predicted from the SRSS mode combination for lateral load forms upper bound in lower stories, and forms lower bound in upper stories compared with those predicted by the other two forms of static lateral loads. This can be expected considering the shape of the static lateral load. It also can be noticed that the maximum story displacements predicted by CSM with three types of lateral load distribution are lower than those computed by time history analysis. However for inter-story displacements, although the results from the static nonlinear analysis are smaller than those from dynamic analysis in the lower stories, the opposite is true in the upper stories.

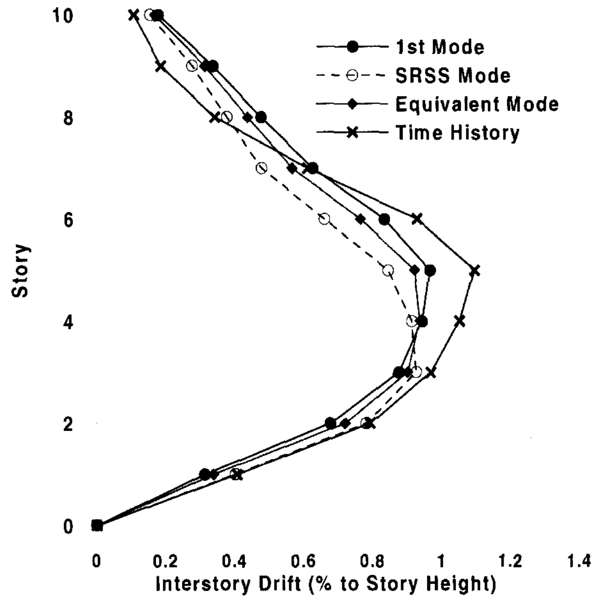


Fig. 16 Maximum interstory drift from each method

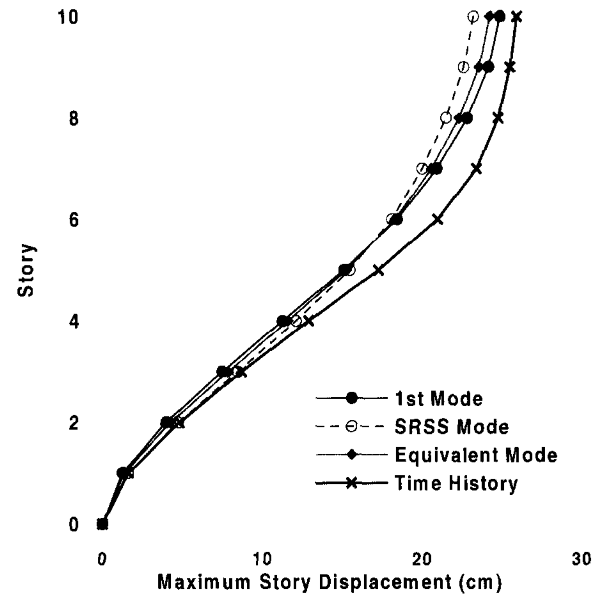


Fig. 17 Maximum story displacements from each method

The inter story drifts shown in the figure indicate that the structure satisfies the collapse prevention limit state prescribed in the reference[9], which is the maximum interstory drift of 2.5% of the story height, but does not satisfy the functional limit state, which is 0.5% of the story height, except at the top two stories.

4.6 Distribution of supplemental damping

The target displacement is set to be 0.5% of the structure height, which is 20cm. It can be seen in Fig. 17 that approximately 5cm of roof displacement needs to be reduced by installing viscous dampers to limit the maximum displacements within the target value. To find out the portion of the damping that should be provided by the dampers the required effective damping β_{eff} corresponding to the target displacement is obtained first in the acceleration-displacement response spectrum. Then the amount of the additional viscous damping ratios required to restrain the top-story displacement within the performance limit can be computed using Eq. (14) and are presented in Table 7 along with the responses at the performance points, the effective damping ratios and the effective periods for each lateral loading pattern. Finally, assuming that the same dampers are used throughout the story, the damping coefficients of the supplemental dampers were evaluated using Eq. (17), which are $C_{1st} = 5100$kgf · sec/cm, $C_{SRSS} = 4956$kgf · sec/cm and $C_{eq} = 5129$kgf · sec/cm for the three story force distribution methods, respectively. The results indicate that the damping coefficients derived for each lateral story force pattern are very close, which demonstrates that the amount of the supplemental damping

Table 7 Supplemental damping ratio required to meet the performance limit

Method	Performance points		Effective damping (%)	Effective period (sec)	Required damping (%)
	Displacement (cm)	Acceleration (g)			
1st mode	14.96	0.159	35.0	1.946	21.6
SRSS combination	14.41	0.169	33.4	1.852	21.5
Equivalent mode	14.41	0.158	35.5	1.921	22.2

required to meet the given performance objectives does not depend significantly on the pattern of the lateral force for the pushover analysis. The maximum top story displacement of the model structure with the added damping computed by dynamic time history analysis turned out to be 16.44cm, which is about 3.5cm short of the target value.

5. Conclusion

In this study seismic performance of a multi-story steel frame was evaluated by capacity spectrum method and a simple procedure was proposed to determine the amount of supplemental viscous damping required to maintain the maximum response within a given target value. The results of the research show that with the addition of the supplemental damping evaluated by the proposed method the performance of the model structures are well restrained within the target point. Therefore it is concluded that the proposed process, which provides the amount of the required additional damping directly without carrying out time-consuming nonlinear dynamic analysis, can be a potential alternative for performance-based design of structures with supplemental dampers.

Acknowledgement

This research is funded by the Korea Science and Engineering Foundation under Grant No. 1999-1-310-001-3. This financial support is gratefully acknowledged.

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