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Multiobjective Decision-Making applied to Ship Optimal Design

Li-Zheng Wang[1], Rong-Fei Xi[1] and Cong-Xi Bao[1]

[1]Wuhan Transportation University, Wuhan, P. R. China; E-mail: wlzg@public.wh.hb.cn

Abstract

Ship optimal design is a multi-objective decision-making process and its optimal solution does not exit in general. It is a problem in which the decision-maker is very interested that an effective solution is how to be found which has good characteristic and is substituted for optimal solution in a sense. In the previous method of multi-objective decision-making, the weighting coefficients are decided from the point of view of individualswhich have a bit subjective and unilateral behavior. In order to fairly and objectively decide the weighting coefficients, which are considered to be optimal in all system of multi-objective decision-making and satisfactory solution to the decision-maker, the paper presents a method of applying the Technology of the Biggest Entropy. It is proved that the method described in the paper is very feasible and effective by means of a practical example of ship optimal design.

Keywords: multiobjective decision-making, biggest entropy, variable tolerate error, optimal ship design

1 Introduction

Ship optimal design is a multi-objective decision-making process and its optimal solution does not exit in general. It is a problem in which the decision-maker is very interested that an effective solution is how to be found which has good characteristic and is substituted for optimal solution in a sense. In a multi-objective decision-making process, the Utility of the decision-making target used to be decided by means of The Utility Theory. Because the Utility had a definite concept only when all thing would be recognized and it is difficult that people describe the concept using very exact quantity, so far The Utility Theory is rarely to be used in practice.

A multi-objective decision-making problem is a mathematical description of the system of objective matter. So far as the system of objective matter is concerned, when it is abstracted as mathematical model or it is quantitatively described its regular is definite, that is to say that there is the objective criterion of evaluation as to system. In the previous methods of multi-objective decision-making, the weighting coefficients are decided from the point of view of individual-swhich have a bit subjective and unilateral behavior. In order to fairly and objectively decide the weighting coefficients, which are considered as optimal solution in all system of multi-objective decision-making and satisfactory solution to the decision-maker, the paper presents a method of applying the Technology of the Biggest Entropy, by which the weighting coefficients are decided. According to the principle of Biggest Entropy, the weighting coefficients can be decided by means of the information given by the target function in the system of ship tech-economic evaluation, using the concept of "Entropy", then the solution of a multi-objective problem is transferred to a single objective problem. So the optimal solution is obtained by means of optimal method of single objective. The principle and application of the Technology of the Biggest Entropy will be described as follows.

2 The basic principle of the technology of the biggest entroy

(Chenting 1986, Gu 1991, Durier 1988)

As far as a problem of multi-objective decision-making (MP) is concerned, it is described as follows:

$$\min F(X) = \{f_1(X), f_2(X), \cdots, f_n(X)\}$$
$$X \in R, \quad R = \left\{ x \left\| \begin{array}{ll} g_i(x) \leq 0, & i = 1, 2, \cdots, l \\ h_i(x) \leq 0, & j = 1, 2, \cdots, m \end{array} \right. \right\} \tag{1}$$

If $\forall X \in R$ and $F(X) \geq F(X_0)$, So X_0 is considered as an optimal solution of (MP). If $X \in R$ and $F(X) \leq F(x_0)$ is not exist, X_0 is an effective solution of (MP). By using weighting coefficients, a problem of multi-objective decision-making process (MP) can be transferred to a single objective problem (WP). It follows that

$$\min \sum_{i=1}^{n} \omega_i f_i(X)$$
$$X \in R$$
$$\sum_{i=1}^{n} \omega_i = 1, \omega_i > 0,$$

Then the solution of (MP) is a normal effective solution of (MP). It is proved that any normal effective solution of (WP) is effective solution of (MP). Supposing that the set of all solutions of (WP) is E and the set of effective solutions of (MP) is M, there is E ⊂ M obviously.

Entropy of system is the metric of indefinite things from the point of view of Information Science. If the Entropy of system is great , the system is disordered. Conversely the system is orderly. Suppose that in the experiment of probability there are possible results such as $X_1, X_2, \cdots, X_n$, the probability of which are $P_1, P_2, \cdots, P_n$ respectively. Before experiment is done, you do not decide which result will be presented. It is illustrated that the things are indefinite and irregular. For example when you throw a coin, the probability appearing the right side is equal to one of reverse side. Before a coin is thrown every time, you do not predict that it is on whether right side or reverse side. But before you take a ball from the bag in which ninety-nine white ball and one black ball are hold, you can almost ascertain that a white ball will be taken. Hence it is concerned with the distribution of probability of the experiment's results that the indefinite metric is either big or small. For a system with even distribution of probability, there is the greatest indefinition. That is to say that the system is in a very irregular state. The indefinition could be described mathematical formula by Shannon as follows:

$$S = -k_0 \sum_{i=1}^{n} P_i \ln P_i \tag{2}$$

where S was defined as Entropy and k_0 is a normal number that was concerned with the metrical unit. If the probability was known, we could measure the indefinite system by means of (2). But how to allot the probability hadn't solved yet, which was solved by Jaynes. He pointed that we must select the distribution of probability having greatest Entropy and obeying all known information. It was only and fair distribution we could do. The statistic and inferable criterion founded by Jaynes is called the criterion of the Biggest Entropy. Although the criterion is subjective in respect to it's characteristic, it is most objective in the whole system.

The normal effective solutions of (WP) are different corresponding to different weighting coefficients. In the light of the whole decision-making system, weighting coefficients of target should be objective values, which are unknown while analyst or decision-maker point out the problems. Only if the 'price' is paid which is exactly negative Entropy of system., the weighting coefficients of target can be decided. So problem (KP) can be given by

$$\min \sum_{i=1}^{n} \omega_i f_i(X) + \frac{1}{k} \sum_{i=1}^{n} \omega_i \ln \omega_i$$

$$\begin{cases} \omega_i > 0, \sum_{i=1}^{n} \omega_i = 1, \quad i = 1, 2, \cdots, n \\ x \in R \end{cases} \tag{3}$$

where k is a positive factor. x in the solution of (KP)such as (x, w) is defined as the solution of Entropy of (MP) in respect to $k > 0$. We can prove that solution of Entropy of (MP) is certainly the effective solution of (MP) and normal effective solution of (MP). Supposing that as far as the problem (MP) given by expression (1) is concerned, the set of the solution of Entropy of which is S_1 , the set of normal effective solution of which is E and the set of effective solution of which is M, then there is $S_1 \subset E \subset M$.

If Lagrange multiplier λ is introduced to (KP), the function founded is

$$L_k(\omega, X, \lambda) = \sum_{i=1}^{n} \omega_i f_i(X) + \frac{1}{k} \sum_{i=1}^{n} \omega_i \ln \omega_i + \lambda(\sum_{i=1}^{n} \omega_i - 1) \tag{4}$$

According to $\frac{\partial L_k}{\partial \omega_i} = 0$, then

$$f_i(X) + \frac{1}{k}(\ln \omega_i + 1) + \lambda = 0, \quad i = 1, 2, \cdots, n$$

By using $\frac{\partial L_k}{\partial \lambda} = 0$ and $\sum_{i=1}^{n} \omega_i = 1$, then the solution obtained is

$$\omega_i = ce^{-kf_i(X)}, \quad i = 1, 2, \cdots, n \tag{5}$$

where $c = 1/\sum_{i=1}^{n}(e^{-kf_i(x)})$.
Transferring equation (5) onto (4), we have

$$L_k(\omega, X, \lambda) = \sum_{i=1}^{n} \omega_i f_i(X) + \frac{1}{k}\sum_{i=1}^{n} \omega_i \ln \omega_i + \lambda(\sum_{i=1}^{n} \omega_i - 1) = \frac{1}{k}\ln c$$

So searching for the extremum of L_k is equivalent to one of the problem (D), described by

$$\max \sum_{i=1}^{n} e^{-kf_i(x)},$$

$$X \in R \tag{6}$$

We can prove that if $k > 0$ any solution of (D) is equivalent to effective solution of (D) and is also the solution of Entropy, if $0 < k_1 < k_2$, assuming that the solution of (KP) in respect to k_1 and k_2 are $(X, \omega), (X^*, \omega^*)$ respectively, we have

$$q \leq P, \quad S_1 \leq d$$

where

$$P = \sum_{i=1}^{n} \bar{\omega} f_i(\bar{X}), \quad q = \sum_{i=1}^{n} \omega_i^* f_i(X^*), \quad S_i = \sum_{i=1}^{n} \bar{\omega}_i \ln \bar{\omega}_i, \quad d = \sum_{i=1}^{n} \omega_i^* \ln \omega_i^*$$

So we come to the conclusion that along with the increase of control factor k, the values of objective function corresponding to the solution of Entropy will reduce. That is to say that the solution of Entropy trend to be optimal in the wake of the increase of k as regards the whole system, which is entirely coincide with physical meaning of the second item of objective function of (KP), i.e. the negative Entropy of system.

When k is very small that is say knowing nothing about the weighting factor, the second item of the value of objective function is very big. If $k \to +\infty$ corresponding to knowing entirely weighting coefficients, the second item of the value of objective function tend to become smallest. The optimal weighting factor can be recognized progressively with the increase of k because Entropy of system decline monotonously.

3 Application

According to the basic principle of the Technology of the Biggest Entropy described above, we can apply it in the ship Optimal Design. Underneath we take an optimal design of self-propelled barges in Wujiang River, a tributary of Changjiang River, for example so as to state the application of this method.

3.1 The basic principle of the Variable Tolerate Error Method
(Ge 1992, Ma 1998, Ma 1998)

In respect to the single objective problem of non-linear plan, it can be described by

$$\begin{aligned} &min f(X), \quad X \in E^n \\ &h_i(X) = 0, \quad i = 1, 2, \cdots, m \\ &g_i(X) \leq 0, \quad i = m+1, m+2, \cdots, p \end{aligned} \tag{7}$$

where X is n dimensional vector, $f(X)$, $h_i(x)$ and $g_i(X)$ are linear or non-linear function. In general we take a long time in satisfying rather strictly feasible demands when above problem (7) is solved. How to solve it effectively and fast, the Variable Tolerate Error Method is introduced. The Variable Tolerate Error Method is a kind of non-linear constrained optimization method, without much preparation and with greater adaptability. The method use feasible points and approximately feasible points to search for the optimal solution of the target function. Meanwhile the qualification of deciding whether the point is feasible or unfeasible is continuously limited, until the optimal solution is found. So the expression (7) can be substituted by

$$\min f(X), \quad X \in E^n$$

$$\Phi^{(k)} - T(X) \geq 0 \tag{8}$$

where

$$\begin{aligned} \Phi^{(k)} &= \min \left[\phi^{(k-1)}, \ \frac{m+1}{r+1} \sum_{i=1}^{r+1} ||X_i^{(k)} - X_{(r+2)}^{(k)}|| \right] \\ \Phi^{(0)} &= 2(m+1)t \end{aligned} \tag{9}$$

in which
$\Phi^{(k)}$ - the value of tolerant error criterion corresponding to search stage number $(k-1)$.
m - size of original polyhedron.
r - $r = n - m$, is the degree of freedom of $f(X)$ in problem (8).
$X_i^{(k)}$ - peak number i of polyhedron in E^n.
$X_{r+2}^{(k)}$ - the ordinates of center of volume in respect to $n = r$ in the variable polyhedron search method by Nelder and Mead.
k - stage number of the completion of search.

Supposing that the second item of big brackets in (9) is defined as $\theta^{(k)}$, then

$$\theta^{(k)} = \frac{m+1}{r+1} \sum_{i=1}^{r+1} ||X_i^{(k)} - X_{r+2}^{(k)}|| = \frac{m+1}{r+1} \left[\sum_{i=1}^{r+1} \sum_{j=1}^{n} (x_{ij}^{(k)} - x_{r+2,j}^{(k)})^2 \right]^{1/2} \tag{10}$$

where $x_{ij}^{(k)}(j = 1, 2, \cdots, n)$ is the peak number i of variable polyhedron in E^n. We will find that $\theta^{(k)}$ indicate the average distance between each peak $x_i^{(k)}(i = 1, 2, \cdots, r+1)$ and center of volume $X_{r+2}^{(k)}$. It can be proved that

$$\begin{aligned} &\Phi^{(0)} \geq \Phi^{(1)} \geq \cdots \geq \Phi^{(k)} \geq \cdots \geq 0 \\ &\lim_{x \to x^*} \phi^{(k)} = 0 \end{aligned} \tag{11}$$

$T(X)$ in (8) is defined as the total metric beyond boundary, which is given by

$$T(X) = \left[\sum_{i=1}^{m} h_i^2(X) + \sum_{i=m+1}^{p} \mu_i g_i^2(X) \right]^{1/2}$$

where

$$\mu_i = \begin{cases} 0, & g_i(X) \geq 0 \\ 1, & g_i(X) < 0 \end{cases} \tag{12}$$

It is proved that the solution fit into $T(X^{(k)}) = 0$ is feasible, it is nearly feasible when $\phi^{(k)} \geq T(X^{(k)}) \geq 0$, and it is infeasible corresponding to $T(X^{(k)}) > \phi^{(k)}$. The approximation from $X^{(k)}$ to $X^{(k+1)}$ is feasible if $0 \leq T(X^{(k+1)}) \leq \phi^{(k)}$ and is infeasible if $T(X^{(k)}) > \phi^{(k)}$. It can be seen that the value of $\phi^{(k+1)}$ can be decided until X is determined either feasible or nearly feasible. So either feasible point or nearly feasible point which is tested and verified step by step is acceptable when we minimize the constrained objective function $f(X)$ by simplex method. In the search process when $X^{(k)}$ is infeasible point as to tolerant error criterion, first we must minimize $T(X)$ until nearly feasible point X fit into $\phi^{(k)} - T(X) \geq 0$ is obtain, then minimization search of $f(X)$ is taken again. It is done continuously until the optimal solution $f(X^*)$ is found, which is satisfied with $T(X)$ and has also enough accuracy.

3.2 The determination of original solution

In order to obtain the optimal solution in the whole system, to avoid the local optimal solution and speed up the time of search, it is very important to decide original solution. In the example above, we use L_{wl}/B, C_b, W_c as the design factor, where L_{wl}/B is the ratio of length on waterline of ship to breadth moulded of ship, C_b is block coefficient, and W_c is weight loading cargoes. According to the range of design factor, i.e.

$$\begin{aligned} 5.5 \leq L_{wl}/B \leq 7.5 \\ 0.5 \leq C_b \leq 0.74 \\ 80\,\text{ton} \leq W_c \leq 200\,\text{ton} \end{aligned}$$

all feasible schemes are founded by the method of Orthogonal Design. Then the values of objective function are calculated progressively by computer. On the basis of orthogonal direct analysis, limitations of waterway and synthetic assessment, the better scheme used as primary solution is gained, where $L_{wl}/B = 6.5, C_b = 0.65$ and $W_c = 140\,\text{ton}$. And the quantitative relationship between design factor and criteria by orthogonal polynomial regression are

$$RFR = 19.35 - 0.498(L_{wl}/B - 7) + 11.55(C_b - 0.62) - 0.0495(W_c - 140)(RMB/\text{ton})$$
$$v = 5.61 + 0.181(L_{wl}/B - 7) - 4.15(C_b - 0.62) - 0.00562(W_c - 140)(m/s)$$
$$p = [38.37 + 0.676(L_{wl}/B - 7) - 27.54(C_b - 0.62) + 0.165(W_c - 140)]\,10^4(RMB)$$

3.3 The determination of optimal solution

It is very difficult to select size of original polyhedron because the design factors of ship have different unit and degree of value respectively. Factors must be taken value transform so that the value of factor ranges from 0 to 1. According to the their maximum value M_i and minimum value m_i in respect to each ship factor, ship design factor after value transform is

$$x_i = (x_i - m_i)/(M_i - m_i), \quad i = 1, 2, \cdots, n$$

Similarly, in order to insure objectivity of optimal results, the objective function after value transform is

$$f_i(X) = (f_i(X) - m_{f_i})/(M_{f_i} - m_{f_i}), \quad i = 1, 2, \cdots, n \tag{13}$$

where M_{f_i}, m_{f_i} are maximum value M_{f_i} and minimum value of objective function respectively.

On the basis of the original solution of orthogonal design combining with orthogonal polynomial regression, the problem of multi-objective programming such as the example described over is presented, i.e.

$$\begin{aligned} \min\ F(X) &= \min\ \{f_1(X), f_2(X), f_3(X)\} \\ X &= \left\{ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \right\}, \quad X \in E^3 \end{aligned} \tag{14}$$

where

x_1 - the value of L_{wl}/B after value transform, i.e. $x_1 = 0.333(L_{wl}/B - 5.5)$.
x_2 - the value of C_b after value transform, i.e. $x_2 = 4.167(C_b - 0.5)$.
x_3 - the value of W_c after value transform, i.e. $x_3 = 0.00833(W_c - 80)$.
$f_1(X)$ - the value of objective function RFR (required freight) after value transform, i.e.
$f_1(X) = 0.153(RFR - 17.534)$.
$f_2(X)$ - the value of objective function ν (velocity) after value transform, i.e.
$f_2(X) = 1.017(-\nu + 5.1)$.
$f_3(X)$ - the value of objective function P (price of ship) after value transform, i.e.
$f_3(X) = 0.0504(P - 28.724) \times 10^4$.

The constraints of the example above are

$$0 \leq x_i \leq 1, \quad i = 1, 2, 3$$
$$L_{wl} \leq 40m, \quad B \leq 7m, \quad \nu \geq 5m/s$$

At the same time requirements of the ship performance as buoyancy, stability, freeboard and so on must be satisfied certainly.

By means of the Technology of the Biggest Entropy combining with the Variable Tolerate Error Method, the optimal solution of the problem (14) is obtained as follows:

$$\begin{aligned} L_{wl}/B &= 6.54m \\ C_b &= 0.655 \\ W_c &= 130\,\text{ton} \end{aligned}$$

4 Conclusion

On the basis of application of the Technology of the Biggest Entropy stated over, It can concluded that

1) Ship optimal design is a multi-objective decision-making process. It is very important to research effective methods in order to obtain objective and satisfactory solutions.

2) As one of method of Multi-objective Decision-making, the Technology of the Biggest Entropy is feasible and effective, by which weighing coefficients are decided.

3) By means of weighing coefficients, a multiobjective problem is transferred to a single objective problem. So far as optimal method of single objective is concerned, the Variable Tolerate Error Method is recommended

4) In order that the optimal solutions in the whole system are obtained more fast, the selection of initial solution gained from information of ship tech-economic evaluation is key.

5) The values of variables and objective function must be non-dimensional and have equal the grade of quantity so as to ascertain objectivity and validity of solutions.

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