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Analysis of the 3-D Stress Wave in a Plate under Impact Load by Finite Element Method

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ABSTRACT

This paper attempt to explore the shape of stress wave propagation of 3-dimensional stress field which is made in the process of time increment. A finite element program about 3-dimensional stress wave propagation is developed for investigating the changing shape of the stress by the impact load. The finite element program, which is the solution for the 3-dimensional stress wave analysis, based on Galerkin and Newmark-β method at time increment step. The tensile stress and compressive stress become larger with the order of the middle, the upper and the opposite layers when the impact load is applied. In a while the shear stress become larger according to the order of the upper, the middle and the opposite layers when impact load applied.

Keywords : Stress Wave, Newmark-β method, Galerkin's method, Impact Load

1. Introduction

A stress wave or shock wave, is defined as the wave motion of stress delivered in a solid medium and is propagated by inertia and elasticity. When certain material is loaded by impact, a stress wave is generated and then propagated into a solid. At that moment, the tensile stress and compressive stress are repeated and the stress is repeatedly dissipated. Stress concentrations such as these can occur without defect of structures, as deformation and fracture behavior is entirely different from the static load situations. The deformation velocity, which governs the deformation behavior of the material, depends on the stress wave, which is also governed by the deformation behavior of a medium.

Stress wave propagation in elastic body was first discussed by Kolsky, Scoch and Miklowitz. Thereafter, only qualitative analyses on the wave propagation in elastic body have been carried out by Brekhovskikh, Lindsay, Morse and Ingard, so that now it is impossible to perform not only qualitative, but quantitative analysis on the stress wave propagation. Therefore, for safety design, it is absolutely necessary to analyze changes according to time increments of stress field and stress wave which are generated in structures by impact. Methods of numerical analysis on stress wave propagation include boundary integration, direct time integration, DtN method, quasi-discretization, and spectrum analysis using the Fourier series. But the analyzing process of these methods are complex, and only qualitative analysis are possible due to the problem of reliability on their analysis results.

Hence, this study was carried out using the 3-dimensional finite element method based on the Newmark-β method to analyze on 3-dimensional stress wave propagation and stress magnitude, which were delivered on a 3-dimensional flat board at particular time increment.

2. The governing equation of stress waves

The governing equation, which satisfies the dynamic balance of 3-dimensional stress wave in the rectangular coordinates, consists of equilibrium equations of inertia force which particle masses have. The governing equation of 3-dimensional stress wave is the second-order partial differential equation with respect to four independent variables including cartesian coordinate x, y and z, and time t.

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} \qquad (1)$$

Equation (1) shows that stress wave moves parallel to a normal line direction on spatial coordinates from the action point of a load at a steady velocity c. The theoretical solution ϕ of the equation is as follows:.

$$\phi(x,y,z,t)=\frac{1}{2}f(lx+my+nz+ct) + f(lx+my+nz-ct)] \qquad (2)$$

For this equation, the function $\varphi=f(lx+my+nz+ct)$ implies moving by ct outward from the action point of impact load; that is, the function $\varphi=f(lx+my+nz+ct)$ moves at the velocity c in the same direction as that of the impact load action. Function $f(lx+my+nz)$, likewise, moves at the velocity c in the opposite direction as that of impact load action. A general governing equation that analyzes longitudinal waves, particularly transversal waves is as follows:

$$\nabla^2\phi=\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} \qquad (3)$$

3. The numerical expression of finite element

For analysis of stress wave behavior at time increments, there is a method of discretization of the analysis field, by which the motion equation of partial differential equation, including the function of time, can be divided into space domain and time domain; an approximate solution can be obtained in the process of direct time integration. Considering the load (f) from outside, the governing equation of stress wave is as follows:

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}-f-k_{tt}\ddot{\phi}=0 \qquad (4)$$

Equation (4) can be expressed numerically as a finite element using the Galerkin method and the matrix equation considering boundary conditions:

$$[K_{tt}]\{\ddot{\phi}\}+[K_{ij}]\{\phi\}+[K_{S_2}]\{\phi\}+\{R_i(t)\}=\{0\} \qquad (5)$$

where

$$K_{tt}=\int_{\Omega^{(e)}}k_{tt}N_iN_j\,d\Omega^{(e)}$$

$$K_{ij}=\int_{\Omega^{(e)}}\left(k_x\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x}+k_y\frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y}+k_z\frac{\partial N_i}{\partial z}\frac{\partial N_j}{\partial z}\right)d\Omega^{(e)}$$

$$K_{S_2}=\int_{S_2}hN_iN_j\,dS_s$$

$$R_{ti}=\int_{\Omega^{(e)}}fN_i\,d\Omega^{(e)}+\int_{S_2^{(e)}}q_i\,dS_2^{(e)}$$

$[K_{tt}]$ is inertia matrix by acceleration, $[K_{ij}]$ is the stiffness matrix by elasticity. $[K_{S_2}]$ is the boundary matrix which depends on the boundary conditions, and $\{R_i(t)\}$is load vector applied from outside. The constant h is the ratio of transmission and reflection of stress wave in boundary surface, and f is the load from outside.

For methods in the use of equilibrium state at random time $t+\Delta t$, there are the Houbolt method, the Wilson θ method, and the Newmark-β method. In this study, the Newmark-β method, which has good convergency and satisfies the equilibrium state, was used. Using the method, the governing equation can be expressed as a time-continuous equation as follows:

$$[K^*]\{\phi\}_{t+\Delta t}=\{F^*\}_{t+\Delta t} \qquad (6)$$

where

$$[K^*]=[K_{ij}]+\frac{4}{\Delta t^2}[M] \quad \text{: effective stiffness matrix}$$

$$\{F^*\}_{t+\Delta t}=\left\{F(t+\Delta t)+[M]\left(\frac{4}{\Delta t^2}\{\phi\}_t+\frac{4}{\Delta t}\{\dot{\phi}\}_t+\{\ddot{\phi}\}_t\right)\right\} \qquad (7)$$

: effective load vector

To solve the above equations, the following process should be carried out step by step.

① From inertia matrix $[K_{tt}]$ and stiffness matrix $[K_{ij}]$ of the element

② Initialize values of $\{\phi\}_0$, $\{\dot{\phi}\}_0$ and $\{\ddot{\phi}\}_0$

③ Select time step Δt and parameter β and γ

④ Assemble $[K^*]$, effective stiffness matrix.

⑤ Modify $\lfloor K^{*} \rfloor$ under the boundary conditions

⑥ From $\{F^{*}\}_{t+\Delta t}$, effective element load vector, at time increments

⑦ Modify the effective load vector $\{F^{*}\}_{t+\Delta t}$, for the boundary conditions.

⑧ Solve the displacement $\{\phi\}_{t+\Delta t}$ at a time increment step $t+\Delta t$

⑨ Calculate the velocity $\{\dot{\phi}\}_{t+\Delta t}$ and acceleration $\{\ddot{\phi}\}_{t+\Delta t}$ at a time increment step $t+\Delta t$

4. Analysis model and boundary conditions of finite element

Table 1 shows the material properties of general steel, the material used in this study, to analyze the finite element for stress wave propagation. Table 2 shows the element size, the number of nodes and elements, and the total amount of freedom. The time increment used for analyzing the finite element is the time for stress wave to propagate by 1 ㎝;

$\Delta t = \frac{l}{c} = \frac{l}{5.015E6} = 2E10^{-7}\ \text{sec}$. During this time, a step load of 10 kg was applied to the center of opposite surface of fixed section, where the value of coordinate axis x, y and z is $xl/2$=5 ㎝, $yl/2$=5 ㎝, and zl=0.75 ㎝, respectively. At that moment, the shape of stress wave propagation of 3-dimensional plate was considered.

Table 1. Material properties of model

Young's Modulus (kg_f/mm2)	Density ($kg_f \cdot sec^2/mm^4$)	Poisson's ratio	Wave propagation Velocity (m/sec)
2.0E4	7.951E-10	0.28	5015

Table 2. The total number of mode point and elements in the finite element analysis

Element Size (mm)	Node Number	Element Number	Total Number of freedom
2.5×2.5×2.5	6,724	4,800	20,121

For this experiment, initial conditions of the analysis subject, $\{\phi\}_0$, $\{\dot{\phi}\}_0$ and $\{\ddot{\phi}\}_0$ were set to 0, and it was assumed that stress wave was reflected totally in the fixed section of both end sides of axis y, where x=0 and 10 ㎝(xl), under the boundary condition, while it was transmitted totally in the other section.

Mesh of the isoparametric cube linear element was used for the analysis of the finite element. The Newmark-β method and finite element program, which was developed by Jin and Hwang's analysis of finite element code for stress wave propagation was also used.

The Newmark parameter, β and γ, used in this study were set to 1/2 and 1/4, respectively, which are well convergent. Also, for the material's geometrical shape and physical properties, the same model which has same values as those used in the theoretical solution were used.

5. Results and consideration

In this study, the time increment step for analysis of finite element was set to 2×10^{-7} sec, and the results were obtained by running the finite element program, making input data with a personal computer, and then carrying out an analysis of finite elements with super-computer crayc-90. Fig. 1-4 show the results of the analysis of finite elements for 3-dimensional stress wave propagation, according to time increments after dividing the material into three parts according to thickness direction.

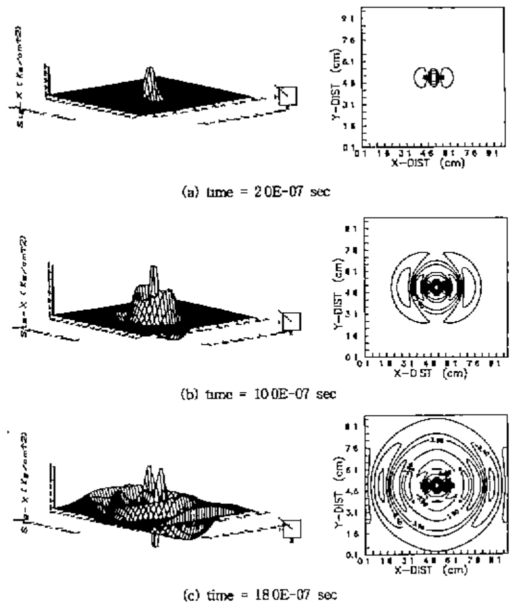


Fig. 1 Results of the stress σ_Y in the impact loaded surface plate

Fig. 1 shows the results of the analysis of x-direction stress(σ_x) generated on the impact-loaded surface according to time increments. When the impact load is applied, tensile stress acts on minute area loaded by impact in the x direction, and compressive stress is generated by the tensile stress around the minute area.

The stress σ_x becomes larger with short time increases than the initial stress of impact-loaded moment. As time increases, the tensile stress becomes smaller and the compressive stress becomes larger; but when the initial shape of stress field remains unchanged, a stress wave is propagated. Also, after the shear of a stress wave reaches the boundary surface, both tensile and compressive stress are reduced; but the shear stress becomes larger in the boundary surface, where the stress wave is reflected, by boundary conditions, while stress becomes zero in the boundary surface, where the stress wave is superposed in front of stress wave. Around the impact-loaded surface, relatively high tensile and compressive stress are obtained repeatedly, compared to those of other surfaces. At that moment, the magnitude of the stress field extends with the value equivalent to the transmission velocity of stress wave in the x- and y-direction.

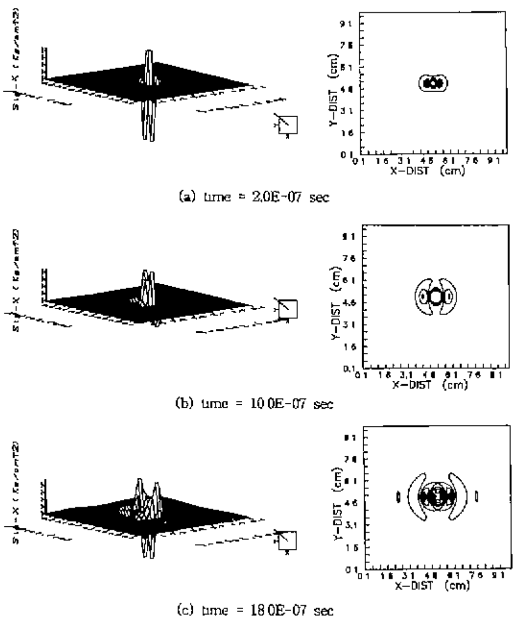


Fig. 2 Results of the stress σ_x in the middle layer plate

Fig.2 shows the results of analyzing x-directional stress (σ_x) which is generated by element of the middle layer among the three layers of the 3-dimensional elements. From the result of analyzing finite element code, σ_x of the middle layer is less than that of impact-loaded surface, and the tensile and compressive stress repeat periodically. At this moment, the magnitude of stress field extends with the value equivalent to the transmission velocity of the shear of stress wave in the x- and y-direction.

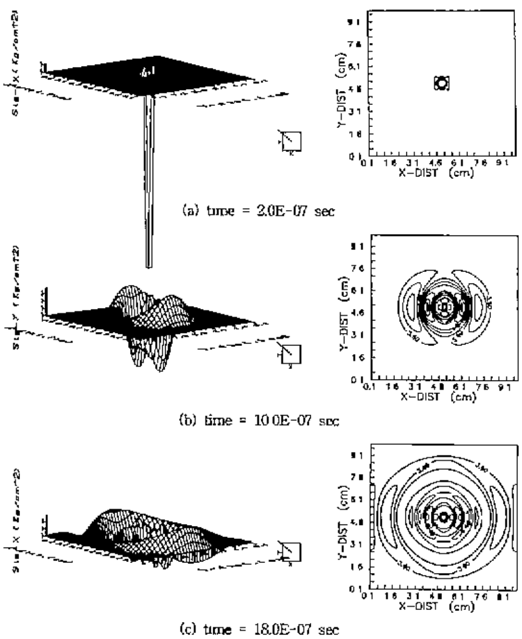
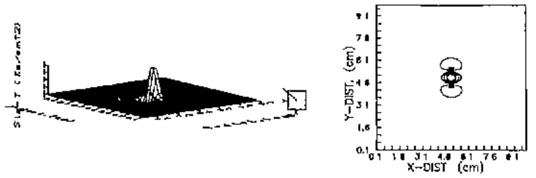
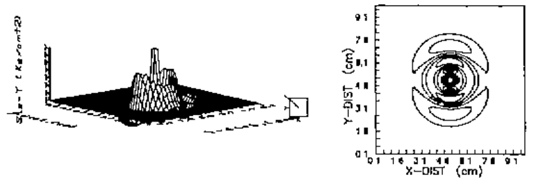


Fig. 3 Results of the stress σ_x in the opposite side of the impact loaded surface plate

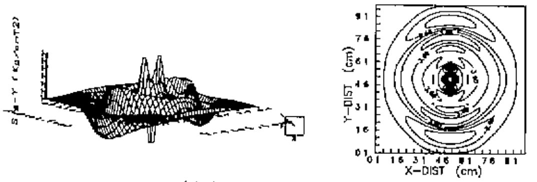
Fig.3 shows the results of analyzing the x-directional stress σ_x which is generated by the element on the opposite side of the impact-loaded surface according time increments. From the results of analyzing the finite element code of Fig. 3, the stress σ_x acts on the impact-loaded surface in the x-direction at the moment of load application; as time increases, the compressive stress becomes smaller and the tensile stress becomes larger. The shear of the stress wave is propagated with the original shape of the stress field. Because of the repetition of tensile and compressive stress around the impact-loaded minute area, the area plays a role in providing a wave source.



(a) time = 2.0E-07 sec

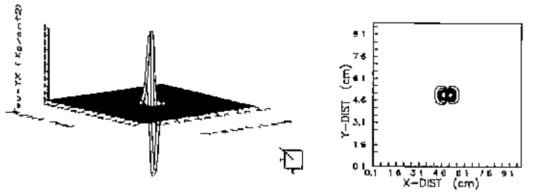


(b) time = 10.0E-07 sec

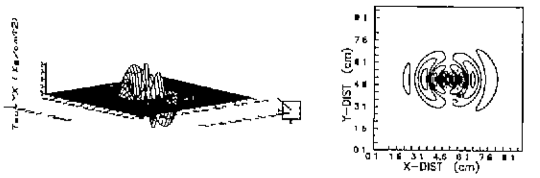


(c) time = 18.0E-07 sec

Fig. 4 Results of the stress τ_{yx} in the impact loaded surface plate



(a) time = 2.0E-07 sec



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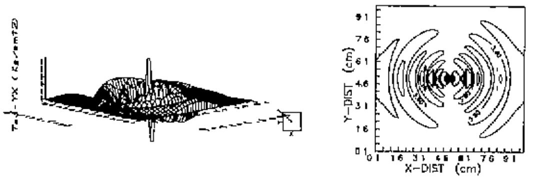
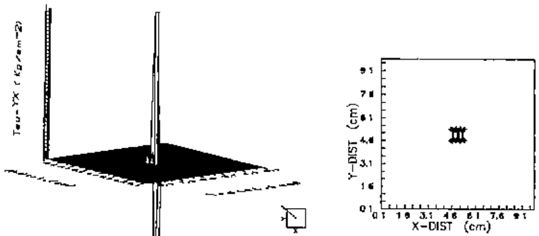


(c) time = 18.0E-07 sec

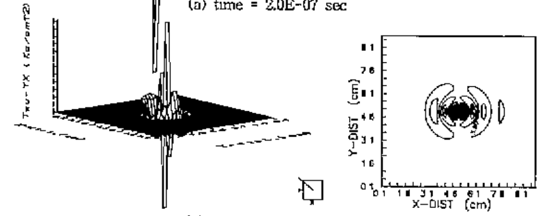
Fig. 5 Results of the shear stress τ_{yx} in the middle layer plate

Fig. 4 is the results of analyzing the shear stress τ_{yx} which is generated by the elemental quality of the impact-loaded surface. The results of analyzing the finite element code of Fig. 4 is that the shear element of stress wave τ_{yx} of the impact-loaded surface is diagonally symmetric to x-axis of the impact-loaded surface, where the right part of the surface is negative and the left part is positive. In addition, the magnitudes of initial and time-increasing shear stress are almost identical. The area in which the shear stress is applied to extends as much as the increasing magnitude of the transmission velocity of stress wave.

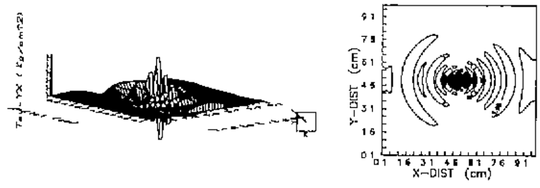
Fig. 5 shows the results of analyzing the shear stress τ_{yx} which is generated by the element the middle layer of the three layers of the impact-loaded 3-dimensional element according to time increments. From the results of analyzing the finite element code, τ_{yx} of the middle layer is larger than that of the impact-loaded surface. The shear stress after time increases becomes remarkably smaller than the initial stress; however, it is generally larger than the shear stress of the impact-loaded surface.



(a) time = 2.0E-07 sec



(b) time = 10.0E-07 sec



(c) time = 18.0E-07 sec

Fig. 6 Results of the shear stress τ_{yx} in the opposite side of the impact loaded surface plate

Fig. 6 is the result of analyzing the shear stress which is generated on the opposite side of the impact-loaded surface according to time increments. The results of analyzing the finite element code of Fig. 6 shows that from the impact-loaded moment until the stress wave's arrival to the boundary surface, the shear stress of the impact-loaded surface is larger than that of the middle layer and the maximum shear stress is generated on the minute impact-loaded surface. When the stress wave is about to reach the boundary surface, the shear stress of the opposite side of the impact-loaded surface becomes smaller than that of the middle layer.

6. Conclusion

In this analysis of the 3-dimensional stress wave, after considering the magnitude of a 3-dimensional stress field according to time increments, the conclusion is as follows.

1. To apply the magnitude of the stress field to safety design, the effects of thickness on the results of analyzing the finite element code can be minimized by dividing the element of thickness direction into more than three layers in analyzing the code.
2. The tensile stress and compressive stress become larger with the order of the middle, the upper and the opposite layers when the impact load is applied.
3. After a period of time, the shear stress becomes larger according to the order of the upper, middle and the opposite layers when the impact load is applied.
4. From the above results, it is deemed that when the impact load is applied to real structures, fracture starts first in the opposite layer of the impact-loaded surface.

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