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Neck Formation in Drawing Processes of Fibers

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Abstract: To better understand the formation of necking in drawing processes of fibers, strain distributions during drawing processes have been analyzed. For simplicity, one-dimensional incompressible steady flow at a constant temperature was assumed and quasi-static model was used. To describe mechanical properties of solid polymers, non-linear visco-plastic material properties were assumed using the power law type hardening and rate-sensitive equation. The effects of various parameters on the neck formation were mathematically analyzed. As material property parameters, strain-hardening parameter, visco-elastic coefficient and strain-rate sensitivity were considered and, for process parameters, the drawing ratio and the process length were considered. It was found that rate-insensitive materials do not reach a steady flow state and the rate-sensitivity plays a key role to have a steady flow. Also, the neck formation is mainly affected by material properties, especially for the quasi-static model. If the process length changes, the strain distribution was found to be proportionally re-distributed along the process line by the factor of the total length change.

Keywords: Necking, Drawing process, Quasi-static, Rate-sensitivity

Introduction

As the last stage of fiber processing, the drawing process introduces large elongation to man-made fibers in a solid state in order to improve their fiber characteristics. In this process, the neck formation (or cold drawing) caused by severe local elongation of fibers is commonly observed, which affects the microstructures and the mechanical properties of fibers[1]. In this work, the neck formation is macroscopically analyzed without considering the microscopic aspect of molecular structures. In order to account for the mechanical behaviors of polymer fibers, non-linear incompressible visco-plastic material properties were assumed using the power law type hardening and rate-sensitive equation[2]. The effect of the material and process parameters on the neck formation was analytically derived, assuming one-dimensional steady flow under the isothermal and quasi-static conditions, for simplicity.

Formulation and Calculation

In the continuous drawing process, a fiber is elongated between feed rolls and end rolls, as schematically illustrated in Figure 1. The position of the drawing region is designated by z along the fiber axis, while its total length is z_L. A fiber is supplied at a constant feed velocity, v_0 and is taken-up by the end rolls at a constant drawing velocity, $v_L(=\alpha_L v_0)$, where α_L is the draw ratio. Here, $v = v(z)$ is the local axial velocity of a fiber to be analyzed with two boundary conditions, v_0 and v_L, which are the velocities of the feed and end rolls, respectively. The drawing of solid polymers performed under the tensile force produces a considerable amount of heat to have a strong effect on the fiber temperature and its material properties. In this work, however, temperature-dependence of solid polymers were ignored because the isothermal condition can be realized by drawing thin fibers in the liquid bath as illustrated in Figure 1.

For a steady flow in the drawing process, kinematics provides the following relations about the local true strain ε and its material time rate:

$$\varepsilon = \ln\left(\frac{dz}{dz_0}\right) = \ln\left(\frac{v}{v_0}\right) \tag{1}$$

$$\dot{\varepsilon} = \frac{d\varepsilon}{dt} = \frac{d\varepsilon}{dz}\cdot\frac{dz}{dt} = \frac{d\varepsilon}{dz}\cdot v = v_0\exp(\varepsilon)\frac{d\varepsilon}{dz} \tag{2}$$

When a fiber is assumed incompressible,

$$Av = A_0v_0$$

so that

$$A = A_0\exp(-\varepsilon) \tag{3}$$

where A and A_0 are the current and initial cross-sectional areas, respectively.

Now, consider

$$F = \sigma \cdot A \tag{4}$$

where F and σ are the force and stress acting on the cross-section of a fiber, respectively. In the laboratory condition where the velocity of a fiber is rather small, a quasi-static condition can be applied so that F is assumed constant along the drawing region because the rheological force is dominant compared with other forces including the inertial force[3,4]. Then, the equilibrium condition ($dF = 0$) gives,

$$\frac{1}{\sigma}\frac{d\sigma}{dz} = \frac{d\varepsilon}{dz}. \tag{5}$$

In order to describe the mechanical properties of solid

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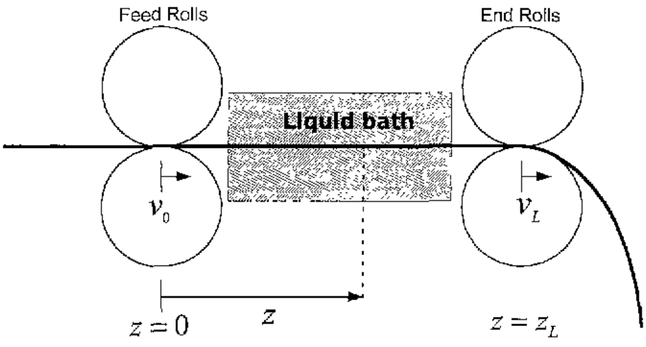

Figure 1. Continuous drawing processes.

semi-crystalline polymers, non-linear visco-plastic material properties were assumed using the power law type hardening and rate-sensitive equation[2,5]: i.e.,

$$\sigma = k[1 - \exp(-w\varepsilon)]\exp(h\varepsilon^2)\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^m \tag{6}$$

where k, w, h, m and $\dot{\varepsilon}_0$ are the scaling factor, visco-elastic coefficient, strain hardening parameter, strain rate sensitivity parameter and reference strain rate, respectively. Equation (6) shows three parts in a multiplicative form, which account for the visco-elastic, strain hardening and strain rate sensitivity effects, respectively.

Considering the equations (1)-(4) and (6), strain profiles can be numerically obtained from the relation between z and ε as[4]

$$z = \left(\frac{v_0}{\dot{\varepsilon}_0}\right)\left(\frac{kA_0}{F}\right)^{1/m}\int_0^{\varepsilon} e^{\varepsilon}[e^{h\varepsilon^2 - \varepsilon}(1 - e^{-w\varepsilon})]^{1/m} d\varepsilon.$$

Results and Discussion

Necking Formation

As a necking criterion, Ziabiki and Tian considered 6 types of fiber radius profiles along the drawing region according to the number of inflexion points[6]. Figure 2 shows the 6 types, which are expressed as strain profiles considering the relation between strain and cross-sectional area as shown in equation (3). The deformation profile, which has coexisting un-drawn and drawn regions such as the profile in Figure 2(e), is often referred to as cold drawing even though the phenomenon is not exclusively associated with the low temperature[1]. This profile typically has three regions divided by two inflexion points.

Note that equation (5) becomes a differential equation for $\varepsilon(z)$ after the constitutive equation in equation (6) and the kinematical relation in equation (2) are substituted into it; i.e.,

$$\frac{1}{\sigma}\frac{d\sigma}{dz} = \frac{we^{-w\varepsilon}}{1 - e^{-w\varepsilon}}\varepsilon' + 2h\varepsilon\varepsilon' + m\left(\varepsilon' + \frac{\varepsilon''}{\varepsilon'}\right) = \varepsilon' \tag{7}$$

where ε' and ε'' are the first and second derivative with respect to z, respectively. The neck formation is identified by considering the second derivative of the strain, which is, from equation (7),

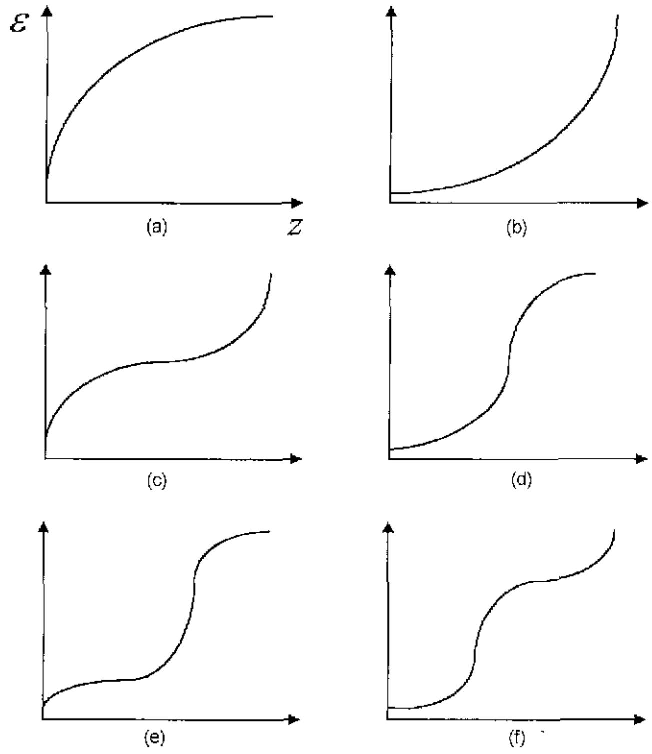

Figure 2. Types of schematic strain profiles.

$$\varepsilon'' = (\varepsilon')^2\left[\frac{1}{m} - 1 - \frac{2h\varepsilon}{m} - \frac{1}{m}\left(\frac{w}{e^{w\varepsilon} - 1}\right)\right] \equiv \frac{1}{m}(\varepsilon')^2\beta(\varepsilon). \tag{8}$$

In equation (8), the sign of ε'' is determined by the function β. Equation (8) shows that β is a function of only material properties, demonstrating that the neck formation is mainly affected by material properties, especially for the quasi-static model.

In equation (7), β is made of two parts:

$$\beta = f(\varepsilon) - g(\varepsilon)$$

where

$$f = 1 - m - 2h\varepsilon$$

and

$$g = \frac{w}{e^{w\varepsilon} - 1}.$$

By equating $\varepsilon'' = 0$, the inflection points are obtained from β, which has at most two solutions, ε^{I} and ε^{II}. As schematically illustrated in Figures 3 and 4, the inflexion points divide the drawing region into three parts. In the middle region, the deformation is usually localized severely

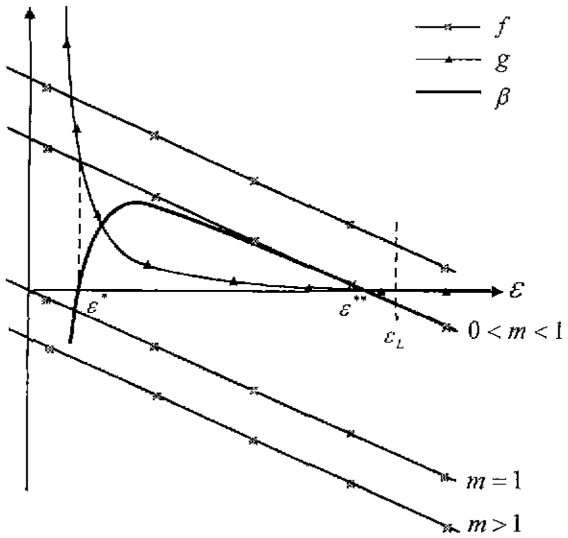

Figure 3. Schematic profiles of f, g, and $\beta(\varepsilon)$ with respect to the strain rate sensitivity parameter.

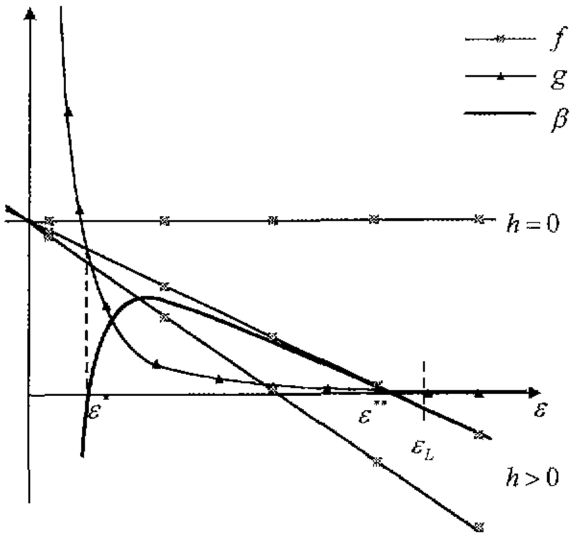

Figure 4. Schematic profiles of f, g, and $\beta(\varepsilon)$ with respect to the strain-hardening parameter.

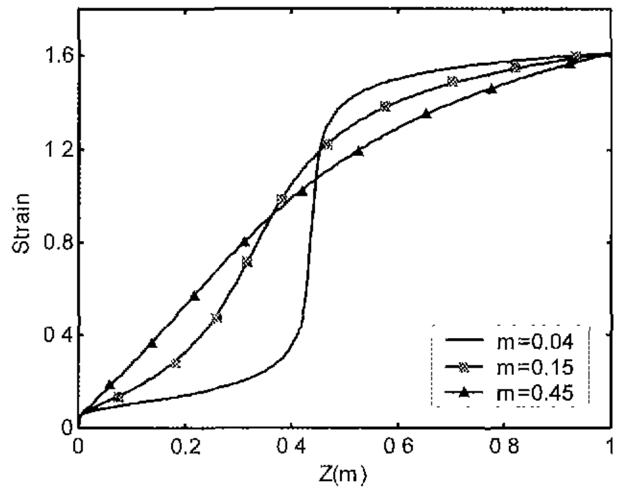

Figure 5. Strain profiles along the process line for various strain rate sensitivity parameters (material properties: k = 63 MPa, w = 35, h = 0.56, process parameters: α_L = 5.2, v_0 = 16.0 m/sec, z_L = 1.0 m, ρ = 0.9 g/cm^3, W = 2.0 g/min).

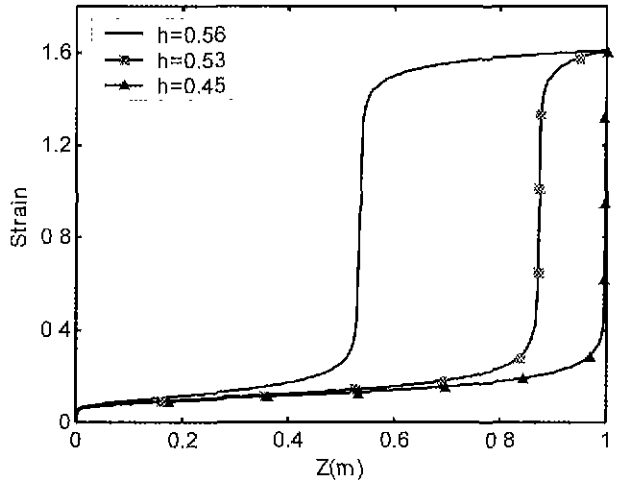

Figure 6. Strain profiles along the process line for various strain hardening parameters (material properties: k = 63 MPa, w = 35, m = 0.04, process parameters: α_L = 5.2, v_0 = 16.0 m/sec, z_L = 1.0 m, ρ = 0.9 g/cm^3, W = 2.0 g/min).

with a positive β value, representing the necking zone. The last region is the stabilizing zone. Note also that it is necessary for the final strain, ε_L, determined by the drawing ratio should exceed ε^* and ε^{**} in order for the necking and stabilization to take place, respectively.

Effect of Material Properties

Equation (8) confirms that the scaling factor and the reference strain rate in equation (6) do not affect the necking behavior. Figure 3 shows the effect of the strain rate sensitivity parameter, m, on the inflection points. Note that, when m = 0, the differential equation obtained from equation (4) does not have solutions, suggesting that the rate-insensitive material does not reach a steady flow state (this is also confirmed in equation (7)). Figure 3 shows that m must have the value below 1.0 in order for β to have two solutions. As m decreases, the strain profile changes type (a) into type (c) and type (e) in Figure 2. The calculated strain distribution along the process line for various m values are shown in Figure 5.

The effect of the strain-hardening parameter, h, which determines the slope of f, is shown in Figure 5. If this value is too small, ε^{**} may go beyond the final strain, ε_L, then there will be no stabilization zones. As this value increases, the

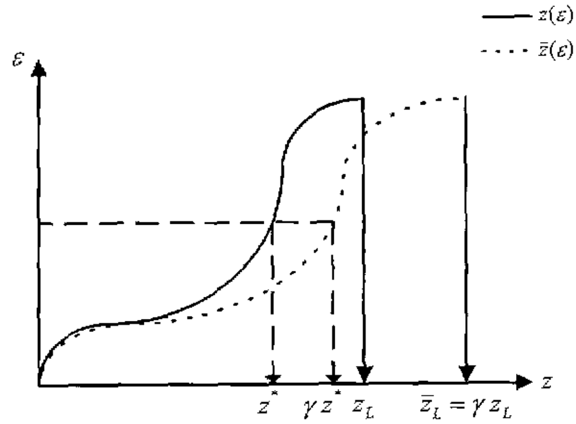

Figure 7. Proportional strain profile change with the process length change.

strain profile changes from type (c) into type (e) and finally into type (a). In Figure 6, the calculated strain distribution shows that a slight increase of h moves the necking zone to the feed roller direction, which can be easily confirmed in Figure 5.

Effect of Process Length

Equation (8) becomes, after mathematical manipulations,

$$\frac{dz^2}{d\varepsilon^2} = -\frac{dz}{d\varepsilon}\left[\frac{1}{m} - 1 - \frac{2h\varepsilon}{m} - \frac{1}{m}\left(\frac{w}{e^{w\varepsilon} - 1}\right)\right] \quad (9)$$

in which z is the dependent variable of ε. If the length of drawing process, z_L, is changed by γ times (γ is a constant) without any changes of the material and process parameters, the boundary conditions become

$$z = 0 \quad \text{at } \varepsilon = 0$$
$$z = \gamma z_L \quad \text{at } \varepsilon = \varepsilon_L. \quad (10)$$

Now, consider the assumed solution,

$\bar{z}(\varepsilon) = \gamma z(\varepsilon)$.

It is easy to confirm that the assumed solution satisfy the boundary conditions, equation (10), as well as equation (9). Therefore, for the process length change, the strain distribution is proportionally re-distributed along the process by the factor of the total length change, γ, as schematically shown in Figure 7.

Conclusion

The neck formation in the continuous drawing process of fibers was analyzed considering the effect of material properties and process parameters such as strain-hardening parameter, visco-elastic coefficient and strain-rate sensitivity as well as the drawing ratio and the process length. It was found that rate-insensitive materials do not reach a steady flow state and the rate-sensitivity plays a key role to have a steady flow. Also, the neck formation is mainly affected by material properties, especially for the quasi-static model. If the process length changes, the strain distribution was found to be proportionally re-distributed along the process by the factor of the total length change.

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