Honam Mathematical J. **23**(2001), No. 1,pp. 67–70

PERSISTENT LAMINATION

SEUNGSANG OH

Dept. of Mathematics,
Chonbuk National University, Chonju, Chonbuk 561-756, Korea,
E-mail : soh@math.chonbuk.ac.kr.

Abstract. Brittenham has shown how an incompressible Seifert surface F for a knot in S^3 can be used to find an infinite class of persistently laminar knots. We generalize this to create larger class of persistently laminar knots which therefore have property P.

In [GO] essential laminations and branched surfaces are introduced to study the topology of manifolds. For example it is shown that a 3-manifold which contains an essential lamination has universal cover $\mathbf{R}^3$. Brittenham [Br2] created infinite class of knots, called *persistently laminar*, associated to an incompressible Seifert surface for a knot in S^3 such that their complements admit *persistent laminations*, that is, laminations which remain essential under all non-trivial Dehn fillings.

In this note we generalize this construction to create larger class. The reader is assumed to be very familiar with the construction and notations of [Br2, §2]. We start with a brief description of Brittenham's construction of a branched surface. As in Figure 1(a), given an incompressible Seifert surface F for a knot K_0, if we attach a tube to a neighborhood of its boundary, running parallel to half of the knot K_0, and then glue the boundary of F to a curve running over the tube, and otherwise following the other

Received April 30, 2001.
1991 AMS Subject Classification : 57M25.
Key words and phrases : persistent lamination, Seifert surface, property P.
This paper was supported (in part) by research funds of Chonbuk National University.

half of K_0, we get a branched surface B_F.

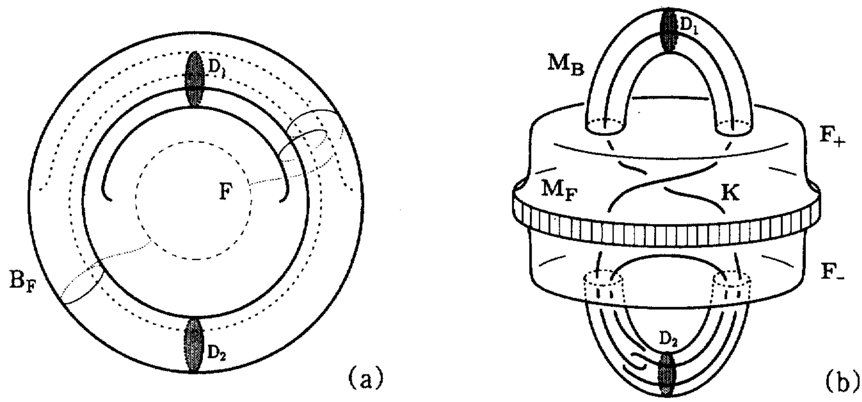

Figure 1. Brittenham's construction

And as in Figure 1(b), we get

$$M_B(= S^3\backslash intN(B_F)) \cong M_F(= S^3\backslash intN(F)) \cup \text{ (two 1-handles)} .$$

Let D_1, D_2 be the meridian disks of the two 1-handles. The main results of [Br2] is that if K is a knot in M_B meeting each of the disks D_1, D_2 in one point, then K is persistently laminar. We have a generalized version;

THEOREM. *Let K be a knot in $M_B = S^3 \setminus intN(B_F)$ meeting D_1 in one point and D_2 in one or more points non-trivially. Then K is persistently laminar.*

Meeting non-trivially means that K is not ambient isotopic into $M_B \setminus D_2$ in M_B. Note that persistently laminar knots have property P and satisfy cabling conjecture.

Proof. The method of proof is to construct an essential branched surface Σ in $M_K = S^3 \setminus K$ by adding extra surfaces to B_F. Let $N(K)$ be the tubular neighborhood of K and $\partial N(K)$ its boundary. Take double copies A_1 and A_2 of an annulus $D_1 \setminus intN(K)$. If we glue the boundary of these into $B_F \cup \partial N(K)$ after slightly changing $\partial N(K)$ near D_1 as in Figure 2(a), we get a desired branched surface Σ which is transversely oriented.

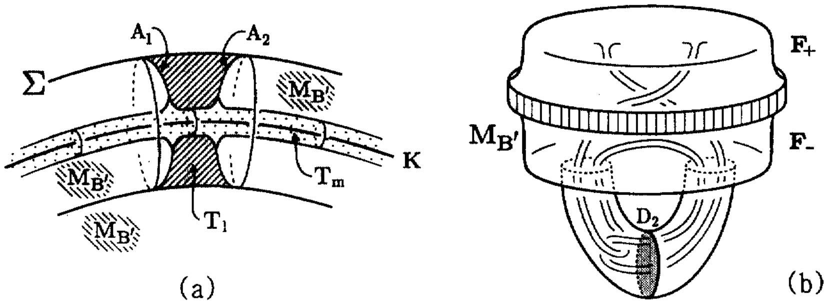

Figure 2. constructing Σ

To verify that Σ is essential in M_K, we begin by checking;

(1) Σ carries a lamination with full support.
(2) Σ does not carry a 2-sphere, and has no disks of contact.
(3) Σ does not carry a compressible torus.
(4) $M_\Sigma = M_K \setminus intN(\Sigma)$ does not have any monogons.
(5) M_Σ is irreducible.
(6) The horizontal boundary $\partial_h N(\Sigma)$ of Σ is incompressible in M_Σ.

An easy exercise for the reader is to figure out (1)-(4). M_Σ has three components T_m, T_l and $M_{B'}$ as shown in Figure 2(a). T_m is a core-deleted solid torus with two meridional cusps, and T_l is a solid torus with two longitudal cusps. Therefore both satisfy (5) and (6). Figure 2(b) shows that $M_{B'} = [M_F \cup$ (a 1-handle)$] \setminus$ (a hole) . Since M_F is irreducible, M_F with a 1-handle is irreducible, so is $M_{B'}$. Therefore only (6) remains to check. Suppose D is a compressing disk for $\partial_h N(\Sigma)$ in $M_{B'}$. If ∂D is on the side of F_+, ∂D must separate two end points of the hole from the cusp, ∂F_+, because F_+ is also incompressible in $M_{B'}$. Hence the hole can be ambient isotopic into $M_{B'} \setminus D_2$ in $M_{B'}$, and we get a contradiction. Otherwise, D must ambient isotopic to D_2. But it is impossible because any ambient isotopy of the hole must meet D_2, so is D.

Now we will show that Σ is essential in $M(\alpha)$, the manifold obtained by any non-trivial α-Dehn surgery along K. Since Σ has not changed after Dehn filling, first four conditions (1)-(4)

immediately hold.

The closure of T_m is a cored solid torus with two meridional cusps. After non-trivial Dehn filling, $T_m(\alpha)$, this component becomes a solid torus with cusps that wind longitudinally at least twice. Therefore $T_m(\alpha)$ is irreducible and its horizontal boundary is incompressible in $T_m(\alpha)$. Note that the first use of this argument may be found in [De]. As a conclusion, (5) and (6) hold again because the other components $M_{B'}$ and T_l have not changed.

References

[Br1]. B. Brittenham, *Persistently laminar tangles, preprint.*

[Br2]. B. Brittenham, *Persistent laminations from Seifert surfaces, preprint.*

[De]. C. Delman, *Essential laminations and Dehn surgery on 2-bridge knots,* Topology Appl. **63** (1995), 201-221.

[GO]. D. Gabai and U. Oertel, *Essential laminations in 3-manifolds,* Annals of Math. **130** (1989), 41-73.