교량의 생애체계신뢰성해석에 기초한 잔존수명예측 연구

Remaining Life Prediction of Deteriorating Bridges Based on Lifetime System Reliability

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요　　약 : 고속도로 건설은 미국을 포함한 여러 국가에서 거의 완료되었으며, 정부나 고속도로 관계기관은 유지관리쪽에 초점을 맞추고 있다. 교량을 효과적으로 유지관리하기 위해, 교량의 실제 내하력과 잔존수명을 예측하는 것은 매우 시급하다. 이러한 목적으로서 시간에 대한 시스템 신뢰성 해석이 필요하다. 이 논문에서는 Lifetime 분포 (함수)를 이용해서, 교량의 잔존수명을 예측하기 위한 전형적인 교량의 모델링기법을 개발하였다. Lifetime의 함수변수를 생성하기 위해서 몬테칼로법을 이용하였다. 결과는 지금 존재하는 교량에 대해 최적의 유지관리 계획에 이용될 수 있다.

ABSTRACT : The construction of highway bridges is almost complete in many countries including the United States. The government and highway agencies change the focus from constructing to maintaining. To maintain the bridges effectively, there is an urgent need to assess actual bridge loading carrying capacity and to predict their remaining life. The system reliability techniques have to be used for this purpose. Based on lifetime distribution (function) techniques, this study illustrates how typical highway bridges can be modeled to predict their remaining life. The parameters of lifetime distribution are generated by Monte. The results can be used for optimization of planning interventions on existing bridges.

핵 심 용 어 : 교량, 시스템신뢰성, 내하력, 잔존수명, 몬테칼로법

KEYWORDS : bridges; system reliability; load carrying capacity; remaining life; Monte Carlo simulation

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1. Introduction

About half of highway bridges in United States are considered to be deficient and therefore are in need of repair or replacement. Half of these are functionally obsolete, and others do not have the required strength [State 1989]. For these bridges, repairs and replacements are needed to increase service life. In order to avoid the high cost of rehabilitation, the rating (evaluation) of these bridges must correctly report the actual load-carrying capacity. The manuals [AASHTO 1983, AASHTO 1994] are used for bridge rating. To predict the remaining lifetime of deteriorating bridges, new tool is needed to model these deteriorating bridges with time.

Resistances and loads are not constant with time. This is because ductility and strength of materials deteriorate with time and are affected by previous loading history, and loads on structures vary with time. Time invariant reliability analysis of structural systems may provide unconservative reliability estimation because it considers only the initial variability of random variables, which may increase with time [Iizuka and Frangopol 1991]. In the case of previous study on bridge degradation, mostly the resistances are the function of time [Kameda and Koike 1975, Estes 1997]. As the time goes on, the resistance (i.e. cross section) decreases. The degradation parameters are used to predict the changing resistance. After the resistances are estimated, the reliability index is computed to assess the bridges with time.

In this paper, the lifetime distributions are used to predict the probability of failure for the components or system. There are several lifetime distributions to describe the evolution of the probability of failure. Mainly, the survivor functions are used in this paper. By using the concept of system reliability and lifetime, this paper addresses the methodology how to model the bridge and how to predict the probability of failure.

2. System Reliability Analysis

2.1 Structure Function

Structure function [Leemis 1995] is a useful tool to describe the state of system with n components. Structure function defines the system state as a function of the component state. A system is assumed to be a collection of n components [Ghosn and Frangopol 1999]. In addition, it is assumed that both components and the system can either be functioning or failed. The state of component i, x_i, is assumed as

$$x_i = \begin{cases} 0 & \text{if component } i \text{ has failed} \\ 1 & \text{if component } i \text{ is functioning} \end{cases} \qquad (1)$$

for $i = 1, 2, \ldots, n$

The n component system can be expressed as a system state vector as following.

$$\mathbf{x} = \{x_1, x_2, \ldots, x_n\} \qquad (2)$$

Structure function, $\phi(\mathbf{x})$, expresses the system state vector **x** to zero or one. The structure function $\phi(\mathbf{x})$ for a given system state vector is

$$\phi(\mathbf{x}) = \begin{cases} 0 & \text{if the system has failed} \\ 1 & \text{if the system is functioning} \end{cases} \tag{3}$$

The most common system is the series and parallel system. For series system, since the any one component failure in the system causes the system failure, the series system is expressed as

$$\begin{aligned} \phi(\mathbf{x}) &= \begin{cases} 0 & \text{if there exists an } i \text{ such that } x_i = 0 \\ 1 & \text{if } x_i = 1 \text{ for all } i = 1, 2, \ldots, n \end{cases} \\ &= \min\{x_1, x_2, \ldots, x_n\} \qquad (4) \\ &= \prod_{i=1}^{n} x_i \end{aligned}$$

For parallel system, all component failures in a system cause the system failure, the parallel system is expressed as

$$\begin{aligned} \phi(\mathbf{x}) &= \begin{cases} 0 & \text{if } x_i = 0 \text{ for all } i = 1, 2, \ldots, n \\ 1 & \text{if there exists an } i \text{ such that } x_i = 1 \end{cases} \\ &= \max\{x_1, x_2, \ldots, x_n\} \qquad (5) \\ &= 1 - \prod_{i=1}^{n} (1 - x_i) \end{aligned}$$

As an example, the structure function is obtained for a 5-component system shown in Fig. 1. Also, Fig. 1 shows the reduction steps. These reduction steps are also expressed as functions through Eq. (6) to Eq. (9).

The first reduction step is a parallel system between components 2 and 3. By first reduction, the subsystem 1 is obtained and expressed as following.

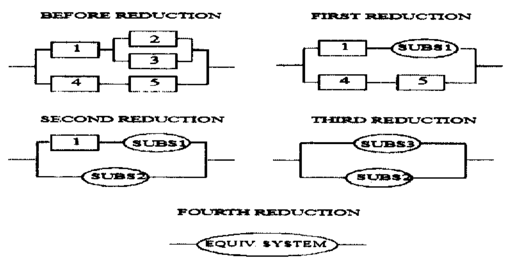


Fig. 1 Sequential Reduction Procedure

$$\phi_{s1}(\mathbf{x}) = 1 - (1 - x_2)(1 - x_3) \tag{6}$$

Where
x_i = State of component i

The second reduction is a series system between components 4 and 5. This is expressed as following.

$$\phi_{s2}(\mathbf{x}) = x_4 x_5 \tag{7}$$

The third reduction is also series system between subsystem 1 and component 1.

$$\phi_{s3}(\mathbf{x}) = x_1 \phi_{S1} \tag{8}$$

By fourth reduction, the structure function for this 5-component system is obtained.

$$\phi(\mathbf{x}) = 1 - \{1 - x_1[1 - (1 - x_2)(1 - x_3)]\}(1 - x_4 x_5) \tag{9}$$

2.2 Reliability Function

The structure function is deterministic.

This assumption may be unrealistic for certain types of components or system. So, reliability functions [Leemis 1995] are necessary to model the structures which are in use. In section 2.1, x_i was defined to be the deterministic state of component i. Now, x_i is a random variable. The probability that component i is functioning is given by

$$p_i = P[x_i = 1] \qquad (10)$$

Where

p_i = Probability that component i is functioning

If there are components, reliability vector of system can be written as

$$\mathbf{p} = \{p_1, p_2, \cdots p_n\} \qquad (11)$$

The system reliability function is defined by

$$r = r(\mathbf{p}) = P[\phi(\mathbf{x}) = 1] \qquad (12)$$

In order to obtain the reliability function for a 5-component system shown in Fig. 1, the same procedure is necessary. But the component reliability function, p, is used in each step instead of component state x.

3. Lifetime Distribution

Reliability function gives the reliability of components or system at specific time t. In this section, the probability of failure is generalized to be a function of time with lifetime distribution. There are several lifetime functions to describe the evolution of the probability of failure. In this paper, one lifetime function is introduced called Survivor function. The lifetime function applies to both discrete and continuous lifetime and is used to describe the distribution of system lifetime, as well as of its components.

The survivor function is the generalization of reliability because the survivor function gives the reliability that a component or system is functioning at one particular time. The survivor function is expressed

$$S(t) = P[T \geq t] \qquad t \geq 0 \qquad (13)$$

It is assumed that when $t \leq 0$, $S(t)$ is one. The survivor function has to satisfy three conditions. These are

1) $S(0) = 1$
2) $\lim_{t \to \infty} S(t) = 0$
3) $S(t)$ is non-increasing without any maintenance

Several distributions are used as survivor functions. The exponential distribution, Weibull distribution, Log-Logistic distribution, and Exponential Power distribution are used in this paper. These survivor functions are shown in table 1.

Table 1 Survivor Function

Distribution	Survivor function
Exponential	$\exp(-\lambda t)$
Weibull	$\exp(-(\lambda t)^{\kappa})$
Log-logistic	$\frac{1}{1+(\lambda t)^{\kappa}}$
Exponential- power	$\exp(1-\exp(\lambda t)^{\kappa})$

Where

λ = Failure rate

κ = Shape factor

t = Time, $t \geq 0$

The failure rate is the frequency of failure in unit time (hours, years, etc.). The failure rate is defined as following.

$$\lambda(t) = \frac{f(t)}{S(t)} \qquad t \geq 0 \tag{14}$$

Where

$$f(t) = -\frac{dS(t)}{dt}$$

Exponential survivor function is only one parameter distribution and has constant failure rate. Others have two parameters (failure rate and shape factor). Depending on shape factor, κ, survivor functions have an increasing failure rate or constant failure rate or decreasing failure rate. The typical trends of each survivor functions are shown in Fig. 2 to Fig. 5.

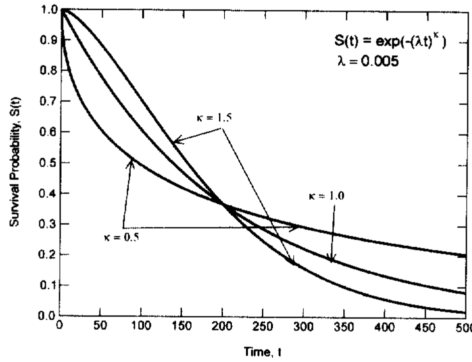


Fig. 3 Weibull Survivor Function

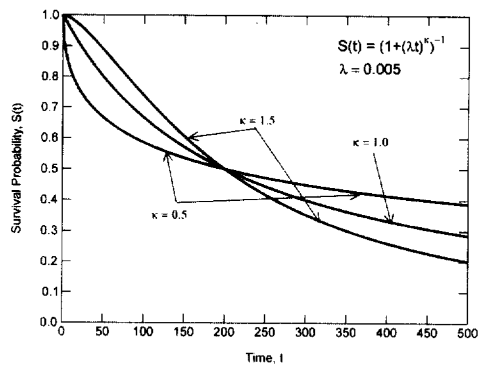


Fig. 4 Log-logistic Survivor Function

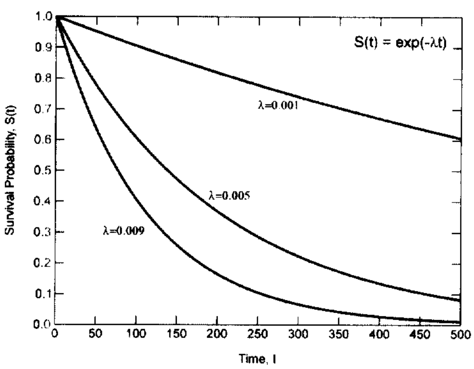


Fig. 2 Exponential Survivor Function

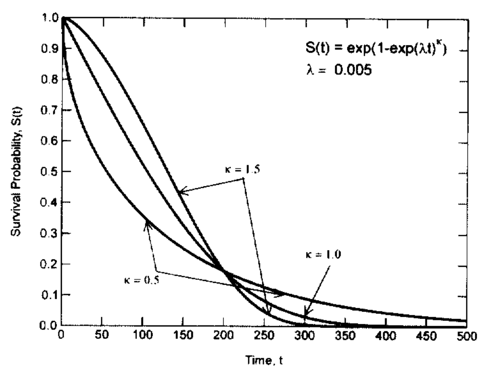


Fig. 5 Exponential Power Survivor Function

In order to find out lifetime function for system, the concept of section 2 and section 3 are used. To make lifetime function for system, the component survivor functions are used as arguments. As an example, if there is a three-component series system with independent relation for each component, the system reliability function is

$$S(t) = S_1(t)S_2(t)S_3(t) \quad (15)$$

Where

$S_i(t)$ = Survivor function of component i

For three-component parallel system, the system lifetime function is

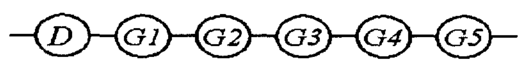
$$S(t) = 1-(1-S_1(t))(1-S_2(t))(1-S_3(t)) \quad (16)$$

4. Modeling Methodology for Real Bridge

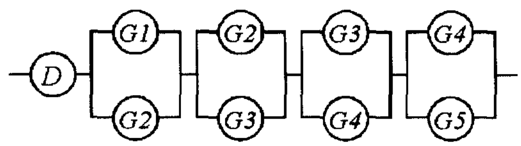
Due to nonlinearity in multi-girder bridge types, single girder failure doesn't cause the bridge failure. When one girder fails on bridge, the load redistribution takes place and the bridge is capable to carry additional loads. The multi-girder bridges are modeled as combination of series and parallel systems in reliability analysis. For example, if the bridge has five girders with concrete deck, the bridge may be modeled as combination of failure model as follows:

- Any one girder failure or deck failure causes the bridge failure.
- Any two adjacent girder failures or deck failure cause the bridge failure.
- Any three adjacent girder failures or deck failure cause the bridge failure.
- Failure of any external girder or any two adjacent internal girders or deck failure cause the bridge failure.

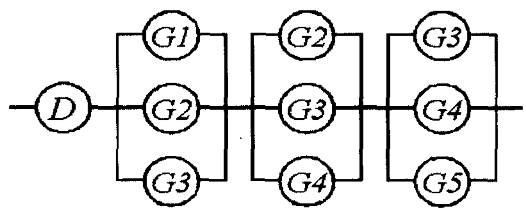
These failure models are shown in Fig. 6. With these failure modes, the reliability analysis will be performed for each bridge.



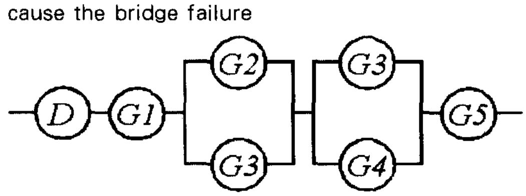
Any one girder failure or deck failure causes the bridge failure



Any two adjacent girder failures or deck failure cause the bridge failure



Any three adjacent girder failures or deck failure cause the bridge failure



Failure of any external girder or any two adjacent internal girders or deck failure cause the bridge failure

Where

D = Deck failure

G1 and G5 = Exterior girder failure

G2, G3, and G4 = Interior girder failure

Fig. 6 Failure Modes

The FORTRAN program is developed to predict the probability of failure by using the concept of system reliability and lifetime function. Since the failure rates of survivor functions are obtained by data analysis, that may have different distribution types for each data. When the program is developed, the Monte Carlo simulation is used to simulate the failure rates of survivor functions.

Fig. 7 and Fig. 8 shows the series system when the failure rate is deterministic and random variable, respectively. It is assumed that each component is independent and has same failure rate. The values of failure rates are on the figures.

Fig. 9 and Fig. 10 are for parallel system. Also, each component is independent.

Fig. 11 and Fig. 12 shows the cumulative-time failure probability of 3-component system. The system is shown on each figure. The failure rate of component 1 changes on Fig. 11 and the failure rate of component 3 changes on Fig. 12. In these figures, each component is independent and

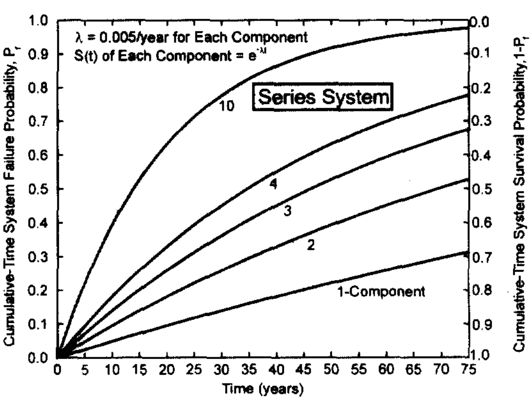


Fig. 7 Effect of Number of Components on Cumulative-Time Failure Probability of Series System

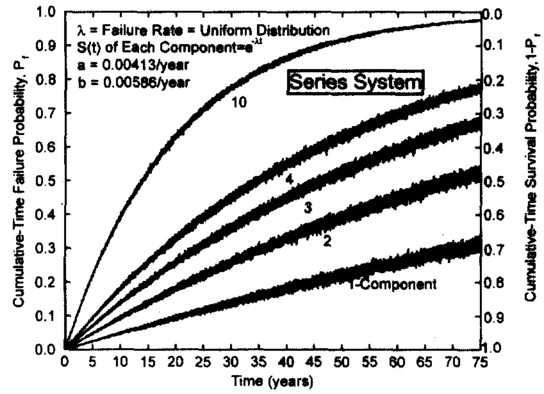


Fig. 8 Effect of Number of Components on Cumulative-Time Failure Probability of Series System

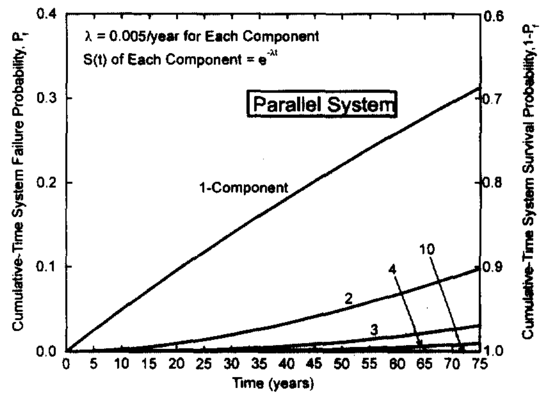


Fig. 9 Effect of Number of Components on Cumulative-Time Failure Probability of Parallel System

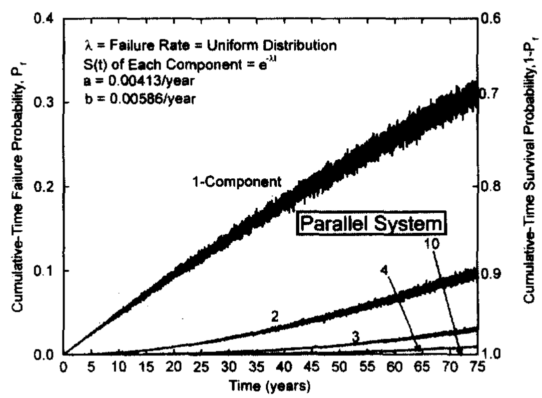


Fig. 10 Effect of Number of Components on Cumulative-Time Failure Probability of Parallel System

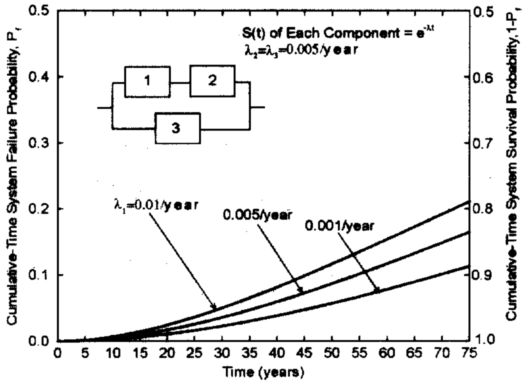


Fig. 11 Effect of Failure Rate of Component 1 on Cumulative-Time Failure Probability.

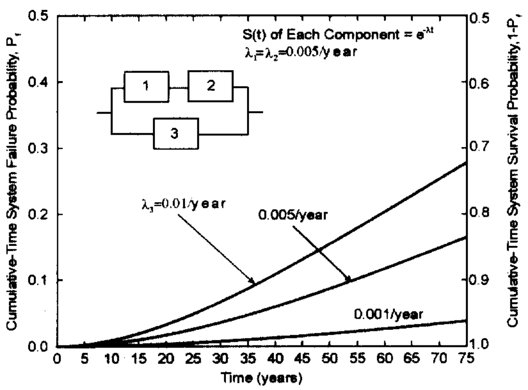


Fig. 12 Effect of Failure Rate of Component 3 on Cumulative-Time Failure Probability.

Table 2 Case Study

Case	Component 1	Component 2	Component 3
A	Exponential Uniform -	Weibull Uniform $\kappa=1.5$	Log-logistic Uniform $\kappa=1.5$
B	Exponential Uniform -	Exponential Uniform -	Exponential Uniform -
C	Weibull Uniform $\kappa=1.5$	Weibull Uniform $\kappa=1.5$	Weibull Uniform $\kappa=1.5$
D	Log-logistic Uniform $\kappa=1.5$	Log-logistic Uniform $\kappa=1.5$	Log-logistic Uniform $\kappa=1.5$

same survivor function.

As a case study, when each component has different survivor function, the probability of failure is predicted for three-component system. The survivor functions for each case are summarized in table 2.

In case A, each component has a different survivor function: component 1 is Exponential, component 2 is Weibull, and component 3 is Log-logistic. For all components, the failure rate is assigned a uniform distribution whose parameters are 0.00413/year and 0.00586/year, and the shape factor is 1.5. In case B, each component has an Exponential survivor function. In case C, each component has a Weibull. In case D, each is Log-ogistic. The parameter values used in cases B, C, and D are the same as that of case A for each survivor function.

The probability of failure for each case is shown in following figures.

Fig. 13 is the result of case A. Fig. 14 is for case B. Fig. 15 and Fig. 16 are for case C and case D, respectively. All cases are compared in Fig. 17. From Fig. 17, the

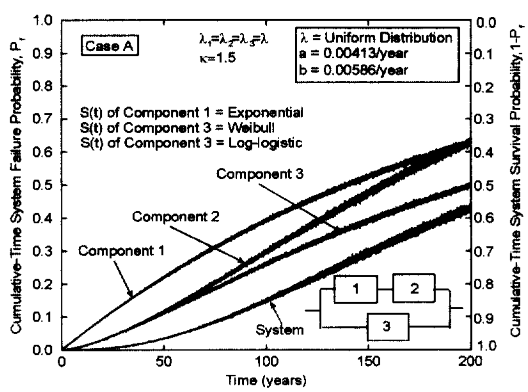


Fig. 13 Cumulative-Time Failure Probability for Case A

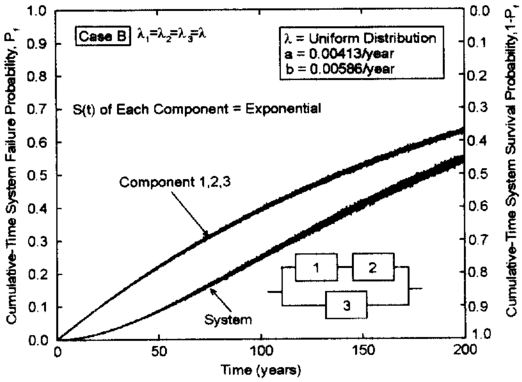


Fig. 14 Cumulative-Time Failure Probability for Case B

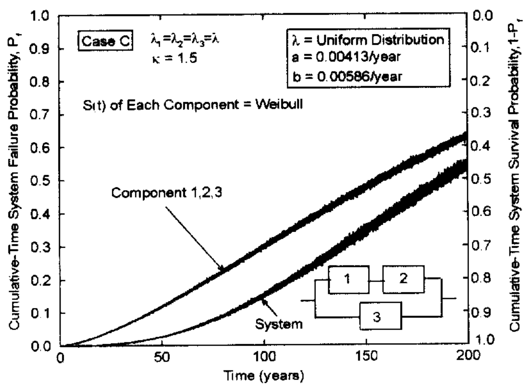


Fig. 15 Cumulative-Time Failure Probability for Case C

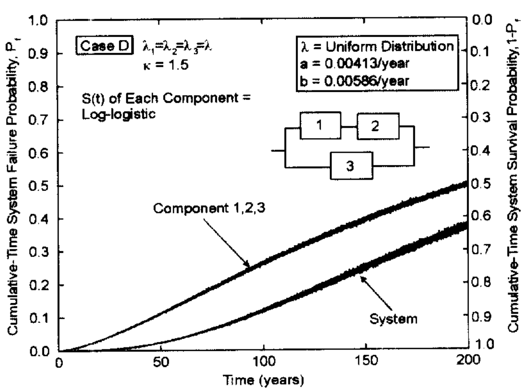


Fig. 16 Cumulative-Time Failure Probability for Case D

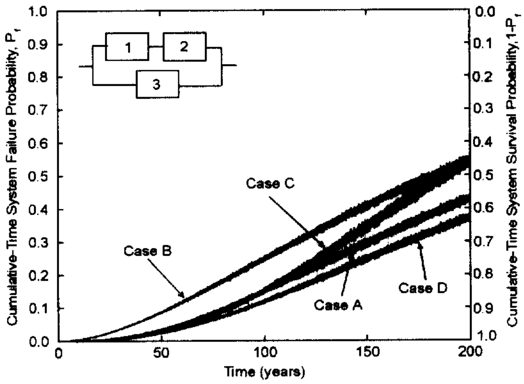


Fig. 17 Cumulative-Time Failure Probability for All Cases

case B is worst case. The reason is that the shape factor used in each survivor function is bigger than 1. When shape factor is bigger than 1, the failure rate is increased from zero to infinity. Except case B, the slops of rest of cases are not steep at the beginning but the slops are steep with time. From the results, it is easy to see and compare the time dependent system reliability.

5. Conclusion

Based on the concept of system reliability and lifetime function, the FORTRAN program was developed and used to predict the probability of failure for the system, which is combination of series and parallel system. Monte Carlo simulation is used to simulate the failure rate. The survivor function is useful tool to compare the survivor patterns of each component or system. As shown in Fig. 17, it is easy to compare survivor patterns. And, because the program gives the probability of failure with time, the optimal intervention time can be computed and intervention plan can be made by using this program.

References

1. American Association of State Highway Transportation Officials, (1983). Manual for Maintenance Inspection of Bridges, Washington, D.C., p 53.
2. American Association of State Highway Transportation Officials, (1994). Manual for Condition Evaluation of Bridges, Washington, D.C., p 153
3. Estes, A. C., (1997). A system reliability approach to the lifetime optimization of inspection and repair of highway bridges, Ph. D. thesis, University of Colorado, p 741.
4. Ghosen, M and Frangopol, D.M., (1999). Bridge Reliability: Components and Systems, Chapter 4 in Bridge Safety and Reliability, D. M. Frangopol, ed., ASCE, Reston, Virginia, pp 83-112.
5. Kameda, H. and Koike, T., (1975). Reliability theory of deteriorating structures, Journal of the Structural Division, Proceedings of ASCE, 10 (ST1), pp 295-310.
6. Iizuka, M., and Frangopol, D.M. (1991). Time Invariant and Time Variant Reliability Analysis and Optimization of Structural System, Technical Report No. 91-02, University of Colorado at Boulder, p 369
7. Leemis, L. M., (1995). Reliability, probabilistic models and statistical methods, Prentice-Hall, New Jersey, p 319.
8. FHWA, U.S Department of Transportation, (1989). The state of nation's highway and bridge: condition and performance and highway bridge replacement and rehabilitation program, Washington, D.C.

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