강접골조의 수직 및 수평하중에 대한 근사해석

Approximate Analysis of Rigid Frames under Vertical and Lateral Loads

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요 약 : 오늘날 컴퓨터가 복잡한 해석을 수행하는 도구로 이용되더라도 해석의 결과값에 대한 적합성을 평가하기 위해서는 공학적 판단이 필요하다. 근사해석법은 이러한 결과값을 검토하기 위한 편리한 도구가 될 수 있으며 골조의 응력분포 그리고 부재 사이즈의 선택을 위한 기초가 될 수 있다. 본 연구의 목적은 유효길이계수와 변곡점을 이용하여 수직하중 및 수평하중을 받는 강접골조를 근사해석하기 위한 방법을 제시하는 것이며 본 연구에서 제안한 방법의 유효성을 검증하기 위해서 기존의 방법과 비교하였고 구조물 거동의 예측에 있어서 향상된 결과를 보여주었다.

ABSTRACT : Even in today's computer-oriented world with all its sophisticated analysis tools, engineering judgement is required to assess the adequacy of computer output. Approximate analysis method can be a feasible tool to check solutions from computer softwares roughly. It can be a simple tool for structural engineer to check force distribution in frame. Also, it can serve as a basis in selecting preliminary member sizes. The objective of this study is to propose an improved approximate method for rigid frame using effective length factor and inflection points. The validity of this method is examined by comparing the results of this method with those of existing methods, showing improvement in the prediction of structural behavior.

핵 심 용 어 : 근사해석, 변곡점, 유효길이계수

KEYWORDS : Approximate analysis, Inflection point, Effective length factor

1. Introduction

Recently structural behavior can be easily visualized with the use of various structural analysis tools. Preliminary design generally start with a rough guess and trial-and-error based on engineers' experience, working towards a finial design. Approximate anal-

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ysis method can be a feasible tool to check a solution from computer softwares roughly. It can be a simple tool for structural engineer to check force distribution in frames.

Also, it can serve as a basis in selecting preliminary member sizes because the design of a structure, no matter how simple or complex, begins with a tentative selection of members. DeWolf[1] addressed approximate analysis of frames subjected to vertical loads and provided guidelines for preliminary member size. Epstein[2] suggested simple formula to locate approximate inflection point. Behr[3],[4] analyzed frames subjected to vertical load using proposed inflection points in beams and columns. Wu[5] proposed an approximate method by revising Epstein's formula about inflection points in beams and Behr's assumptions about inflection points in columns, and analyzing frames storywisely with moment redistribution approach.

For frames subjected to lateral loads, the portal or cantilever method[6] are still widely used to find member forces. Haris[7] proposed a method for the analysis of frames subject to lateral load based on the approximate stiffness of frames and the assumption that inflection points in all columns are in the middle height.

The objective of this study is to propose an improved approximate method for rigid frame analysis using effective length factor and inflection points. The validity of this method is examined by comparing the result of this method with those of existing methods and structural analysis program MIDAS[8]. To compare the approximate method presented in this paper with the existing methods and MIDAS, six frames are analyzed.

2. Development of Approximate Analysis

Generally adopted assumptions for the approximate analysis of frames subjected to vertical loads are that the inflection points are located at a distance of 0.1L from each end of the girder[6]. This may lead to poor prediction of moment in column. Because moments in columns can be affected significantly by the location of inflection points in beams, which varies due to relative stiffness of beam and column. Thus, varying inflection point in beam, such as Epstein's formula, may lead to better result. Epstein's formula adopted in this study is as follows:

$$\frac{x}{l} = \left(\frac{1-d}{\beta-\alpha d}\right)\alpha\beta \qquad (1)$$

where, x=location of inflection points of beams, d= moment distribution factor, l= span of beam, α= factor corresponding to location of inflection points, and β= 2/3 (β= 1 when the far end is pinned)

For braced frames, the French rules[9] proposes the following approximate solution for the effective length factor:

$$K=\frac{3\,G_A\,G_B+1.4(G_A+G_B)+0.64}{3\,G_A\,G_B+2.0(G_A+G_B)+1.28} \qquad (2)$$

where $G_A = \frac{\sum_A \left(\frac{EI}{L}\right)_c}{\sum_A \left(\frac{EI}{L}\right)_b}$ $G_B = \frac{\sum_B \left(\frac{EI}{L}\right)_c}{\sum_B \left(\frac{EI}{L}\right)_b}$

In this study, procedure for assessing moments in beams and columns is as follows:

(1) Locate inflection point in beam according to eq.(1)

(2) Compute K according to eq.(2)

(3) Isolate column with hinged-fixed boundary condition and find moments at the top and bottom in column

(4) Apply releasing moment at the bottom of the column and distribute it to the beam and the column

For the frames under lateral load, the location of inflection points in beams is generally at the mid-span. In the portal and cantilever methods, the location of inflection points in all columns is 0.5H, where H is the column height. However, in this study, after examining the results of Behr's study[4], the location of inflection points at the first-and top-story is assumed at 0.3H and 0.65H, respectively. The procedure for the calculation of member forces is the same as the cantilever method.

For a normally proportioned rigid frame, the total drift at level i can be conceived as the sum of lateral deformation of all the columns at floor level i, the rotation of all joints, and the axial deformation of columns, and it is expressed as[7]

$$D_i = \sum_{j=1}^{i} \frac{M_{cj} h_j^2}{6E_c I_{cj}} + \left[\theta_i \frac{h_i}{2} + \sum_{j=1}^{i-1} \theta_j \left(\frac{h_j + h_{j+1}}{2}\right)\right] + \sum_{k=1}^{i} \alpha_k \left(\frac{h_k}{2} + \sum_{j=k+1}^{i} h_j\right) \quad (3)$$

In eq.(3), the rotation of the joint θ and the frame vertical rotation from its center line α are determined as

$$\theta = \frac{M_b L_b}{6E_b I_b}, \alpha = \frac{e_1 - e_2}{L_d}, e = \frac{P_c h}{E_c A_c}$$

where, b and c stand for beam and column, respectively and e_1, e_2 are the elastic axial deformation of two columns departed by the length of L_d.

3. Numerical Examples

To compare the accuracy of approximate methods six frames are analyzed.

3.1. Frames subject to Vertical Load

Example 1.

The frame shown in Fig. 1 is analyzed by Wu[5]. In Table 1 moments in beams and columns are compared. Considering the average error which was obtained by using the absolute value of error, the proposed method showed a little improved result. However, the maximum error of the Wu

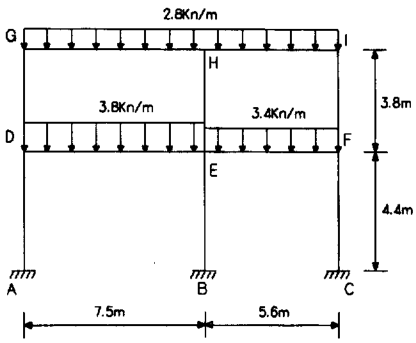

Fig. 1 Frame geometry and loading

Table 1. Comparison of moments (unit: t・m)

Member	Joint	Wu (A)	This study (B)	MIDAS (C)	Error(%)	
					(A-C)/C	(B-C)/C
1	A	-3.74	-2.61	-1.95	91.8	33.8
	D	7.48	4.99	5.53	35.3	-9.8
2	B	-1.24	-1.66	-1.68	-26.2	-1.2
	E	2.49	2.30	2.35	5.96	-2.1
3	C	-1.08	-1.60	-1.30	-16.9	23.1
	F	2.15	2.23	1.91	12.6	16.8
4	D	-5.88	-2.94	-4.85	21.2	-39.4
	G	7.01	8.24	6.38	9.9	29.2
5	E	-2.79	-1.72	-3.11	-10.3	-44.7
	H	-2.16	2.93	4.30	-150.2	-31.9
6	F	-1.38	-1.12	-1.65	-16.4	-32.1
	I	1.60	1.90	1.78	-10.1	6.7
7	D	11.47	12.07	13.46	-14.8	-10.3
	E	17.40	17.04	19.25	-9.6	-11.5
8	E	12.96	12.93	14.25	-9.1	-9.3
	F	3.10	4.25	3.56	-12.9	19.4
9	G	5.68	8.24	6.38	-11.0	29.2
	H	13.44	13.07	15.38	-12.6	-15.0
10	H	10.93	11.07	11.08	-1.4	-0.1
	I	1.28	1.90	1.78	-28.1	6.7
Average(%)					25.3	18.6

Note: The average error was obtained using the absolute value of error.

method was 150% at the joint H in member 5, while that of the proposed method was 45% at the joint E in member 5. The error range of this method is much narrower than the Wu's method.

Example 2.

To find out overall performance of the proposed method mid-and high-rise frames are analyzed. The members of the frames are listed in Table 2.

In a building frames, considerable amounts of the differential columns shortening is accumulated in the members of the upper stories, and so are the bending moments and shear forces when the vertical load analysis for the frame is performed by an ordinary method, such as the finite element analysis of complete frame as a whole. These differential column shortenings and bending moments due to the dead weight may be overestimated and considered incorrect because the ordinary frame analysis methods do not take into account the sequential nature of the construction and of the application of its weight(10).

In Fig. 2, inflection point in columns are compared with other methods, where MIDAS(A) is the result obtained with considering sequential application of vertical load. While MIDAS(B) is tantamount to

Table 2. Frame configuration and member properties

	15story-3bay frame	30story-4bay frame
H(m)	1story : 4.5	1story : 4.5
	2~15story : 3.5	2~30story : 3.5
L(m)	8	8
Column $A(m^2)$ $I(m^4)$	1~5story: A=0.36 I=0.0108 6~10story: A=0.25 I=0.0052 11~15story: A=0.16 I=0.0021	1~10story: A=1.00 I=0.083 11~20story: A=0.64 I=0.0341 21~30story: A=0.36 I=0.0108
Beam	1~5story: A=0.50 I=0.041 6~10story: A=0.40 I=0.021 11~15story: A=0.24 I=0.0072	1~10story: A=0.96 I=0.115 11~20story: A=0.70 I=0.058 21~30story: A=0.40 I=0.021

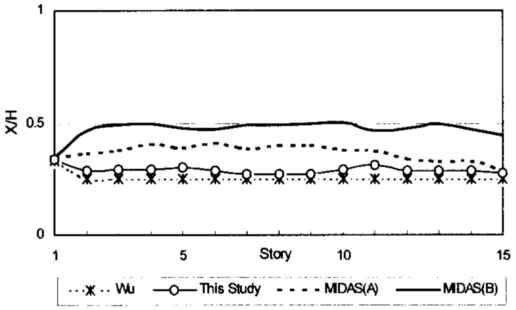

Fig. 2 Location of inflection points

the application of simultaneous loading to the frame. The proposed method is a little closer to MIDAS(A) than the Wu's method. The fact that this method and the Wu's methods are close to MIDAS(A) results from the analysis procedure. These two methods analyze frame from the top to the bottom, which is similar to the step in MIDAS(A).

In Fig. 3, end moments in columns are compared. The average error of the proposed method was the half of that of the Wu's method when compared to MIDAS(A). However, this advantage of this study does not exist when compared to MIDAS(B). Similar trend can be found in the shear force of column shown in Fig. 4.

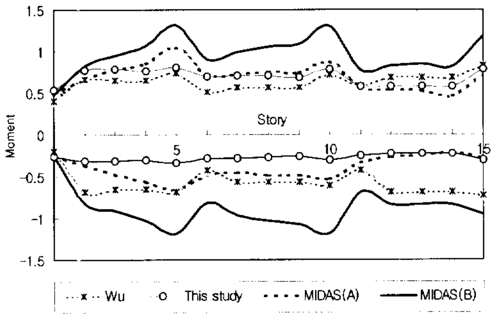

Fig. 3 End moments in column

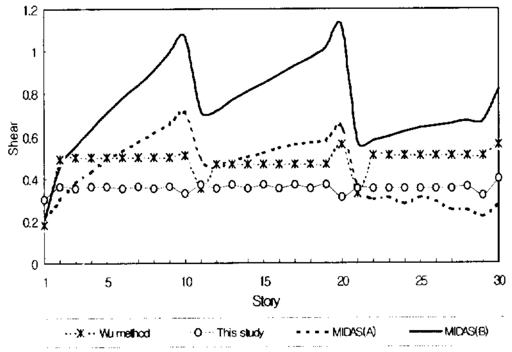

Fig. 4 Shear in column

Example 3.

In the 30-story frame the inflection point by approximate methods shown in Fig. 5 is deviated a lot from MIDAS except at the top and bottom of the frame. This causes erroneous prediction of column moments as can be seen in Fig. 6.

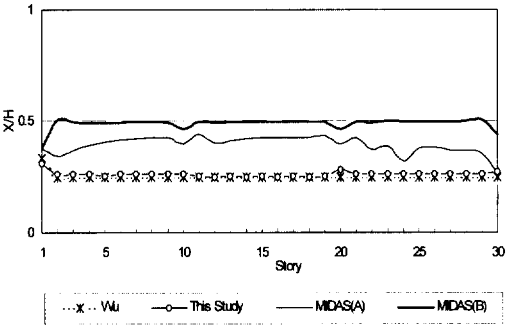

Fig. 5 Location of inflection points

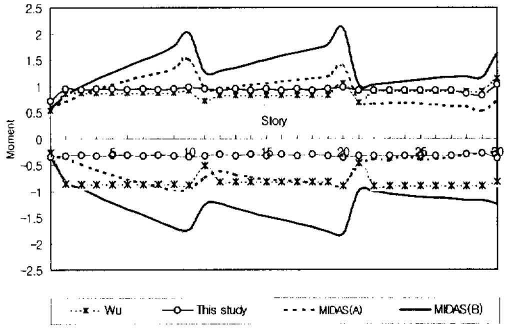

Fig. 6 End moments in column

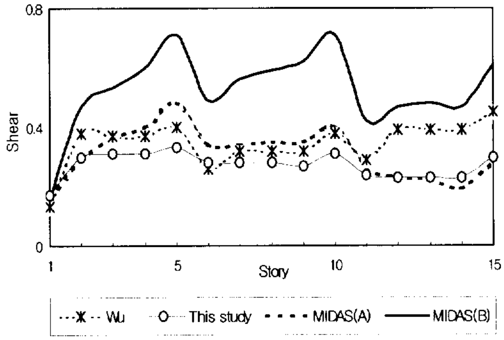

Fig. 7 Shear in column

Shear force computed from column moment in approximate method could not be predicted correctly as seen in Fig. 7. Both approximate methods could not represent well structural characteristics at the location of the stiffness change, where error was maximized.

3.2 Frames subject to Lateral Loads

Example 1.

To investigate the applicability of the proposed method to frame under lateral loading 4-story frame shown in Fig. 8 is analyzed.

As can be found from Table 3 and 4, the proposed method shows somewhat improved prediction.

Example 2.

This example is 15story-3bay frame in Table 2 with concentrated lateral load of 4 ton at every floor level. Cantilever and proposed methods show very similar result in moment and shear shown in Fig. 9 and 10. Maximum error of moment by cantilever method was 64% at the top story while that of the proposed method was 51% at the first story.

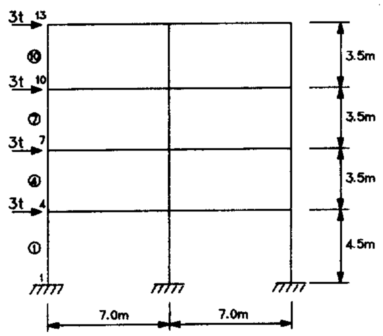

Fig. 8 Frame geometry and loading

Table 3. Comparison of end moment in column

Member	Joint	Cantilever method (A)	This study (B)	MIDAS (C)	Error(%)	
					(A-C)/C	(B-C)/C
1	4	6.31	6.23	7.39	-14.6	-15.7
	1	-6.31	-11.29	-9.49	-33.5	-19.0
4	7	3.95	3.44	3.65	8.2	-5.8
	4	-3.95	-3.44	-4.64	-14.9	-25.9
7	10	2.63	3.14	2.85	-7.7	10.2
	7	-2.63	-3.14	-3.23	-18.6	-2.8
10	13	1.31	1.86	1.70	-22.9	9.4
	10	-1.31	-0.80	-1.11	18.0	-27.9
Average(%)					17.3	14.6

Table 4. Comparison of shear in column

Story	Cantilever method (A)	This study (B)	MIDAS (C)	Error(%)	
				(A-C)/C	(B-C)/C
1	-2.80	-3.89	-3.75	-25.33	3.73
2	-2.26	-1.97	-2.37	-4.64	-16.87
3	-1.50	-1.79	-1.66	-9.64	7.83
4	-0.74	-0.76	-0.80	-7.50	-5.0
Average(%)				11.78	8.36

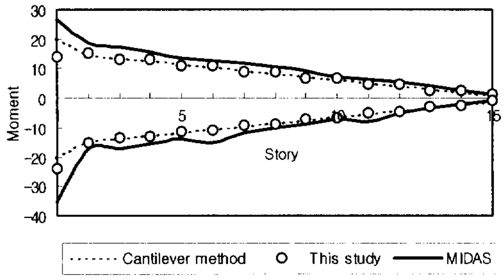

Fig. 9 End moment in column

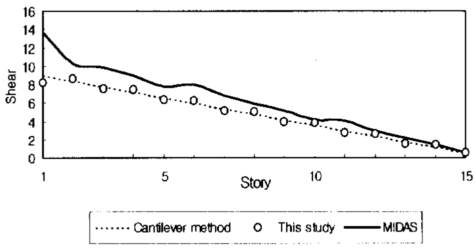

Fig. 10 Shear in column

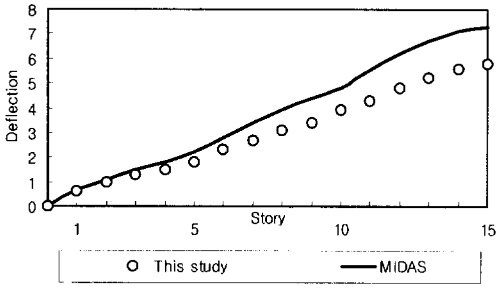

Fig. 11 Story drift

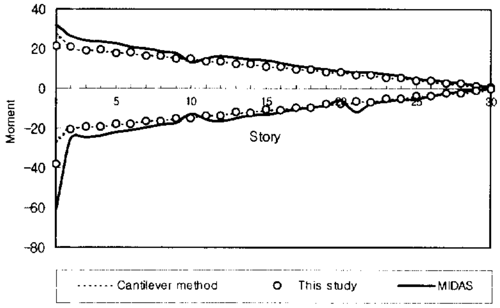

Fig. 12 End moments in column

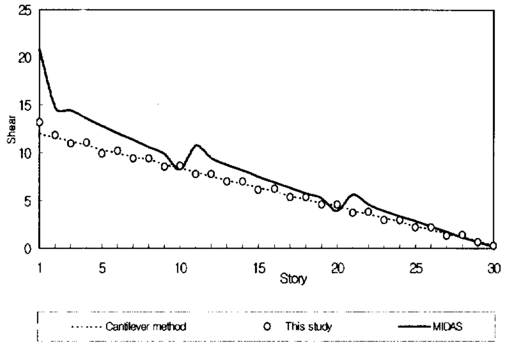

Fig. 13 Shear in column

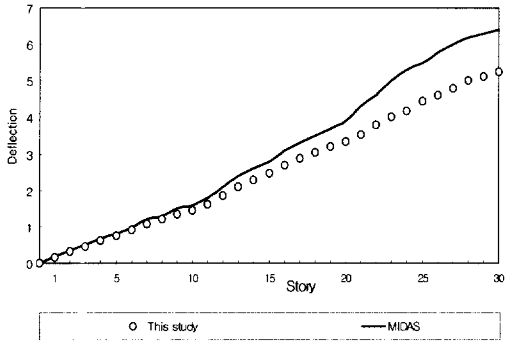

Fig. 14 Story drift

Story drift based on eq.(3) is shown in Fig. 11 and it was underestimated by 20% at the top story.

Example 3.

This example is 30story-4bay frame in Table 2 with concentrated lateral load of 4 ton at every floor level. Accuracy of moment prediction for this frame(Fig. 12) was similar to that of 15story frame. The maximum error of cantilever method was 112% while that of the proposed method was 70% at the 29th story. The shear prediction(Fig. 13) showed no difference between the two approximate methods in overall, but the cantilever method had maximum error of 78% at the top story while this study produced only 17% error.

Story drift based on eq.(3) is shown in Fig. 14 and it was underestimated by 13% at the top story.

4. Conclusion

This study is focused on improving the accuracy of approximate method for rigid frame analysis while maintaining the simplicity of computational procedure. As mentioned earlier, approximate method can be a useful tool for preliminary design or be instrumental in studying force distribution in rigid frames.

The findings of this study can be summarized as follows:

For the vertical load

- the proposed method which is simple and easy to apply showed better performance than the existing method when the sequential application of vertical load is considered.

- the error range of the proposed method was much narrower than the existing method, which means improved reliability in estimating forces with simple calculation.
- the proposed and the Wu's methods were unable to represent well the structural behavior at the location of the stiffness change in frame.

For the lateral load

- the proposed method showed a little better result than the cantilever method, especially, at the top and the base of the frames
- story drift was underestimated over 13%. To lessen this error moments in beam and column have to be predicted more accurately.
- the error range of the proposed method was much narrower than the existing method, but, force prediction by approximate methods at the base and the top of frame still requires refinement.

REFERENCES

1. John T. DeWolf, "Preliminary Member Sizes : Rigid Frames", J. of Structural Division, ASCE, Vol. 107, No. ST5, May, 1981
2. Howard I. Epstein, "Approximate Location of Inflection Points", J. of Structural Engineering, ASCE, Vol. 114, No. 6, June, 1988
3. R. A. Behr, C. H. Goodspeed, and R. M. Henry, "Potential errors in approximate method of structural analysis,"J. of Structural Engineering, Vol.115, No.4, April, 1989
4. R. A. Behr, E. J. Grotton, and C. A. Dwinal "Revised Method Of Approximate Structural Analysis", J. of Structural Engineering, ASCE, Vol.116, No. 11, 1990.
5. JingHai Wu, "Approximate Analysis Of Building Frames For Vertical Loads", J. of Structural Engineering, ASCE, Vol. 121, No. 4, 1995
6. Aslam Kassimali, Structural Analysis, 2nd ed. Brooks/Coke Publishing Co., 1999.
7. Ali A. K. Haris, "Approximate Stiffness Analysis of High-Rise Buildings", J. of Structural Division, ASCE, Vol. 104, No. ST4, April, 1978
8. MIDAS/GEN Ver.3.5 User's Manual, POSCO Engineering & Construction Co., Ltd, 1999.
9. P. Dumonteil, "Simple Equations for Effective Length Factors," Engineering Journal, AISC, Vol. 9, No. 3, pp.111-115, 1992.
10. Chan K. Choi and E D. Kim, "Multistory frmaes under sequential gravity loads", J. of Strucutral Engineering, ASCE, Vol. 111, No.11, November, 1985.

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