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Experimental Investigation for Flexural Stiffness of Paperboard-stacked Structure

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Abstract

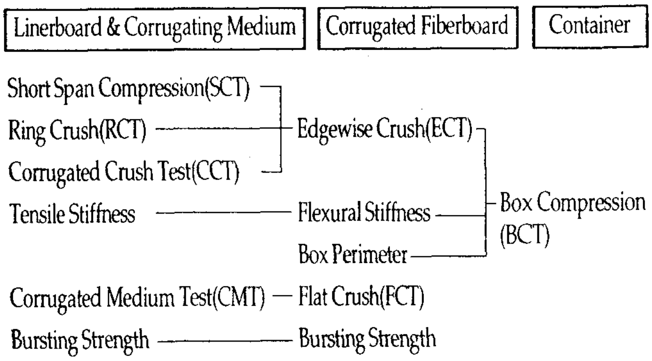
Top-to-bottom compression strength of corrugated fiberboard boxes is partly dependent on the load-carrying ability of the central panel areas. The ability of these central areas to resist bending under load will increase the stacking strength of the box. The difference of box compression strengths, among boxes which are made with identical dimensions and fabricated with same components but different flute sizes, is primarily due to difference of the flexural stiffness of the box panels. Top-to-bottom compression strength of a box is accurately predicted by flexural stiffness measurements and the edge crush test of the combined boards. This study was carried out to analyze the flexural stiffness, maximum bending force and maximum deflection for various corrugated fiberboards by experimental investigation. There were significant differences between the machine direction (MD) and the cross-machine direction (CD) of corrugated fiberboards tested. It was about 50% in SW and DW, and 62%~74% in dual-medium corrugated fiberboards(e.g. DM, DMA and DMB), respectively. There were no significant differences of maximum deflection in machine direction among the tested fiberboards but, in cross direction, DM showed the highest value and followed by SW, DMA, DMB and DW in order. For the corrugated fiberboards tested, flexural stiffness in machine direction is about 29%~48% larger than cross direction, and difference of flexural stiffness between the two direction is the lowest in DMA and DMB.

Keywords: corrugated fiberboard box, flexural stiffness, maximim bending force, maximum deflection, top-to-bottom compression strength

Introduction

The investigations to evaluate mechanical relations from liners and medium to corrugated board and box need characteristics of the final products to be understood and very various test methods should be followed for those evaluations(1~8).

The compression strength of corrugated fiberboard box is determined by edgewise compression strength (ECS) and flexural stiffness of corrugated fiberboard and perimeter of the box(3, 6, 7, 9~11). Especially, the flexural stiffness of corrugated fiberboard is directly related with the buckling of box and this is the important characteristics that is required for practical use of corrugated fiberboard box.



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The test methods for flexural stiffness of corrugated fiberboard are classified by the number of points on which the specimen is loaded along the span. The four-point bending test is the most common methods and authorized standardization related with the corrugated fiberboard also adapts this methods*(12)*.

The purpose of this study is to analyze relations among the flexural stiffness, bending force and bending deflection of corrugated fiberboard which are tested by four-point bending test for various geometrical structures of corrugated medium, and to analogize the main constructional factors for improvement of compression strength.

Theoretical review for flexural stiffness

1) The relations for flexural stiffness and compression strength of container

Fiberboard is composed of cellulose fibers which are arranged with the same direction as the fiberboard manufactured. The direction is classified with machine direction (MD) and cross-machine direction (CD). Therefore, fiberboard is orthotropic material that has totally different strength properties at each direction and especially, at machine direction, the mechanical properties of fiberboard is superior.

Normally, the direction of linerboard and medium is correspond with producing direction of corrugated fiberboard (Fig. 1). When the corrugated box is loaded, the compression strength is endured by four corners and panels of box. Four vertical corners of box depend on ECS of combined board on vertical flute and panels depend on flexural stiffness at both principal directions*(1, 3, 5, 6, 7, 13)*. Therefore, ECS and flexural stiffness of the corrugated board are the main factors to determine the stacking strength of the corrugated fiberboard box.

The flexural stiffness is effected by linerboard at MD and corrugated medium at CD. Machine direction flexural stiffness means force works at perpendicular to MD of corrugated fiberboard and the span of tested specimen is parallel to MD of the board (Fig. 7).

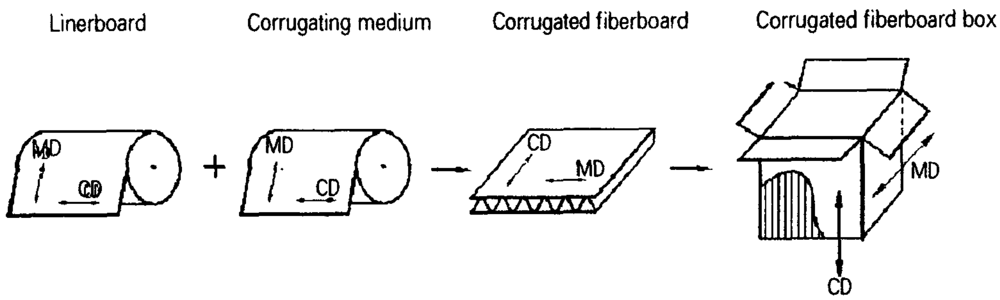

Fig. 1. Machine direction and cross-machine direction according to the manufacturing processes of corrugated fiberboard box

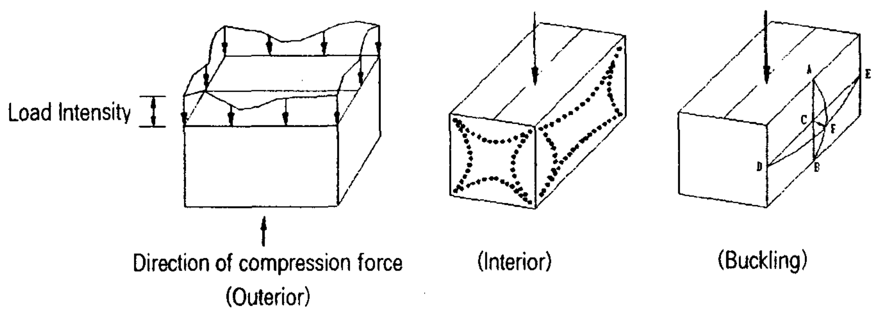

Fig. 2. Stress distribution and buckling configuration in a corrugated fiberboard box

2) Theory of flexural stiffness for flexural force

The analysis for flexural stiffness of corrugated fiberboard have several test methods - two point, three point and four-point bending test. Those test methods are classified by load applied point. It has been recognized that four-point bending test is the most confident test method among those(6, 14~16).

The characteristics of four-point bending test is that bending stresses without shear stresses act over the central span, L, between supporting point as the loading arrangement diagrammed in Fig. 3. Therefore, the corrugated fiberboard is in a state of pure bending (BMD in FIG. 3) and bent as a circular arc with no shear (pure moment beam, SFD in Fig. 3). These enable to get the relation of pure bending moment and deflection for corrugated fiberboard.

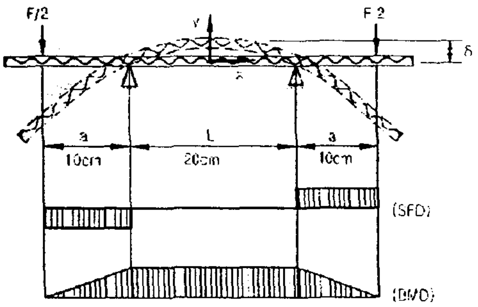

Fig. 3. Loading principle and distribution of bending moment for four-point bending test

A differential equation of elastic line for neutral axis as shown at Fig. 3 is

$$EI\frac{d^2y}{dx^2} = -M \quad (1)$$

At four-point bending moment, the maximum bending moment is $M = \frac{F}{2}a$, therefore, a differential equation of elastic line is integrated below.

$$EI\frac{dy}{dx} = -\frac{Fa}{2}x + C_1 \quad (2)$$

$$EIy = -\frac{Fa}{4}x^2 + C_1x + C_2 \quad (3)$$

Boundary condition at Fig. 3 is $\frac{dy}{dx}=0$ in $x = a + \frac{L}{2}$ and y=0 in $x = a$. From these two conditions, therefore, integral constant C_1, C_2 of Equation (2) and (3) are derived.

$$C_1 = \frac{Fa}{2}\left(a + \frac{L}{2}\right),\ C_2 = \frac{Fa^3}{4} - \frac{Fa^2}{2}\left(a + \frac{L}{2}\right)$$

From Equation (3), flexural stiffness, EI(E=modulus of elasticity, I=monent of inertia) is represented by Equation (4).

$$EI = \frac{1}{y}\left[-\frac{Fa}{4}x^2 + \frac{Fa}{2}\left(a + \frac{L}{2}\right)x + \frac{Fa^3}{4} - \frac{Fa^2}{2}\left(a + \frac{L}{2}\right)\right] \quad (4)$$

Supposed deflection at $x = a + \frac{L}{2}$, center of specimen within proportional limit diagrammed at Fig. 3, is δ, Equation (4) is expressed as:

$$EI = \frac{1}{\delta}\left[\frac{Fa}{4}\left(a + \frac{L}{2}\right)^2 + \frac{Fa^3}{4} - \frac{Fa^2}{2}\left(a + \frac{L}{2}\right)\right],\ Nm^2 \quad (5)$$

Supposed $a = \frac{L}{2}$, S_b (Nm), flexural stiffness per unit width of specimen can be denoted by grouping the same meaning items as Equation (6).

$$S_b = \frac{EI}{W} = \frac{FL^3}{32w\delta} = \frac{1}{16}\left(\frac{F}{\delta}\right)\left(\frac{L^3}{W}\right)\left(\frac{a}{L}\right),\ Nm \quad (6)$$

where, F= maximum bending force within proportional limit, N

W= width of specimen, m

L= distance between two supporting point, m

a= distance between supporting point and loading point, m

δ= maximum deflection of central span within proportional limit, m

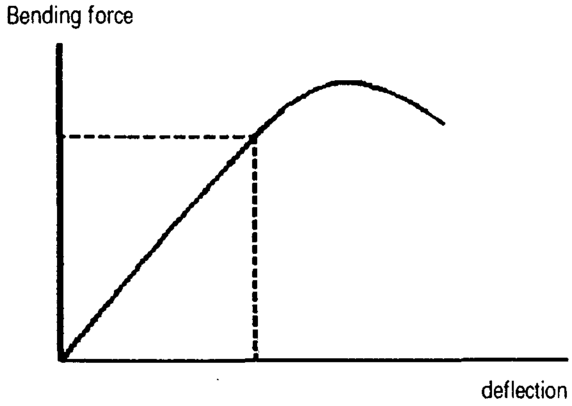

Fig. 4. Relationship between bending force and deflection

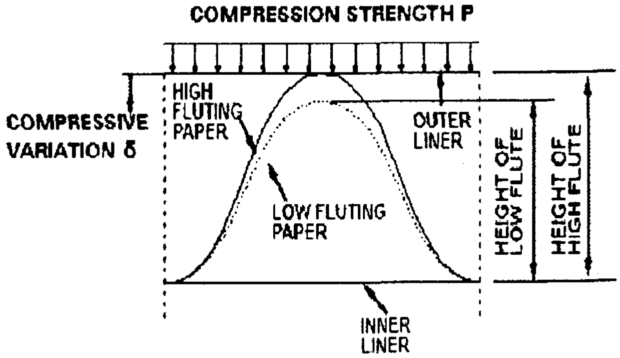

Fig.5. Structure of the dual medium corrugated fiberboard

Test Method

The corrugated fiberboards used for flexural stiffness test are 5 kinds of boards which have different constructional shapes. The boards are single wall (SW), double wall (DW), dual medium (DM), dual medium+A/F (DMA) and dual medium+B/F (DMB) corrugated fiberboard. The dual medium corrugated fiberboards differ from the regular corrugated boards by having the design structure of two flute levels. (Fig. 5).

The dual medium corrugated fiberboard is designed with the 2 layers of medium papers, with A/F one on top of the lower A'/F. Dual medium corrugated fiberboards have the thickness similar to A/F single wall boards, but its strength is much higher. Table 1 shows physical dimensions and board combinations of these corrugated boards.

The specimen of corrugated fiberboards is 500mm × 50mm for length and width. Distance between loading anvils which force the compression on the specimen is 400mm and distance between the supporting anvil is 200mm that is the half of loading anvil distance(*12*).

Table 1. Physical dimensions of the corrugated fiberboards used for bending test

Kinds	Flute type	Board combination	Total basis weight, g/m^2	Take-up factor
SW	A	SK210/$K_2$180/SK210	700.80	(A/F)1.560
DW	AB	SK210/$K_2$180/SK210/$K_2$180/SK210	1167.12	(A/F)1.560 (B/F)1.424
DM	AA'	SK210/$K_2$180/$K_2$180/SK210	961.62	(A/F)1.560 (A'/F)1.449
DMA	AA'+A	SK210/$K_2$180/$K_2$180/SK210/$K_2$180/SK210	1452.42	(A/F)1.560 (A'/F)1.449
DMB	AA'+B	SK210/$K_2$180/$K_2$180/SK210/$K_2$180/SK210	1427.94	(A/F)1.560 (A'/F)1.449 (B/F)1.424

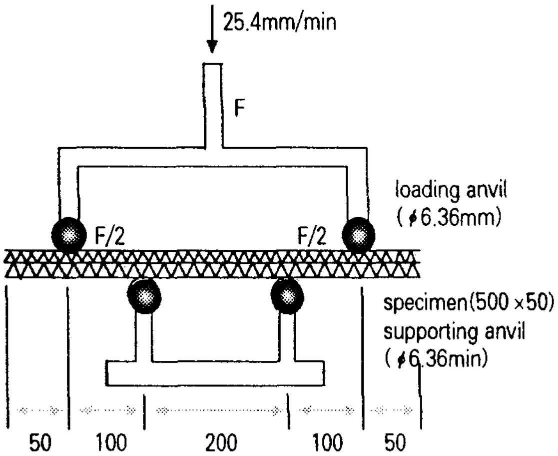

Fig. 6. Four-point bending test [unit: mm]

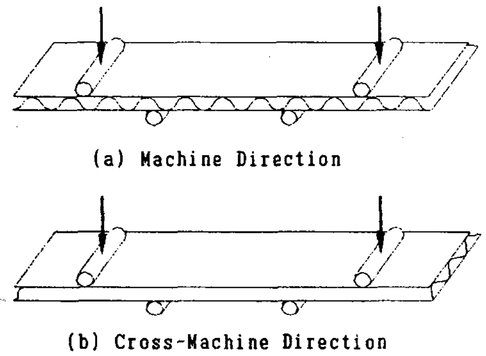

Fig. 7. Orientation of specimen for determination of flexural stiffness of corrugated fiberboard

Flexural stiffness of corrugated board at MD and CD is important factor to determine the compression strength of corrugated fiberboard box. In this study, therefore, the bending test of corrugated fiberboard at MD and CD was acted as shown in Fig. 6 and the contact condition between anvil and specimen were not considered specially.

The loading rate by UTM(TSH-50) was set at 25.4mm/min and the specimen was pre-conditioned at standard condition (23±1℃, RH 50±2%) for 48 hours. The test was performed at room where the temperature and RH was kept regularly*(17, 18)*.

The load direction of corrugated board in the bending test was applied for which the outside of corrugated board was forced by tension and inside of the board endured compression because loaded container get bent inside to outside. (Fig. 8)

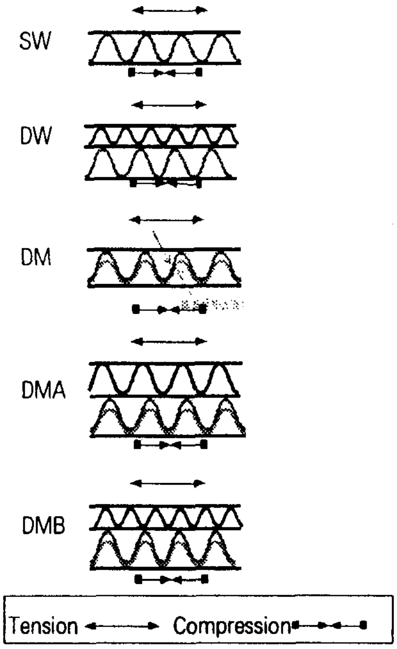

Fig. 8. Applied load direction in a bending test of corrugated fiberboards

At equation (6), is the deflection of specimen on central span, which is ®ˉ of cross-head movement for UTM as shown in Equation (7) and (8) calculated from Equation (4).

In forcing point, $y_x = 0 = \frac{1}{EI}\left(-\frac{3FL^3}{32}\right)$ .... (7)

In cenral point, $\delta = y_x = a + L/2 = \frac{1}{EI}\left(\frac{FL^3}{32}\right)$ .... (8)

Test results and analysis

1) Maximum bending force and deflection

Fig. 9 describes specimen and forced loading points in detail when the bending test for corrugated fiberboard is applied. Test results of maximum bending force and deflection for MD and CD of the corrugated fiberboards are shown at Table 2. Fig. 10 and Fig. 11. Except SW among tested corrugated fiberboard, the difference of test data by change of forcing direction (inside linerboard ⇆ outside linerboard) is huge. Bending force-Deflection curve for CD of corrrugated fiberboard shows there is one yield point before failure point but two or more yield points for MD.

The maximum bending force in CD is much higher than in MD. The difference between MD and CD is about 50% at SW and DW, and 62%~74% at DM, DMA and DMB. Compared the maximum bending force between DW and DM or between DW and DMB, there was very little difference between DW and DM but little higher difference between DW and DMB. DW had higher bending force than DM.

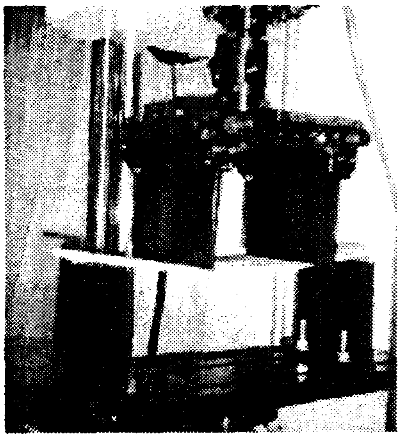

Fig. 9. Experimental arrangement for four-point bending test

Maximum deflection of the tested corrugated boards was also higher in CD than MD. Especially difference between two directions was biggest at DM. Maximum deflection in MD had no distinctive difference among those corrugated fiberboard but there were differences in CD as the order follows: MD → SW, DMA, DMB → DW.

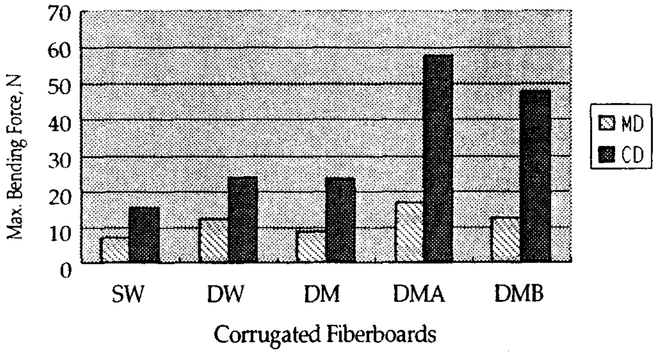

Fig. 10. Maximum bending forces for various corrugated fiberboards

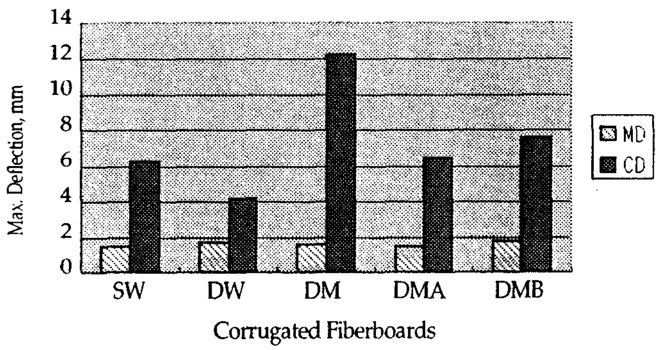

Fig. 11. Maximum Deflections for various corrugated fiberboards

2) Flexural Stiffness

Generally flexural stiffness of corrugated fiberboard is higher in MD than CD. This test results also show flexural stiffness at MD is 29%~48% higher than CD and difference of flexural stiffness between two

Table 2. Maximum bending force and deflection for various corrugated fiberboards

Kinds	Max. bending force, N			Max. deflection, mm			Flexural stiffness, Nm		
	MD	CD	MD/CD,%	MD	CD	MD/CD,%	MD	CD	MD/CD,%
SW	7.01	15.34	54	1.46	6.28	77	27.16	14.05	93
DW	12.26	23.94	49	1.71	4.17	59	43.12	25.15	71
DM	8.77	23.27	62	1.56	12.27	87	39.25	21.32	84
DMA	16.71	57.98	71	1.50	6.40	77	89.18	63.72	40
DMB	12.44	47.45	74	1.79	7.52	76	64.04	41.85	53

directions is the smallest at DMA and DMB.

Flexural stiffness of the tested board at MD and CD is as follow in order: DMA→DMB→DW→DM→SW. Flexural stiffness per unit paperboard required for manufacturing corrugated board is also higher at DMA and DMB than any other boards.

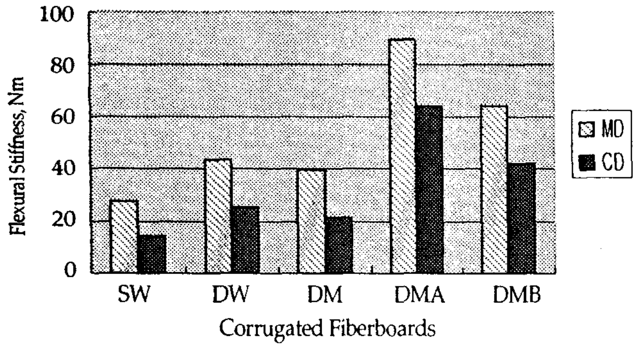

Fig. 12. Flexural stiffness for various corrugated fiberboards

Conclusion

This study analyzed flexural stiffness, bending strength and deflection tested by four-point bending test for corrugated fiberboards constructed with geometrically different flutes. The results of this study are follows:

1) The difference of maximum bending strength between MD and CD was 50% in SW and DW, but it was as much as 62%~74% in DM, DMA and DMB.

2) Differences of maximum deflection among tested corrugated fiberboards were just small in MD, but in CD the differences are as follow in order: DM →SW, DMA, DMB → DW. DM has the largest deflection value.

3) Flexural stiffness of all tested corrugated fiberboards in MD are 29%~48% higher than CD and the difference between both directions is smallest in DMA and DMB. Flexural stiffness at both directions are as follow in order: DMA→DMB→DW→DM→SW.

References

(1) Gartaganis, P. A. Strength properties of corrugated containers. Tappi 58(11): 102-108(1975)

(2) Koning, J. W. Compressive properties of linerboard as related to corrugated fiberboard containers: theoretical model verification. Tappi 61(8): 69-71 (1978)

(3) Kutt, H. and B. B. Mithel. Structural strength characteristics of containers. Tappi 52(9): 1685-1690 (1969)

(4) Little, J. R. A theory of box compressive resistance in relation to the structural properties of corrugated paperboard. Paper Trade Journal 116(24): 31-34 (1943)

(5) Maltenfort, G. G. Compression failure analysis. Paperboard Packaging 172 (1979)

(6) Maltenfort, G. G. Corrugated shipping containers: an engineering approach. Jelmar Publishing Co. (1988)

(7) McKEE, R. C., J. W. Gander and J. R. Wachuta. Compression strength formula for corrugated boxes. Paperboard Packaging (Aug.): 144-159 (1963)

(8) Nordman, L., E. Kolhonen, M. Toroi. Investigation of the compression of corrugated board. Paperboard Packaging 48 (1978)

(9)Johnson, M. W., T. J. Urbanik, W. E. Denniston. Maximizing top-to-bottom compression strength. Paperboard Packaging 98 (1980)

(10) Kutt, H. and B. B. Mithel. Studies of compressive strength of corrugated containers. Tappi Journal 51(4) (1968)

(11) Seth, R. S. Relationship between edgewise compressive strength of corrugated board and its components. Tappi Journal(March): 98-101 (1985)

(12) TAPPI T820. Flexural stiffness of corrugated board

(13) Maltenfort, G. G. Compression load distribution on corrugated boxes. Paperboard Packaging 98 (1980)

(14) Billerad Handbook. Testing of corrugated board and its components

(15) Lorentzen & Wettre. Testing instruments

(16) McKEE, R. C., J. W. Gander and J. R. Wachuta. Flexural stiffness of corrugated board. Paperboard Packaging 47(12): 111-118 (1962)

(17) TAPPI T402. Standard conditioning and testing atmospheres for paper, paperboard, pulp handsheets, and related products.

(18) ASTM D685. Practice for conditioning paper and paper products for testing