Collapse of Magnetised, Singular Isothermal Toroids

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ABSTRACT

This poster summarizes numerical collapse calculations of non-rotating and rotating singular, isothermal toroids that employed the zeus2d (Norman and Stone 1992) magnetohydrodynamics package. In the non-rotating collapse calculations, it is seen that infall proceeds at a constant rate and magnetically supported, high density pseudo-disks form in the equatorial plane. With rotating clouds, however, toroidal magnetic fields grow as infall proceeds, teaming with angular momentum to slow the inflow to the center and generate outflow.

Key Words : methods: numerical — MHD — collapse

I. Introduction

It is known that low mass stars form in the dense cores of molecular clouds. It is known that molecular cloud cores are typically flattened. Although gas pressure and turbulence can provide isotropic support against gravitational collapse, rotation or magnetic support play a role in the support of isolated, non-spherical clouds. Observed rotation rates (Goodman et al. 1993) are too small to account for the flattening. So, it is reasonable to examine models of molecular clouds that have magnetic field support.

Li and Shu (1996) calculated a pivotal state at the end of quasi-static core evolution at which point the central isothermal density becomes infinite. This pivotal state may be used as initial conditions for numerical collapse. Like the collapse of the singular, isothermal sphere (Shu 1977), the collapse of the (magnetised) singular isothermal toroids is self similar.

II. Simulations

(a) Initial Conditions

Initial conditions were set from the pivotal states of Li and Shu (1996) and from rotating pivotal states calculated by the same authors. The initial values were given by

$$\vec{B} = \frac{1}{2\pi}\nabla \times \left(\frac{\Phi}{r\sin\theta}e_\phi\right) = \frac{2a^2}{\sqrt{G}r\sin\theta}\left[\frac{d\varphi}{d\theta}\hat{r} - \varphi\hat{\theta}\right] \tag{1}$$

$$\rho(r,\theta) = \frac{a^2}{2\pi Gr^2}R(\theta) \tag{2}$$

$$\Phi(r,\theta) = \frac{4\pi a^2 r}{G^{\frac{1}{2}}}\varphi(\theta) \tag{3}$$

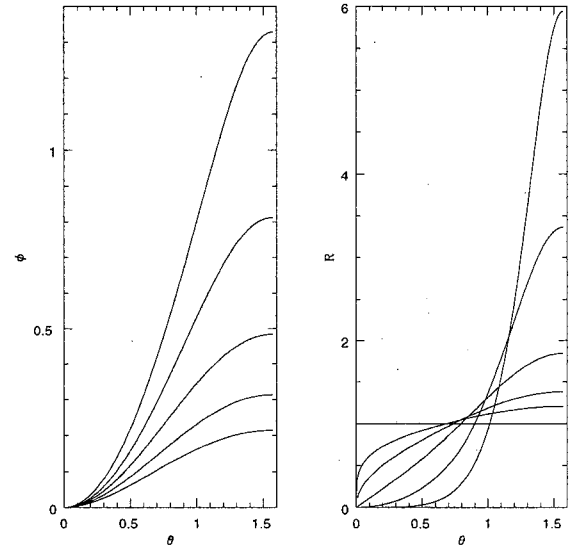$$v(r,\theta) = aV(\theta) \tag{4}$$

Fig. 1.— $\varphi(\theta)$ (left) and $R(\theta)$ (right) for $n = 0, .125, .25, .5, 1, 2, 3, 4$ (from bottom to top on the left and from top to bottom on the right, where $n = 4H_0$).

where $R(\theta)$ and $\varphi(\theta)$ are shown for a few values of H_0 in Figure 1. H_0 is a measure of magnetic field support.

(b) Numerical Method

Zeus2d provided the backbone for the simulations. Astrophysically, the collapse should remain isothermal through radiation. Radiation was not included in the simulations, but energy was removed at each timestep to maintain isothermality. Also, self gravity was modified to handle the diverging potential of an inverse square law density distribution. For cases with stronger magnetic support, a (Alfvén) timestep tending toward
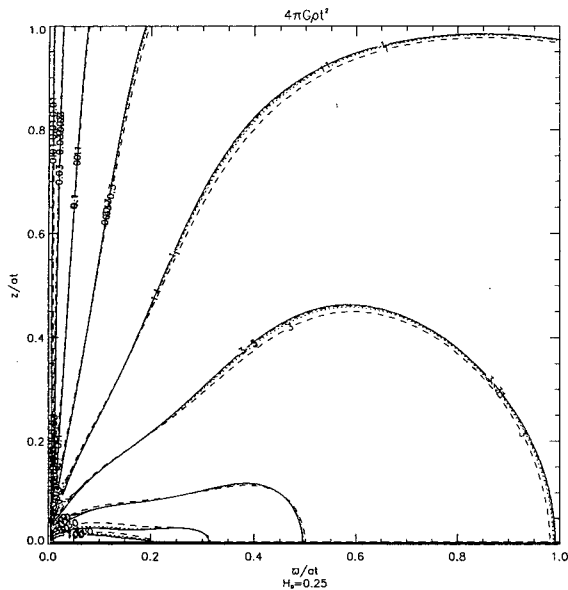
Fig. 2.— Isodensity contours at four equal spaced time intervals (dashed lines are earliest and the solid lines are latest) showing convergence to the self-similar solution for the $H_0 = 0.25$ toroid.

zero in low density regions developed, requiring inclusion of the displacement current with an artificially lowered speed of light (Boris 1970, Miller and Stone 1999, Allen 2000) to advance the simulation. For this work, the speed of light was lowered to $10^6 cm/s = 50a$. This trick allows the simulation to proceed at the cost of having incorrect physics in the regions where the trick is needed (But these regions are not of interest in this work). The simulations were carried out in ϖ, θ, and t cylindrical coordinates and the results converted to x and θ (where $x = r/at$) similarity coordinates for analysis and display.

III. Results

(a)   Non-rotating Collapse

For the non-rotating collapse calculation, two initial field strengths are shown corresponding to $H_0 = 0.25$ and $H_0 = 0.5$. For both cases, a high density pseudo-disk forms in the equatorial plane and infall to the origin occurs at a constant $\dot{M} = \mathcal{M}_0(1 + H_0)\frac{a^3}{G}$.

The higher magnetic field strength $H_0 = 0.5$ case exhibited cells of very low density but finite field, requiring the inclusion of the Alfvén correction. For this reason, the solutions may be incorrect in regions of low density (regions close to the polar axis). Since there is almost no matter along the polar axis, nothing observable is lost.
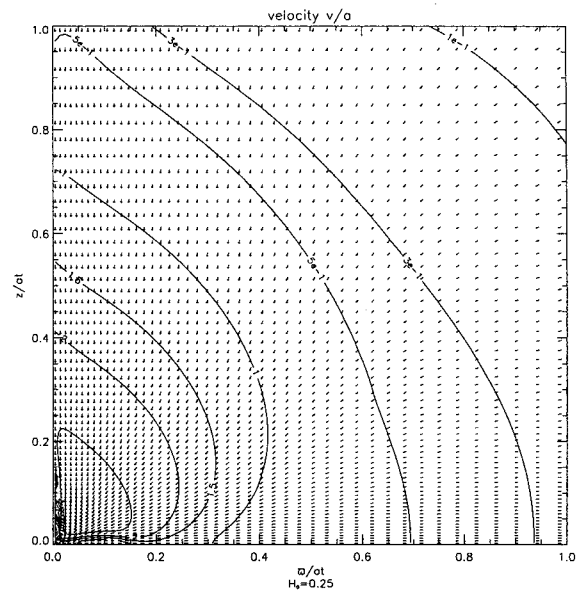

Fig. 3.— Contours of constant speed overlaid with flow direction unit vectors for the $H_0 = 0.25$ toroid.

(b) Rotating Collapse

For the case of rotating collapse, the simulations did not run until convergence to a self-similar solution was reached, due to appearance of another timestep problem near the polar axis. However, the simulations have run long enough to see the absence of the high density pseudo-disks present in the non-rotating cases and appearance of outflow driven by the growing toroidal magnetic field.

IV.   Conclusion

Gravitational collapse of magnetised molecular clouds have been studied. For the non-rotating clouds studied, magnetically supported pseudo-disks form and infall onto a central pointmass occurs at a constant rate. For the rotating clouds studied, central accretion diminishes and an outflow is driven.

REFERENCES

Allen, A., PhD. Thesis, University of California, Berkeley

Boris, J., 1970, NRL Memo Report MR-2167

Goodman, A., *et al.* 1998, ApJ, 504, 223

Li, Z. Y., & Shu, F. H. 1996, ApJ, 472, 211

Miller, K. A. & Stone, J. M. 2000, ApJ, 534, 398

Shu, F. H. 1977, ApJ, 214, 488

Shu, F. *et al.* , 1999, in The Origin of Stars and Planetary Systems, ed. C. J. Lada & N. D. Kylafis (Dordrecht: Kluwer), 193
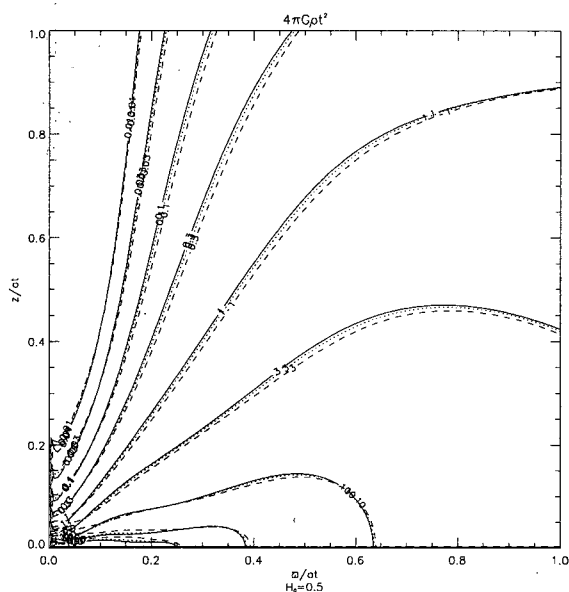
Fig. 4.— Isodensity contours at four equal spaced time intervals (dashed lines are earliest and the solid lines are latest) showing convergence to the self-similar solution for the $H_0 = 0.5$ toroid.
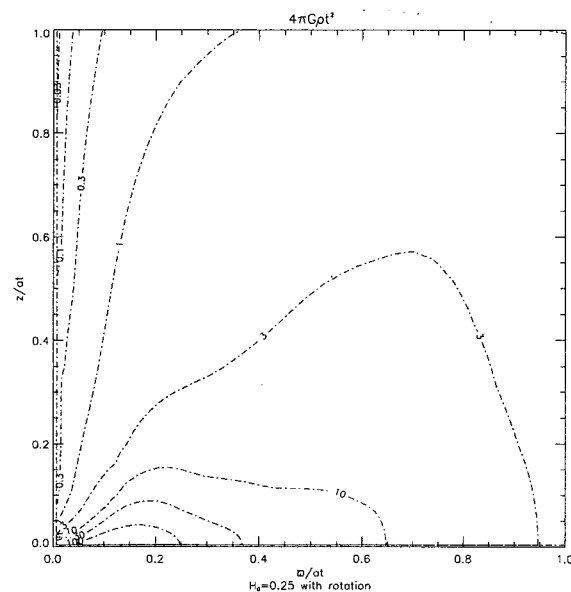

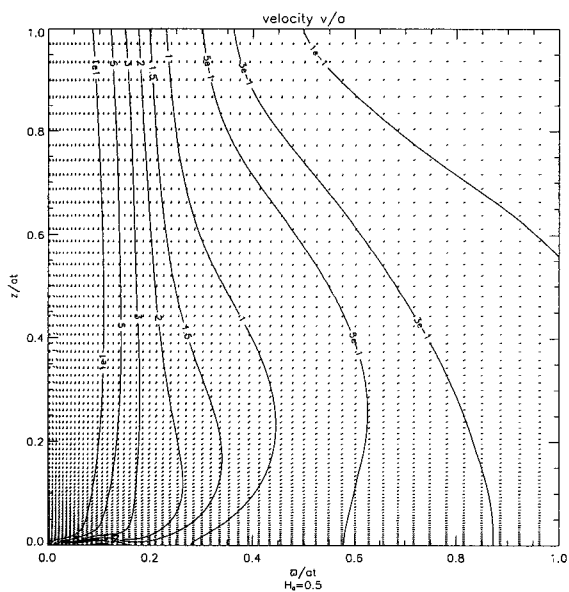**Fig. 6.**— Isodensity contours for the rotating, $H_0 = 0.25$ toroid.

Fig. 5.— Contours of constant speed overlaid with flow direction unit vectors for the $H_0 = 0.5$ toroid. The results cannot be trusted along the polar axis (see text for details).
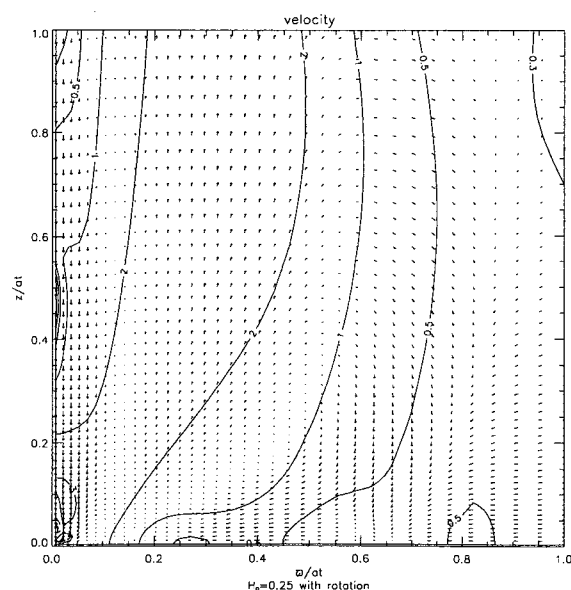

Fig. 7.— Isospeed contours overlaid with flow direction unit vectors for the rotating, $H_0 = 0.25$ toroid.