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Relativistic Radiation Hydrodynamics of Spherical Accretion

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ABSTRACT

Radiation hydrodynamics in high velocity or high optical-depth flow should be treated under rigorous relativistic formalism. Relativistic radiation hydrodynamic moment equations are summarized, and its application to the near-critical accretion onto neutron star is discussed. The relativistic effects can dominate the dynamics of the flow even when the gravity is weak and the velocity is small. First order equations fail to describe the intricate relativistic effects correctly.

Key Words : accretion — black hole physics — radiative transfer

I. Introduction

The major components of accretion system are matter and radiation, and they are bound to interact one way or another. In freely falling spherical accretion flow onto neutron stars and black holes, this interaction becomes important when the mass accretion rate is above the Eddington accretion rate, defined as

$$\dot{M}_E \equiv \frac{L_E}{c^2} = 2.2 \times 10^{-9} \left(\frac{M}{M_\odot}\right) \ \mathrm{M_\odot \ yr^{-1}}, \qquad (1)$$

where L_E is the Eddington luminosity and $M_\odot$ the mass of the Sun. Below this mass accretion rate, matters simply cools and photons freely stream out. Above it, the electron scattering optical depth becomes larger than 1, and outgoing photons interact with matter. Momentum and energy are exchanged between matter and radiation via scattering and absorption. Momentum transfer from radiation to matter produces radiation force, which becomes comparable to the gravity when the luminosity approaches L_E. Dynamics of accretion flow can also be altered by the energy transfer. Hard photons produced in the inner part of the accretion flow can heat the outer part of the flow via scattering or absorption, changing the entropy of the accretion flow. In this supercritical accretion, $\dot{M} \geq \dot{M}_E$, photons have to diffuse out of the incoming flow thorough scattering or absorption. When the optical depth, τ, is high and infall velocity, v_r, is large, the diffusion velocity of photons, $v_{\rm diff} \sim c/\tau$, can become lower than v_r. When this condition is met, most photons advect with the flow and radiation is trapped. Such flow requires careful treatment because of relativistic radiative transfer and hydrodynamic effects.

II. Relativstic Radiation Hydrodynamics Equations

Thomas (1930) first laid out the fully special relativistic theory of radiative transfer. Thirty years later, Lindquist (1960) extended the theory to the curved spacetime, but limited to the spherically symmeric case and diffusion regime. General radiation moment equations are derived by Thorne (1981) in projected symmetric trace-free tensor formalism. This formalism exclusively uses the radiation quantities in comoving frame, a frame comoving with the matter. Park (1993) rederived the relativistic radiation moment equations in mixed-frame formalism for Schwarzschild spacetime. Radiation quantities and their derivatives are expressed in fixed coordinate while the interaction between radiation and matter are expressed in comoving frame. The equations are simpler to understand and the interaction terms are intuitively described. Here, I summarize the equations and definitions suitable for spherical accretion onto Schwarzschild black holes. This mixed-frame radiation moment equations can be easily extended to other spacetime and non-spherical accretion. But in such non-symmetric cases, closure relations among radiation moments are not well-known and application to real accretion is not straightforward. Therefore, I limit the following discussion only to the spherically symmetric accretion in Schwarzschild geometry

$$d\tau^2 = (1 - \frac{2m}{r})dt^2 - \frac{dr^2}{1-2m/r} - r^2(d\theta^2 + \sin^2\theta d\phi^2). \qquad (2)$$

I choose $c = 1$.

In relativistic framework, radiation quantities in fixed frame and in comoving frame are defined in each tetrad. Energy density is defined as

$$E = 2\pi \iint I_\nu d\nu d\mu, \quad E_{co} = 2\pi \iint I_{\nu_{co}} d\nu_{co} d\mu_{co}, \qquad (3)$$

where I is the specific intensity, μ and μ_{co} are the direction cosines between the photon trajectory and the radial direction in each tetrad, and ν and ν_{co} the photon frequency in each tetrad. The radiation flux is

$$F = 2\pi \iint I_\nu \mu d\nu d\mu, F_{co} = 2\pi \iint I_{\nu_{co}} \mu_{co} d\nu_{co} d\mu_{co}, \qquad (4)$$

and the radiation pressure

$$E = 2\pi \int\int I_\nu \mu^2 d\nu d\mu, E_{co} = 2\pi \int\int I_{\nu_{co}} \mu_{co}^2 d\nu_{co} d\mu_{co}. \quad (5)$$

These quantities constitute the tetrad components of the radiation stress tensor. Applying the Lorentz transformation between the tetrads components in fixed frame and comoving frame yields the transformaton law between comoving frame radiation moments and fixed frame radiation moments:

$$E_{co} = \frac{1}{1-v^2}[E - 2vF + v^2 P], \quad (6)$$

$$F_{co} = \frac{1}{1-v^2}[(1+v^2)F - vc(E+P)], \quad (7)$$

$$P_{co} = \frac{1}{1-v^2}[P - 2vF + v^2 E], \quad (8)$$

where v is the proper velocity of the flow, the velocity measured by the observer fixed to the coordinate system. For the Schwarzschild metric with dimensionless mass $m \equiv GM/c^2$, v is related to the radial component of the four-velocity U^r as

$$v = \frac{U^r}{y}, \quad (9)$$

where

$$y \equiv [1 + (U^r)^2 - 2m/r]^{1/2}. \quad (10)$$

Now, the transfer of momentum and energy from the radiation to the matter is easily expressed by comoving radiation moments. The particle number conservation is

$$\frac{1}{1-2m/r}\frac{\partial}{\partial t}(yn) + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 n U^r) = 0, \quad (11)$$

where n is the proper number density of particles conserved, e.g., baryon. The relativistic Euler equation is

$$\frac{1}{2}\frac{\partial}{\partial r}(U^r)^2 + \frac{y}{1-2m/r}\frac{\partial U^r}{\partial t} + \frac{y^2}{\omega_g}\frac{\partial P_g}{\partial r} + \frac{yU^r}{1-2m/r}\frac{1}{\omega_g}\frac{\partial P_g}{\partial t} + \frac{m}{r^2} = \frac{y}{\omega_g}\chi_{co}F_{co}, \quad (12)$$

where P_g is the gas pressure, $\omega_g \equiv P_g + e_g$ the gas enthalpy, e_g the gas internal energy including the rest mass energy, and χ_{co} is the linear opacity measured in comoving frame. It is evident from this equation that the comoving flux is directly related to the acceleration, or rather deceleration, of the matter. The energy equation is

$$\frac{\partial e_g}{\partial r} - \frac{\omega_g}{n}\frac{\partial n}{\partial r} + \frac{\partial y}{1-2m/r}\frac{1}{U^r}\left[\frac{\partial e_g}{\partial t} - \frac{\omega_g}{n}\frac{\partial n}{\partial t}\right] = \frac{\Gamma_{co} - \Lambda_{co}}{U^r}, \quad (13)$$

where Γ_{co} and Λ_{co} are the heating and cooling functions per unit proper volume in comoving frame.

For spherically symmetric radiation field, two radiation moment equations are derived for three moments. This is general characteristics of any radiative transfer equation reduced in moment form: radiation moments are always under-determined and additional constraint like closure relation should be provided. The energy equation for the radiation field is

$$\frac{1}{1-2m/r}\frac{\partial E}{\partial t} + \frac{1}{1-2m/r}\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\left(1 - \frac{2m}{r}\right)F\right] = -\frac{y}{1-2m/r}[\Gamma_{co} - \Lambda_{co} + v\chi_{co}F_{co}]. \quad (14)$$

We can see that the gravitational redshift-corrected luminosity $4\pi r^2(1-2m/r)F$ is conserved in the absence of interaction with matter. The generalized form of the momentum equation for the radiation field is

$$\frac{1}{1-2m/r}\frac{\partial F}{\partial t} + \frac{\partial P}{\partial r} - \frac{E-3P}{r} + \frac{1}{1-2m/r}\frac{m}{r}\frac{E+P}{r} = -\frac{y}{1-2m/r}[\chi_{co}F_{co} + v(\Gamma_{co} - \Lambda_{co})]. \quad (15)$$

Although only terms of up to v^1 appear in the equations, they are relativistically exact, i.e., correct up to arbitrary order in v. The above two equations (14) and (15) are usually supplemented by variable Eddington factor relation for closure.

III. Application to Near Critical Accretion onto Neutron Stars

As a concrete example, I apply the above relativistic radiation hydrodynamics equations to the steady-state, near critical accretion onto neutron star (Park & Miller 1991). The spacetime around the neutron star is described by the Schwarzshild metric and the gas is assumed not to contain any pressure or internal energy. When the luminosity is some fraction of the Eddington luminosity

$$\epsilon \equiv 1 - \frac{L}{L_E}, \quad (16)$$

the gas infall roughly follows the modified free-fall

$$v_{mff} = \left(\epsilon\frac{2m}{r}\right)^{1/2}. \quad (17)$$

However, as ϵ approaches 0, the slower velocity makes the optical depth much higher than 1 and the relativistic radiation transport effects become important. Even in the Newtonian calculation with corrections up to $(v/c)^1$, the flow velocity deviates significantly from $v_{mff} \propto r^{-1/2}$ close to the neutron star surface as shown in Fig. 1 (*dotted curve*) for $\epsilon = 0.2$ accretion (Miller 1990). The correct general relativistic treatment shows even pronounced decrease of velocity (*solid curve* in Fig. 1) and increase of radiation field. The dynamics of the flow is determined by the relativistic Euler

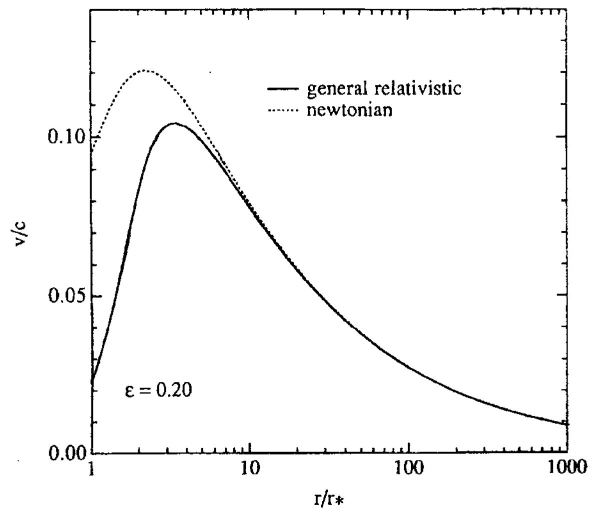

Fig. 1.— The flow velocity measured by the fixed observer for spherical accretion onto neutron star with $\epsilon \equiv 1-L/L_E = 0.2$. The radius is in units of neutron star radius r_*, which is assumed to be 10 km. *Dotted curve* is from the Newtonian calculation of Miller (1990) and the *solid curve* is the correct relativistic solution of Park & Miller (1991).

equation (12). If we define a critical flux

$$F_{cr} \equiv \frac{1}{y} \frac{L_E}{4\pi r^2} \tag{18}$$

as an extension of Eddington flux, the flow decelerates whenever the comoving flux is larger

$$F_{co} > F_{cr}. \tag{19}$$

The factor y (Eq. 10) is a pure relativistic factor. The gravitational redshift factor between the luminosity measured at infinity and that measured locally at r is incorporated in F_{co} through eqs. (7) and (14). The ratio F_{co}/F_{cr} is shown in Fig. 2, showing that this ratio becomes larger than 1 around $r \simeq 3.5r_*$ which corresponds to the start of deceleration in Fig. 1.

In the extreme case when the luminosity is independent of the accretion flow, it can be shown that the steady-state accretion flow does not exist when the luminosity from the central source is larger than 76% (measured at infinity) of the Eddington luminosity for spherical accretion 1.4 $M_\odot$ with 10 km radius neutron star. This limit value is significantly smaller than the classical Eddington limit.

As shown in this specific example, we conclude that the rigorous relativistic radiation hydrodynamics equations should be used whenever $v \to c$, $2m/r \to 1$, or $\tau v \to c$. These situations most likely to arise in the accretion onto neutron stars or black holes, and especially even at large radius for near-critical or super-critical accretion.

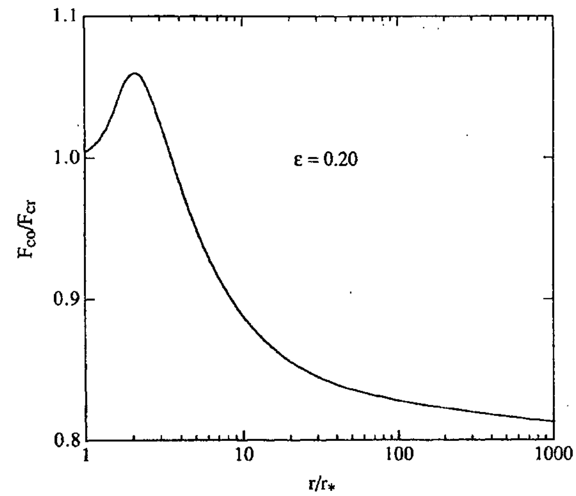

Fig. 2.— The comoving flux, flux measured by the observer comoving with the flow, in units of the critical flux defined in the main text. In Newtonian treatment the ratio is always equal to $1-\epsilon = 0.8$ in this case.

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REFERENCES

Thosmas, L. H. 1930, Quat. J. Math., 1, 239

Lindquist, R. W. 1966, Ann. Phys., 37, 487

Thorne, K. S. 1981, MNRAS, 194, 439

Park, M.-G. 1993, A&A, 274, 642

Miller, G. S. 1990, ApJ, 356, 572

Park, M.-G. & Miller, G. S. 1991, 371, 708