

## Diffusive Shock Acceleration with Self-Consistent Injection

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(Received Aug. 31, 2001; Accepted Nov. 15, 2001)

### ABSTRACT

A numerical scheme that incorporates a self-consistent cosmic-ray (CR, hereafter) injection model into the combined gas dynamics and CR diffusion-convection code has been developed. The hydro/CR code can follow in a very cos-effective way the evolution of CR modified shocks by adopting subzone shock-tracking and multi-level Adaptive Mesh Refinement techniques. The injection model is based on interactions of the suprathermal particles with self-generated MHD waves in quasi-parallel shocks. The particle injection is followed numerically by filtering the diffusive flux of suprathermal particles across the shock to upstream region according to a velocity-dependent transparency function, which represents the fraction of leaking suprathermal particles. In the strong shock limit of Mach numbers  $\geq 20$ , significant physical processes such as the injection and acceleration seem to become independent of  $M$ , while they are sensitively dependent on  $M$  for  $M < 10$ . Although some particles injected early in the evolution continue to be accelerated to higher energies, the postshock CR pressure reaches a time asymptotic value due to balance between acceleration and diffusion of the CR particles.

*Key Words* : acceleration of particles – cosmic rays – hydrodynamics – methods: numerical

### I. INTRODUCTION

Two of main physical processes in the diffusive shock acceleration (DSA, hereafter) theory are the particle injection at shocks and the particle diffusion through scatterings with MHD waves. According to quasi-linear theory as well as plasma simulations of strong quasi-parallel shocks, the streaming motion of the CR particles against the background fluid can induce wave generation leading to strong MHD waves that scatter particles and prevent them from leaking upstream (e.g., Bell 1978; Quest 1988). So due to these self-generated MHD waves thermal particles are confined and advected downstream. Some suprathermal particles in the high energy tail of the Maxwellian velocity distribution, however, may have velocities that allow them to re-cross the shock. This so-called “thermal leakage” injection process is important to reaching an understanding of the efficiency of DSA of the CRs. Malkov (1998) presented a self-consistent, analytic, nonlinear calculations for ion injection based on this process. By adopting Malkov’s analytic solution, a numerical treatment of this injection model has been devised and incorporated into the combined gas dynamics and the CR diffusion-convection code (Gieseler et al. 2000). According to their simulations, the injection process seems to be self-regulated in such a way that the injection rate reaches and stays at a nearly stable value after quick initial adjustment, but well before the CR shock reaches a steady state structure. They found that about  $10^{-3}$  of incoming thermal particles are injected into the CRs, roughly independent of Mach numbers. Due to severe computational requirement, however, their simulations were carried out only until the maximum momentum of  $(p_{\max}/m_p c) \sim 1$  was achieved.

In this contribution we re-examine the same problem by implementing their numerical injection model into our newly developed CR/AMR code.

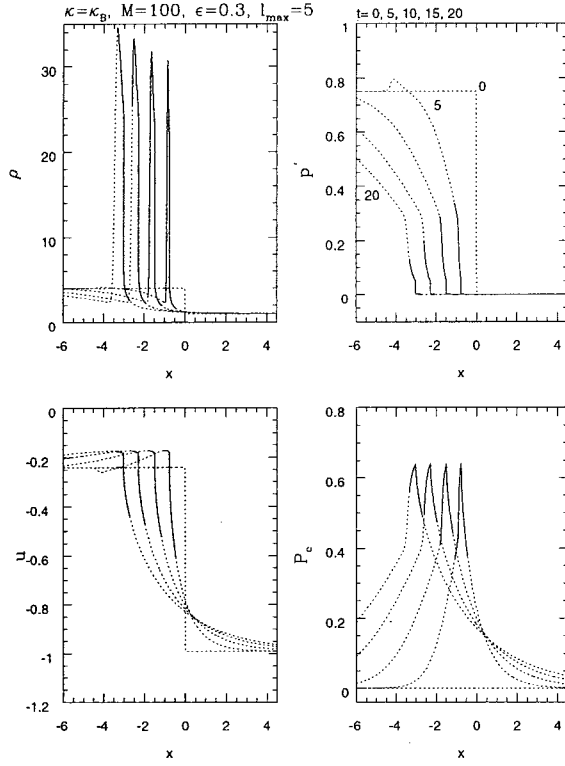
### II. Numerical Methods

#### (a) CR/AMR Hydrodynamics Code

The diffusion-convection equation for nonlinear DSA includes an extremely wide range of length scales for a Bohm type diffusion. In order to follow accurately the evolution of a CR modified shock, it is necessary to resolve the precursor structure upstream of the subshock and, at the same time, to solve correctly the diffusion of the low energy particles near the injection pool. So a large dynamic range of resolved scales is required for CR shock simulations. To solve this problem generally we have successfully combined a powerful “Adaptive Mesh Refinement” (AMR) technique (Berger & Le Veque 1998) and a “shock tracking” technique (Le Veque & Shyue 1995), and implemented them into a hydro/CR code based on the wave-propagation method (Kang et al. 2001). The AMR technique allows us to “zoom in” inside the precursor structure with a hierarchy of small, refined grid levels applied around the shock.

#### (b) Self-Consistent Injection Model

In “thermal leakage” injection model, most of downstream thermal protons would be confined by the waves and only particles with higher velocity in the tail of the Maxwellian distribution are able to leak through the shock. In order to include self-consistently the injection of the CR protons according to the analytic solution of (Malkov 1998), we have adopted the “transparency



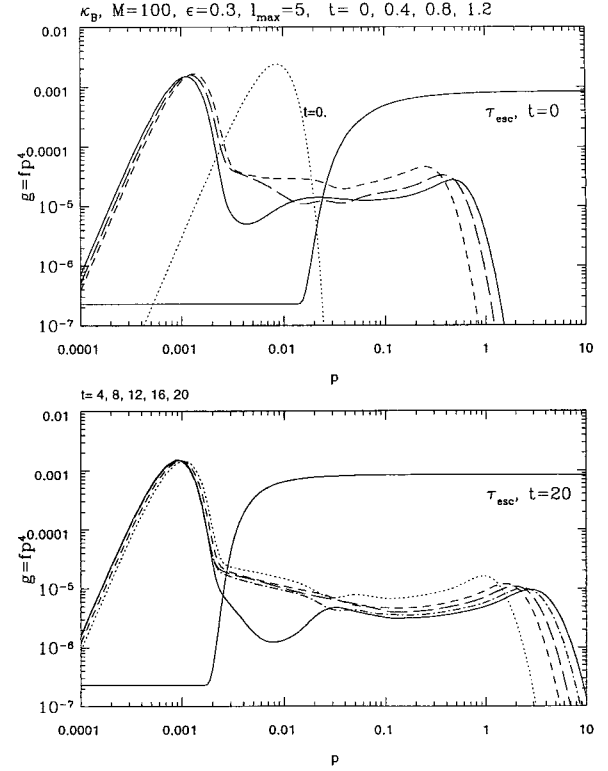
**Fig. 1.**— Evolution of  $M = 100$  shock. The solid lines are for the refinement region at  $l_g = 1$  grid, while the dotted lines represent the shock structure on the base grid. Five levels of refinements ( $l_{\max} = 5$ ) were used.

function"  $\tau_{\text{esc}}(v, u_2)$ , which expresses the probability that supra-thermal particles at a given velocity can leak upstream through the magnetic waves, based on non-linear particle interactions with self-generated waves (Gieseler et al 2000). The only free parameter of this model is rather well constrained, since  $0.3 \leq \epsilon \leq 0.4$  for strong shocks. Due to the exponential cut off in a thermal velocity distribution, however, the injection rate depends rather sensitively on the value of  $\epsilon$ .

In the new CR/AMR code the diffusive flux of supra-thermal particles across the shock to the upstream region is filtered by the transparency function so that the probability for leakage is zero below the injection pool, then increases to unity above the injection pool. The rate of particles injected into the Fermi process is then proportional to the convolution of  $\partial\tau_{\text{esc}}/\partial p$  with  $f(p)$  of Maxwellian tail.

### III. RESULTS

We have adopted the same definitions for the injection and acceleration efficiencies as in Gieseler et al. (2000). The injection efficiency,  $\xi(t)$ , is given by the fraction of particles that has been swept through the shock after the time  $t$ , and then injected into the CR

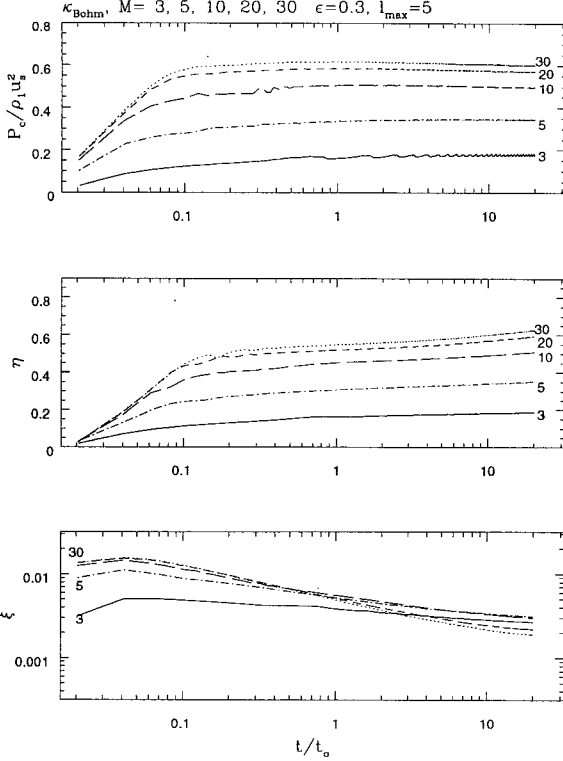


**Fig. 2.**— Time evolution of the CR distribution function,  $g(p) = f(p)p^4$ , at the shock is shown for  $M = 100$  shock. The transparency function at normalized time  $t=0$  and 20 is also plotted for reference.

distribution. The acceleration efficiency,  $\eta(t)$ , is defined as the fraction of initial total energy flux through the shock that is transferred to CRs.

The dynamics of the CR modified shock depends on four parameters: the gas adiabatic index,  $\gamma_g = 5/3$ , gas Mach number of the shock,  $M = V_s/c_s$ ,  $\beta = u_o/c$ , and the diffusion coefficient,  $\kappa$ . Here  $u_o$  is the velocity normalization constant corresponding the upstream flow velocity. For all simulations we present here  $u_o = 3000 \text{ kms}^{-1}$ . We assume a Bohm type diffusion coefficient,  $\kappa_B = \kappa_o p^2 / (p^2 + 1)^{1/2}$ , where  $p$  is expressed in units of  $m_p c$ . We considered five values for Mach number,  $M = 5, 10, 20, 30$  and 100 for the initial shock jump by adjusting the preshock gas pressure. The simulations were carried out on a base grid with  $\Delta x_0 = 3.2 \times 10^{-3}$  using  $l_{\max} = 5$  additional grid levels.

Fig. 1 shows time evolution of the CR modified shock structure for a  $M = 100$  shock with  $\epsilon = 0.3$ . This shows how the precursor grows and the flow structure is modified as CRs are accelerated. Evolution of the CR distribution function, at the shock, represented as  $g = p^4 f(p)$  is given for the same shock in Fig. 2. Just above the injection pool, the distribution function changes smoothly from a Maxwell distribution to



**Fig. 3.**— Top: Postshock CR pressure in units of ram pressure, Middle: Acceleration efficiency  $\eta(t)$ , Bottom: Injection efficiency  $\xi(t)$  for  $M = 3, 5, 10, 20$ , and  $30$  shock for  $\epsilon = 0.3$ .

an approximate power-law whose index is close to the test-particle slope. As the postshock temperature decreases due to energy transfer to CRs, the Maxwell distribution shifts to lower momenta, but the transparency function shifts less to lower momenta. As a result, the injection rate decreases over time following an initial “start-up” increase when the postshock temperature is high. Fig. 3 shows how the CR pressure at the shock, and the acceleration/injection efficiencies,  $\eta$  and  $\xi$ , evolve for shocks with different Mach numbers when the inverse wave-amplitude parameter  $\epsilon = 0.3$  is adopted. For all Mach numbers the postshock  $P_c$  increases until  $p_{\max}/m_p c \sim 1$ , and then stays at a steady value afterwards.

#### IV. SUMMARY

We have shown through self-consistent numerical simulations that the “thermal leakage” injection process at quasi-parallel CR shocks is highly efficient and regulated by the convolution of the population in the high energy tail of the Maxwell velocity distribution and the transparency function  $\tau_{\text{esc}}(p, u_2)$ . Thus it depends strongly on the physical properties of the subshock and postshock gas, that is, the postshock flow

speed relative to the subshock and the gas temperature. This requires one to follow accurately the evolution of the subshock and the development of the precursor. For high Mach number cases, however, this becomes a numerical challenge since the flow is highly supersonic in the precursor and so the entropy conservation is difficult there with conventional hydro codes which solve for the total energy instead of the thermal energy.

Our results can be summarized as follows:

1. The suprathermal particles can be injected very efficiently into CR population via thermal leakage process. As the subshock weakens, the injection rate of new particles decreases.
2. In the strong shock limit of  $M \geq 20$ , significant physical processes such as the injection and acceleration seem independent of the shock Mach number, while they are sensitively dependent on  $M$  for  $M < 10$ .
3. Although some particles injected early continue to be accelerated, postshock CR pressure reaches a time asymptotic value due to balance between acceleration and diffusion of the CR particles.
4. For  $M \geq 10$ , the shock structure is strongly modified by the CR pressure.
5. Self-consistent injection and time-dependent simulations are important to understanding the DSA theory.

#### ACKNOWLEDGEMENTS

This work was supported by Korea Research Foundation Grant (KRF-2000-051- DP0448) and by “The 2nd Supercomputing Application Support Program” of the KISTI (Korea Institute of Science and Technology Information).

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