

Hydrodynamic approach to cosmic ray acceleration

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ABSTRACT

To study the structure and dynamics of a cosmic-ray-plasma system, hydrodynamic approach is a fairly good approximation. In this approach, there are three basic energy transfer mechanisms: work done by the plasma flow against pressure gradients, cosmic ray streaming instability and stochastic acceleration. The interplay between these mechanisms gives a range of structures. We present some results of different version of the hydrodynamic approach, e.g., flow structure, injection, instability, acceleration with and without shocks.

I. Introduction

Energetic charged particles or cosmic rays and magnetized thermal plasma are coupled together by the embedded magnetic field. The magnetic irregularities or hydromagnetic waves act as scattering centres for the cosmic rays. As a result besides advecting with the plasma flow, cosmic rays also diffuse through the plasma. If the plasma flow is converging cosmic rays gain energy systematically and is called first order Fermi process. Hydromagnetic waves are excited by cosmic ray streaming instability when cosmic rays stream through the plasma. The excited waves in turn scatter cosmic rays and thus forming a self-consistent system called cosmic-ray-plasma system. When waves of different phase velocity exit, cosmic rays also diffuse in momentum space and is called second order Fermi process or stochastic acceleration. The coupling between the cosmic rays and hydromagnetic waves depends on the magnitude of the waves, hence advection and diffusion (in both real and momentum space) are functions of the magnitude of the waves.

In our Galaxy (and probably other galaxies, plausibly galaxy clusters) the energy density of the thermal plasma, cosmic rays, and magnetic field (regular and irregular) are of the same order of magnitude. The back-reaction of the cosmic rays and waves on the plasma ought to be taken care of properly. Hydrodynamic approach of cosmic ray propagation is a fairly good approximation to study the structure and dynamics of the cosmic-ray-plasma system. For two-, three- and four-fluid versions, see e.g., Axford et al. (1977), McKenzie & Völk (1982), Jiang et al. (1996), Ko (1992).

II. Hydrodynamic approach

In hydrodynamic approach all constituents are described by fluid quantities. The most comprehensive version we know of is the four-fluid version put forward by Ko (1992). The model comprises thermal plasma, cosmic rays and two Alfvén waves (which propagate along the magnetic field in opposite directions). The thermal plasma is described by a mass density, a velocity, an energy density, while the cosmic ray and waves

are treated as massless fluids which are described by an energy density accordingly. The governing equations consists of the total mass and momentum equations, the energy equations for the thermal plasma, cosmic rays and waves, and if necessary supplemented by the equations for the magnetic field.

In Ko (1992), the energy equation for the cosmic rays is obtained by taking the energy moment of the cosmic ray propagation equation by Skilling (1975a, b). The energy equations for the waves are obtained by integrating the wave power spectrum equations by Dewar (1970) with an additional term describing cosmic ray streaming instability.

Since the cosmic rays and waves are described by energy densities, the dynamics of the system will be best discussed in terms of energy transfer between various constituents. Basically, there are three energy transfer mechanisms: work done by the plasma flow against pressure gradients, cosmic ray streaming instability and stochastic acceleration. If large scale magnetic field is considered explicitly, then the usual coupling between plasma and magnetic field must also be taken care of.

(a) Evolution and instability

Let's start from a simple example, namely, a spatially uniform four-fluid model. It turns out that the total energy of the cosmic rays and waves is a constant. The thermal plasma is decoupled from the rest. If the system starts with both forward and backward waves, both waves decay until one withers. The system evolves towards a uni-directional wave system. It looks like oppositely propagating waves are killing each other and transfer the energy to cosmic rays. This is stochastic acceleration (Ko 1992).

However, when one works out the linear stability analysis of the uni-directional wave system, one finds that the system is susceptible to the slow magneto-acoustic instability (McKenzie & Webb 1984, Zank 1989; Ko & Jeng 1994). Stochastic acceleration tries to drive the cosmic-ray-plasma system towards a uni-directional wave state, which is unstable. So what the final state would be awaits further investigations.

(b) Two-fluid model

When we neglect the waves and focus on the thermal plasma and the cosmic rays only, we have the two-fluid model. The one-dimensional steady state of the two-fluid model has been studied quite thoroughly under the name cosmic-ray-modified shocks (e.g., Drury & Völk 1981; Axford et al. 1982; Ko et al. 1997). First of all, solutions are monotonic because the model can be reduced to one first order autonomous ordinary differential equation. For physically admissible solutions, the plasma flow speed has three generic forms: (i) spatially uniform, (ii) smoothly decreasing, and (iii) decreasing in the upstream region, then through a subshock (a genuine discontinuity), and then a uniform downstream state.

Flows with subshock are more likely to occur when the upstream Mach number is intermediate and the cosmic ray partial pressure is not large. Furthermore, it is possible to have three downstream states for one upstream state. Two of the three possibilities have a subshock, while the third one either is smooth or has a subshock.

Injection is a notorious problem in cosmic ray acceleration. How do some lucky thermal particles become energetic particles? In fact, thermal plasma and cosmic rays belong to a grand distribution function which describes particles from the very low energy to the very high. The artificial separation of particles into two parts demands a careful treatment on the distribution function. Recently, there are some progress on self-regulating injection (Malkov 1998; Gieseler et al. 2000). However, to have a rough idea on injection, one may introduce phenomenological models in the hydrodynamic approach.

From the deviation of the cosmic ray energy equation, if we interpret the contribution from the lower cutoff momentum as injection, then it is proportional to the negative of the divergence of the plasma flow (e.g., Zank et al. 1993; Ko et al. 1997). Moreover it is reasonable to assume that injection depends on the pressure of the thermal plasma. For instances, when injection is linear in thermal pressure, the problem can be formally recast into one without injection. The upshot is injection softens the equation of state of the plasma and makes it less likely to have subshocks (Ko et al. 1997).

Note that the one-dimensional steady state cosmic-ray-modified shock is also unstable (see e.g., Drury & Falle 1986; Kang et al. 1992; Zank et al. 1993).

(c) Three-fluid model

McKenzie & Völk (1982) worked out a three-fluid model which comprised thermal plasma, cosmic rays and one Alfvén wave. The one-dimensional steady state gives similar results as previous subsection, because the system can also be reduced to a first order autonomous ordinary differential equation.

We want to discuss another three-fluid model put forward by Jiang et al. (1996). In this case, the three fluids are cosmic rays and two oppositely propagating Alfvén waves. The thermal plasma is treated as a momentum and energy reservoir. Although this is only a test particle picture, we can systematically study stochastic acceleration as both waves are present. In their paper, the coupling between the cosmic rays and waves depends on the energy density of the waves.

For one-dimensional steady state in a uniform background flow, the system can be reduced to a set of two first order autonomous ordinary differential equations. The solutions can be classified completely. In general, for physically admissible solutions, the cosmic ray energy density vanishes far upstream but is finite far downstream, i.e., cosmic ray is accelerated efficiently. On the other hand, the energy density of both waves are finite far upstream but vanish far downstream. It can be shown that both wave energy densities decrease monotonically in the direction of the flow, while the cosmic ray energy density may have a local maximum. In most cases, the major acceleration mechanism is the work done by the flow against pressure gradients. Stochastic acceleration acts like a trigger.

In a shocked background, we can study the contribution of shock acceleration from the gain in cosmic ray energy density and energy flux (Ko 1998, 2000). (The gain means the difference between the far downstream value and the far upstream value.) The acceleration efficiency of different flow profiles can then be compared by their gains. For example, consider two super-Alfvénic flows: (i) shock transition: change discontinuously from a uniform upstream flow to a uniform downstream flow; (ii) rapid but smooth transition: change continuously but rapidly from the same upstream velocity to the same downstream velocity as in the shocked flow. (Rapid but smooth means the transition length is small compare to the system but is large compare to the longest wavelength of the waves.) To our surprise, the gains in cosmic ray energy flux are the same in both cases, but the gain in cosmic ray energy density is always smaller in the shocked flow (Ko 2000). How should we interpret the result? Can we say that shock decelerates cosmic rays in the hydrodynamic approach?

The reason behind this unexpected result is the shock (a genuine discontinuity) generates backward propagating waves. Nevertheless, one can conceive that in a self-consistent model the major acceleration or gain in energy density is expected to come from the high compression precursor upstream of the subshock (if the shock exists). Therefore, the question raised here may not be relevant.

Furthermore, we may interpret some of the results in the self-consistent two-fluid model as shock deceleration. Consider the case where one far upstream state gives three downstream states in which one of them is smooth and the other two are with subshock. It can be

shown that the gains in cosmic ray energy density and flux by the shocked flows are always less than the corresponding gains by the smooth flow (Ko 2001a). Indeed the difference in gains is rather significant (see Ko et al. 1997). In hindsight the result is kind of obvious. The overall compression ratio of the smooth flow is always larger than the corresponding values of both shocked flows. Moreover, part of the shock energy is channeled to heating up the downstream plasma.

Nonetheless, we deem that multiple downstream states produced by one upstream state is not physical. Recent investigation by the more sophisticated hybrid approach indicates that cosmic ray shocks are capable of self-organization (Malkov et al. 2000). That means only one unique state exists, namely, the self-organized critical state. The uniqueness renders the comparison procedure described above futile, because there is nothing to compare with.

(d) Four-fluid model

The last example is the self-consistent four-fluid model. The one-dimensional steady state problem can be reduced to a set of two first order autonomous ordinary differential equations. However, the mathematics is too tedious to analyze and we use numerical method instead. We seek continuous (smooth) physically admissible solutions (Ko 2001b).

The interplay between the three energy transfer mechanisms mentioned in the beginning of this section produces various steady state structures. Flow velocity, energy densities of cosmic rays and waves can be non-monotonic. We found two types of solutions so far: one resembles uniform flows and the other resembles modified-shocks. The uniform flow type solutions have both waves far upstream but they vanish far downstream (similar to the test particle picture in previous subsection). The modified-shock type solutions also have both waves far upstream. However, in the far downstream region the forward wave dies and the backward wave survives.

III. Remarks

Hydrodynamic approach is a good starting point to study the dynamics of magnetized plasma involving cosmic rays. It should be useful in studying the evolution and structure of molecular cloud, interstellar medium and even intra-cluster medium. We deem that cosmic ray is an under-rated player in this area, although its energy density is commonly considered to be of the same order as other constituents, say, thermal gas, magnetic field.

Having said that, hydrodynamic approach is only a singular limiting case of the more elaborated hybrid approach (Malkov 1997a, b). In hybrid approach cosmic ray is described by a distribution function and the waves are described by their power spectra, while the thermal plasma is still treated as a fluid. For

other shortcomings of and comments on the hydrodynamic approach, the reader is referred to, e.g., Heavens (1984), Achterberg et al. (1984), Jones & Kang (1990), Jones & Ellison (1991), Ko (1995).

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