JOURNAL OF THE KOREAN ASTRONOMICAL SOCIETY
34: S281 ~ S283, 2001

An Isothermal Mganetohydrodynamic Code and Its Application to the Parker Instability

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(Received Aug. 31, 2001; Accepted Nov. 15, 2001)

ABSTRACT

As a companion to an adiabatic version developed by Ryu and his coworkers, we have built an isothermal magnetohydrodynamic code for astrophysical flows. It is suited for the dynamical simulations of flows where cooling timescale is much shorter than dynamical timescale, as well as for turbulence and dynamo simulations in which detailed energetics are unimportant. Since a simple isothermal equation of state substitutes the energy conservation equation, the numerical schemes for isothermal flows are simpler (no contact discontinuity) than those for adiabatic flows and the resulting code is faster. Tests for shock tubes and Alfvén wave decay have shown that our isothermal code has not only a good shock capturing ability, but also numerical dissipation smaller than its adiabatic analogue. As a real astrophysical application of the code, we have simulated the nonlinear three-dimensional evolution of the Parker instability. A factor of two enhancement in vertical column density has been achieved at most, and the main structures formed are sheet-like and aligned with the mean field direction. We conclude that the Parker instability alone is not a viable formation mechanism of the giant molecular clouds.

Key Words : methods: numerical — MHD — ISM: clouds — ISM: magnetic fields — ISM: structure — shock waves

I. INTRODUCTION

The dynamics of astrophysical flows in general depends on detailed cooling and heating processes. Good numerical models should take into account of the energetics of the processes in the flows. There are, however, two extreme cases for which the energetics can be simplified. One case is for the so-called "adiabatic flows" where the energy in the system is not exchanged with outside. Such adiabatic approximation is good for flows, in which cooling time is far longer than dynamical time. The other case is for the "isothermal flows" where the temperature is kept constant, temporally and spatially. The isothermal approximation describes the energetics of flows, in which cooling time is far shorter than dynamical time. An example of such case is the flows in molecular clouds. The isothermality can also be applied, if the detailed energy budget is not an important issue such as in turbulence or dynamo simulations.

Usually, numerical simulations of isothermal flows are made with adiabatic codes by setting the adiabatic index, γ, close to unity. Yet, it is desirable to build codes specifically for isothermal flows, since those codes are *simpler* and *faster* than adiabatic ones. That is because the energy conservation equation need not to be solved in isothermal codes. As the result, the entropy mode, which carries the contact discontinuity, need not to be considered.

Motivated by the above astrophysical and numerical reasons, we have built an isothermal magnetohydrodynamic (MHD) code based on the total variation diminishing (TVD) scheme (Harten 1983). In this proceeding, we briefly describe the building procedures of the code, and show two performance tests devised to measure the shock capturing ability and numerical dissipation of the code. For complete descriptions, refer Ryu & Jones (1995) and Ryu, Jones, & Frank (1995) for the adiabatic version, and Kim et al. (1999) for the isothermal version.

As an application for an inherently important astrophysical problem, we have simulated the three-dimensional nonlinear evolution of the Parker instability. Based on the simulation results, we address the question on whether the Parker instability forms giant molecular clouds (GMCs). Unabridged descriptions on this issue were published in Kim et al (1998) and Kim, Ryu, & Jones (2001).

II. ISOTHERMAL MHD CODE

The isothermal MHD equations are composed of a set of conservation equations for mass, momentum, and magnetic flux. In addition, the isothermal equation of state ($p = \rho a^2$, where a is an isothermal sound speed) is applied, which makes the set of equations closed. Here, we mention briefly the building procedures of the code devised to solve the isothermal MHD equations. First, a one-dimensional TVD code is constructed, based on a second-order extension of the Roe-type upwind scheme (Roe 1981; Harten 1983). The left and right eigenvectors of a Jacobian matrix of the isothermal MHD

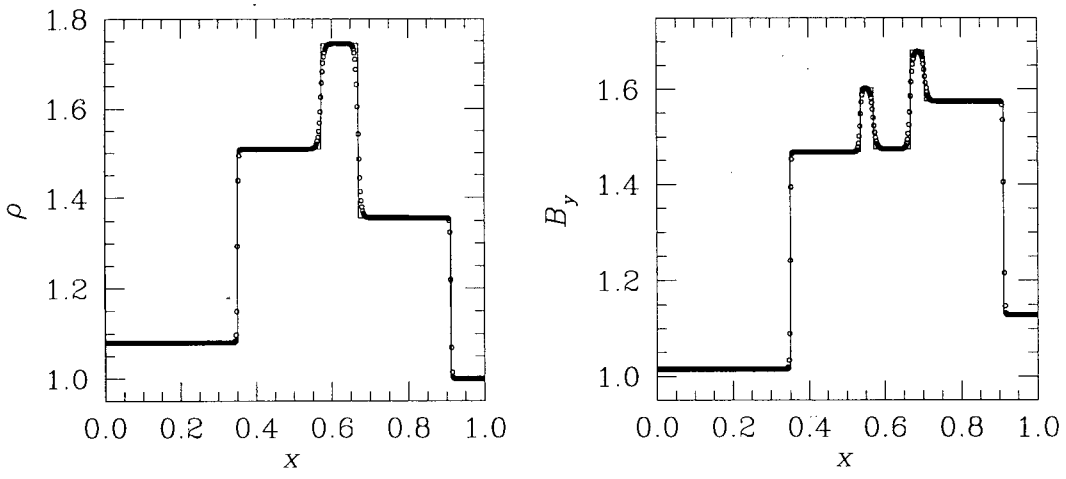

Fig. 1.— A snapshot at $t = 0.2$ from one-dimensional isothermal magnetohydrodynamic shock tube test. The initial condition is $(\rho, v_x, v_y, v_z, B_y, B_z) = (1.08, 1.2, 0.01, 0.5, 3.6/\sqrt{4\pi}, 2/\sqrt{4\pi})$ in the left region, $(\rho, v_x, v_y, v_z, B_y, B_z) = (1, 0, 0, 0, 4/\sqrt{4\pi}, 2/\sqrt{4\pi})$ in the right region, $B_x = 2/\sqrt{4\pi}$ and $a = 1$ for the whole computational interval. Open circles represent the numerical solution, while lines represent the analytic solution obtained from an exact nonlinear Riemann solver. The calculation has been done with 512 cells.

equations, which are the essential ingredients of the Roe-type scheme, were presented explicitly in Kim et al. (1999). Then, the one-dimensional code is extended to a multi-dimensional code through a Strang-type dimensional splitting. In the multi-dimensional code, an explicit cleaning step is included to eliminate non-zero $\nabla \cdot B$ at every time step.

In order to test the shock capturing ability, we have simulated a battery of isothermal MHD shock tube problems, which encompass all the structures formed by the three MHD wave families. Fig. 1 shows one example of them. After setting up the left half of the computational domain [0,1] with, $(\rho = 1.08, v_x = 1.2, v_y = 0.01, v_z = 0.5, B_y = 3.6/\sqrt{4\pi}, B_z = 2/\sqrt{4\pi})$, the right half with $(\rho = 1, v_x = 0, v_y = 0, v_z = 0, B_y = 4/\sqrt{4\pi}, B_z = 2/\sqrt{4\pi})$, and $B_x = 2/\sqrt{4\pi}$ in the whole domain, we have followed the evolution up to $t = 0.2$. Other numerical parameters are Courant number 0.8, $a = 1$, and the number of cells 512. Two fast shocks propagate outermost and two slow shocks interior to those. Two rotational discontinuities lie between the fast and slow shocks in the right panel of Fig. 1. Without any stiffener, strong shocks are resolved within 3 or 4 cells and other discontinuities spread a bit wider.

Another important aspect of a code is its numerical dissipation. In isothermal MHD codes, the dissipation comes only from numerical viscosity and resistivity (no thermal conduction). A simple way to quantify the dissipation is to follow up a standing Alfvén wave numerically, measure the decay rate of peak amplitudes, and finally estimate a Reynolds number. For this purpose, we have set up an initial condition that $\rho_0 = 1$, $\delta v_z = v_{\rm amp} \sin(k_x x + k_y y)$, $B = 1 \cdot \hat{x}$, and all other quantities are equal to zero. The calculations have been done in a square periodic box with size $L = 1$ using cells from 8×8 to 128×128. $k_x = k_y = 2\pi/L$ has been set. In Fig. 2 the resulting normalized decay rates as well as Reynolds numbers (see Ryu et al. 1995 for the definitions of the decay rate and the Reynolds number) are shown. Our numerical Reynolds numbers scale almost as $R \propto n_{\rm cell}^2$, indicating the code has a second-order accuracy. Compared to the adiabatic MHD code, the isothermal MHD code has smaller (up to 50%) numerical dissipation. This is mostly because it does not have the entropy mode.

III. PARKER INSTABILITY

The fact that a vertically stratified interstellar magnetized gas under a uniform gravity is unstable was proved by Parker (1966) through a linear stability analysis. Numerical simulation of the Parker instability is an interesting problem in both numerical and astrophysical points of view. Numerically, the Parker instability might be one of good astrophysical application problems for isothermal MHD codes. In the typical environment of our Galactic disk, where the instability takes place, the turbulent speed of gas is far larger than its thermal speed and is almost constant. So by regarding the turbulent speed as the isothermal speed, the energetics of the gas can be described by the isothermal equation of state. Astrophysically, it is an important issue whether the Parker instability itself can form the GMCs. In order to address this issue, we have carried out three-dimensional numerical simulations of the Parker instability with and without being included the effects of the Galactic rotation.

The initial equilibrium system is composed of exponentially stratified gas and field (along the azimuthal

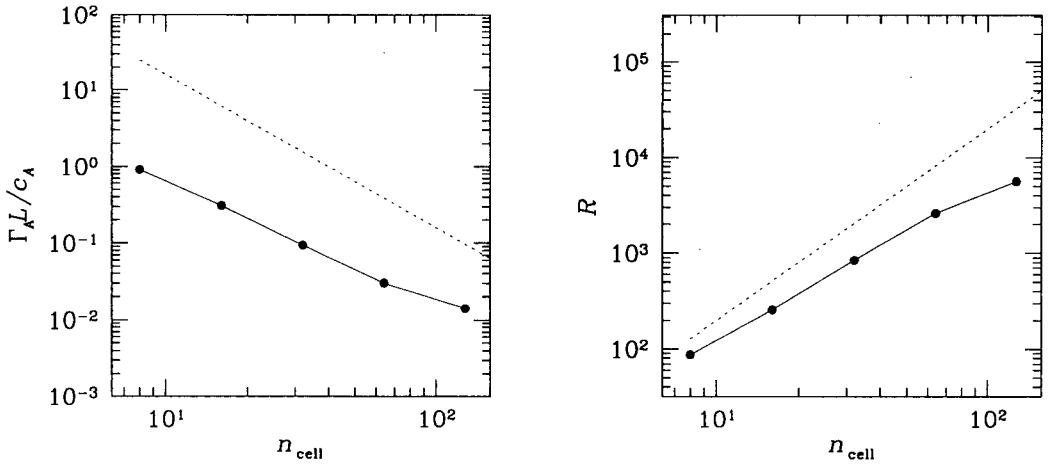

Fig. 2.— Normalized decay rate, $\Gamma_A L/c_A$, and magnetic Reynolds number, R, as a function of the number of cells along one direction of the computation domain. At a given resolution, the peak-to-peak decay rate of the root-mean-square of z-velocity (left) and the corresponding Reynolds number (right) are plotted with filled circles, respectively. The calculations have been done with 8×8, 16×16, 32×32, 64×64, and 128×128 cells. For comparison, dotted lines of $(\Gamma_A L/c_A) \propto n_{\rm cell}^{-2}$ and $R \propto n_{\rm cell}^2$ are drawn.

direction) in a uniform gravity (along the downward vertical direction). For the rotation model, the computational box is set to rotate around the Galactic center with a constant angular speed. To initiate the instability random velocity perturbations are added. Here, we enumerate some of the important findings from the numerical simulations. First, from resolution study, the density structure seen in high-resolution simulations is somewhat different from that in coarse-resolution ones. This is due to the inherent property of the Parker instability that the most unstable mode has an infinite wavenumber along the radial direction. In numerical experiments, the size of the smallest resolvable structure is limited by numerical resolution. Second, the maximum enhancement factor of vertical column density is at most ~ 2, and sheet-like structures form along the direction of the initial magnetic field. These results enable us to conclude that the Parker instability alone is not a viable formation mechanism of GMCs. Third, in the rotation model, Coriolis force makes the field lines in the valley regions helically-skewed. Hence, we suggest that the Coriolis force plays an important role in converting uniform field into random component.

IV. CONCLUSION

We have built an isothermal MHD TVD code based on the second order extension of the Roe-type upwind scheme. The code has not only a good shock capturing ability but also has low numerical dissipation. Through nonlinear simulations of the Parker instability with the code, we have demonstrated that the Parker instability alone is not a viable mechanism for the formation of the GMCs.

ACKNOWLEDGEMENTS

The work by DR was supported in part by KRF through grant KRF-2000-015-DS0046. The work by TWJ was supported in part by NASA grant NAG5-5055, by NSF grant AST00-71167, and by the University of Minnesota Supercomputing Institute. The work by JK and SSH was supported in part by the bilateral agreement between KOSEF-Korea and CONACYT-Mexico through 2000-113-01-2.

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