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Radiation Hydrodynamics of 2-D Accretion Disks

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ABSTRACT

To examine the structure and dynamics of thick accretion disks, we use a two-dimensional viscous hydrodynamic code coupled with radiation transport. The α -model and the full viscous stress-tensor description for the kinematic viscosity are used. The radiation transport is treated in the gray, flux-limited diffusion approximation. The finite difference methods used are based on an explicit-implicit method. We apply the numerical code to the Super-Eddington black-hole model for SS 433.@The result for a very small viscosity parameter α reproduces well the characteristic features of SS 433, such as the relativistic jets with $\sim 0.26c$, the small collimation degree of the jets, the mass-outflow rate of $\geq 5\times10^{-7}M_{\odot}\mathrm{yr}^{-1}$, and the formation of the X-ray iron emission lines.

Key Words : ACCRETION DISKS – HYDRODYNAMICS – RADIATION – X-RAY SOURCE: individual (SS 433)

I. INTRODUCTION

Radiation generally plays an important role in hydrodynamics of the accretion disk , particularly when we consider highly luminous disks. In the luminous disks, we expect geometrically thick disk rather than thin disk. The thin disks have been generally treated as one-dimensional standard accretion disk models by Shakura & Sunyaev (1973). However, for the highly luminous disks, we need a two-dimensional disk model coupled with radiation transport. Since it is very difficult for us to solve exactly the radiation transfer equation itself, we often use some approximations for the radiation transport and the flux-limited diffusion approximation is adopted here due to its simple but general applicability. Then the radiation hydrodynamic code of the two-dimensional accretion disks is applied to the X-ray source SS 433, because SS 433 is considered to have a thick accretion disk with super-Eddington accretion rate. We present briefly the numerical methods of the radiation hydrodynamics and examine the super-Eddington black hole models for SS 433. Details of the numerical results will appear elswhere in the near future.

II. NUMERICAL METHOD

(a) Basic Equations

The basic equations for mass, momentum, gas energy, and radiation energy are written in two-dimensional spherical polar coordinates (r,ζ,φ) where ζ is the polar angle measured from the equatorial plane of the disk :

$$\frac{\partial\rho}{\partial t}+\mathrm{div}(\rho\boldsymbol{v})=0, \tag{1}$$

$$\frac{\partial(\rho v)}{\partial t}+\mathrm{div}(\rho v\boldsymbol{v})=\rho\left[\frac{w^2}{r}+\frac{v_\varphi^2}{r}-\frac{GM_*}{(r-r_\mathrm{g})^2}\right] -\frac{\partial p}{\partial r}+f_r+\mathrm{div}\boldsymbol{S}_r+\frac{1}{r}S_{rr}, \tag{2}$$

$$\frac{\partial(\rho rw)}{\partial t}+\mathrm{div}(\rho rw\boldsymbol{v})= -\rho v_\varphi^2\tan\zeta-\frac{\partial p}{\partial\zeta}+\mathrm{div}(r\boldsymbol{S}_\zeta)+S_{\varphi\varphi}\tan\zeta+f_\zeta, \tag{3}$$

$$\frac{\partial(\rho r\cos\zeta v_\varphi)}{\partial t}+\mathrm{div}(\rho r\cos\zeta v_\varphi\boldsymbol{v})=\mathrm{div}(r\cos\zeta\boldsymbol{S}_\varphi), \tag{4}$$

$$\frac{\partial\rho\varepsilon}{\partial t}+\mathrm{div}(\rho\varepsilon\boldsymbol{v})=-p\,\mathrm{div}\boldsymbol{v}+\Phi-\Lambda, \tag{5}$$

$$\frac{\partial E_0}{\partial t}+\mathrm{div}\boldsymbol{F_0}+\mathrm{div}(\boldsymbol{v}E_0+\boldsymbol{v}P_0)=\Lambda-\rho\frac{(\kappa+\sigma)}{c}\boldsymbol{v}\cdot\boldsymbol{F}_0, \tag{6}$$

where ρ is the density, $\boldsymbol{v}=(v,w,v_\varphi)$ are the three velocity components, G is the gravitational constant, M_* is the central mass, p is the gas pressure, ε is the specific internal energy of the gas, E_0 is the radiation energy density per unit volume, and P_0 is the radiative stress tensor. It should be noticed that the subscript "0" denotes the value in the comoving frame. Here we adopt the pseudo-Newtonian potential (Paczynski & Wiita 1980) in equation (2), where r_g is the Schwarzschild radius. The force density $\boldsymbol{f}_\mathrm{R}=(f_r,f_\zeta)$ exerted by the radiation field is given by

$$\boldsymbol{f}_\mathrm{R}=\rho\frac{\kappa+\sigma}{c}\boldsymbol{F}_0, \tag{7}$$

where κ and σ denote the absorption and scattering coefficients and $\boldsymbol{F_0}$ is the radiative flux in the comoving frame. The full viscous stress tensor $S=(\boldsymbol{S}_r,\boldsymbol{S}_\zeta,\boldsymbol{S}_\varphi)$

is included here because we expect that radial and vertical motions of the disk gas may not be negligible compared with azimuthal one. $\Phi = (S\ \nabla)\boldsymbol{v}$ is the viscous dissipation rate per unit mass. The quantity Λ describes the cooling and heating of the gas, i.e. the energy exchange between the radiation field and the gas due to absorption and emission processes

$$\Lambda = \rho c \kappa (S_* - E_0), \tag{8}$$

where S_* is the source function and c is the speed of light. For this source function, we assume local thermal equilibrium $S_* = aT^4$, where T is the gas temperature and a is the radiation constant. For the equation of state, the pressure is given by the ideal gas law $p = R_G \rho T/\mu$ where μ is the mean molecular weight and R_G is the gas constant. The temperature T is proportional to the specific internal energy ε by the relation $p = (\gamma - 1)\rho\varepsilon = R_G \rho T/\mu$ where γ is the specific heat ratio. To close the system of equations, we use the flux-limited diffusion approximation (Levermore & Pomraning 1981) for the radiative flux:

$$\boldsymbol{F}_0 = -\frac{\lambda c}{\rho(\kappa + \sigma)} \text{grad}\, E_0, \tag{9}$$

and

$$\boldsymbol{P}_0 = E_0 \cdot T_{\text{Edd}}, \tag{10}$$

where λ and T_{Edd} are the *flux-limiter* and the *Eddington Tensor*, respectively, for which we use the approximate formulas given in Kley (1989). The formulas fulfil the correct limiting conditions in the optically thick diffusion limit and the optically thin streaming limit, respectively.

The kinematic viscosity ν is given by

$$\nu = \alpha\, c_s \min\left[H_p, H\right], \tag{11}$$

where α is a dimensionless viscosity parameter , usually α=0.001–1.0, c_s the local sound speed, H the disk height, and $H_p = p/\mid \text{grad}\, p \mid$ the pressure scale height on the equatorial plane.

(b) Methods

The numerical schemes used are basically the same as that described by Kley (1989) and Okuda et al. (1997). The methods are based on an explicit-implicit finite difference scheme. The numerical procedure consists of the following four steps.

(1) Advective transport : The first step is the calculations of the advective terms which describe the change of mass, energy, or momentum in a given grid cell as a consequence of the transport of these quantities through the surfaces. Here, we use the van Leer's (1977) second-order monotonic transport algorithm which is an extension of the simple first-order upwind scheme. The high accuracy of the monotonic interpolation scheme was demonstrated by Hawley et al. (1984) and Norman & Winkler (1986).

(2) Force terms : Second step is the explicit finite difference method for the force terms such as the gravitational, centrifugal, and pressure-gradient forces. The scheme is treated by the two-step method, which achieves a better accuracy in the difference equations.

The treatment of the explicit finite difference equations for viscous terms and energy equations sometimes induces troublesome problems in their stability and accuracy, so we use an implicit one for these terms.

(3) Viscous terms : Thirdly we write the finite difference equations for the viscous terms of momentum equations in an implicit form and solve the relevant coupled linear equations.

(4) Energy equations : Finally we solve the coupled implicit difference equations for gas energy and radiation energy.

These coupled linear equations can be solved by the Modified Incomplete LU decomposition & Bi-Conjugate Gradient method which is based on the successive overrelaxation procedure.

III. Application to SS 433

Even at present, SS 433 is a very puzzling and interesting X-ray object. It has remarkable observational features, such as two oppositely directed relativistic jets, its expected too high energy, and the precessing motion of the jets.

(a) Model Parameters

For the central source of SS 433, we consider a Schwartzschild black hole with mass $M_* = 10M_\odot$ and mass accretion rate $\dot{M}_* = 5\dot{M}_E(\sim 10^{20}\text{g s}^{-1})$ and examine the structure and dynamics of accretion disk around the black hole and its surrounding atmosphere. $\dot{M}_E$ is the Eddington critical accretion rate given by $\dot{M}_E = 48\pi G M_*/(\kappa_e c)$, where κ_e is the electron scattering opacity. This high accretion rate $\dot{M}_*$ is suggested from the X-ray observation (kotani et al. 1996). The model parameters used are given in table 1 where α, r_{in} , and r_{out} are the kinematic viscosity parameter, the inner boundary radius of the computational domain, and the outer boundary radius, respectively.

(b) Initial and Boundary Conditions

Initial conditions consist of a Shakura-Sunyaev standard disk and a rarefied hot atmosphere around the disk. Variables at the inner boundary r_{in} are given

Table 1. Model parameters

model	$\dot{M}_*(\text{g s}^{-1})$	α	r_{in}	$r_{\text{out}}/r_{\text{in}}$
BH-1	10^{20}	0.001	$2r_g$	220
BH-2	10^{20}	0.1	$2r_g$	220

by extrapolation of the variables near the boundary but we prohibit outflow at the inner boundary. At the outer boundary $r_{\rm out}$ except the disk region, the freely-floating conditions are imposed and constant inflow of gas is kept at the outer boundary of the disk.

(c) Numerical Results

i) model BH-1 with $\alpha = 10^{-3}$

The initial disk thickness H/r based on the standard disk model is ~ 1 at the inner region of the disk and ~ 0.1 at the outer boundary of $r/r_{\rm in} \sim 200$. The ratio β of the gas pressure to the total pressure at the outer boundary of the disk is ~ 0.27 .

We performed a time evolution of the disk until $t = 4 \times 10^3 P_{\rm d}$ where a quasi-steady state is attained. Here, $P_{\rm d}$ is the Keplerian orbital period at the inner boundary. The total luminosity L at the final phase emitted from the whole system is 1.6×10^{39}erg s^{-1} and $L/L_{\rm E}$ is ~ 1 which is far smaller than $\dot{M}_*/\dot{M}_{\rm E}(= 5)$, where $L_{\rm E}$ is the Eddington luminosity. In spite of the highly super-Eddington input matter, most of the matter is blown off from the system through the jets and the disk wind, and the luminosity is comparable to the Eddington luminosity.

At the initial stage of the evolution, matter in the inner region of the disk is ejected strongly outward. Some of the ejected gas hit on the rotational axis and others propagate outward. The gas hit on the axis leads to a high-temperature region along the axis and an anisotropic radiation field whose equ-contour lines are vertical to the radial direction. This results in outward radiation forces which dominate gravitational forces and a high-velocity jets region is formed along the axis. Although, at the beginning of the evolution, the high-velocity region is confined to a narrow region along the rotational axis, it expands gradually outward and settles down to a quasi-steady state at the final phase with a collimation angle of $\sim 30°$ to the axis. On the other hand, the outwardly ejected matter of the disk becomes to occupy most of the computational domain considered here with increasing time. At the final phase, this dense region has a cone-like shape with an opening angle $\sim 60°$ to the equatorial plane and is optically thick. We regard this optically and geometrically thick, dense, and cold region as the accretion disk. The boundary region between the high-velocity region and the disk is characterized by sharp gradients of density and temperature like a shock discontinuity.

The densities are high as $\leq 10^{-7}$g cm^{-3} in the disk but small as $\sim 10^{-8}$ to 10^{-26}g cm^{-3} in the high-velocity region above the disk and decrease abruptly towards the rotational axis. The temperatures range from 2×10^6 to $\sim 10^7$ K in the disk region, jump to $\sim 10^8$ K above the upper disk, again distribute gradually between $\sim 10^8 - 10^9$ K in the high-velocity region, and are as very high as $10^9 - 10^{11}$ K near the rotational axis. The calculations show that the total integrated emission of the X-ray iron lines is attributed mostly to a confined high-velocity region with velocities of $\sim 0.2c$ at $60° \leq \zeta \leq 70°$. Although the maximum velocity $\sim 0.4c$ is obtained at $\zeta \sim 80°$, the high-velocity gas does not contribute to the X-ray iron lines because the densities at the maximum velocity region are very low as $\leq 10^{-18}$g cm^{-3} compared with $\sim 10^{-10}$g cm^{-3} in the region of $\sim 0.2c$. This would explain the small collimation degree of the observed relativistic jets.

ii) model BH-2 with $\alpha = 0.1$

The initial disk thickness in this model is similar to that of model BH-1. But the densities and the temperatures in the initial disk are roughly more than one order of magnitude and by a few factor larger than those in model BH-1, respectively. This model shows very different time evolutions from model BH-1. The gas ejected initially from the disk mostly fades out from the outer boundary but partly is intercorporated into the outer part of the disk. The final disk becomes fatter at the outer disk than the initial disk. As a result, a well-collimated high-velocity jets region along the rotational axis is not established but an accretion disk with an opening angle $\sim 10°$ to the equatorial plane is formed. The temperatures and the densities at the atmospheric region around the disk are as hot as $\geq 10^8$ K and low as $\sim 10^{-9}$ – 10^{-13} g cm^{-3}, respectively, but the velocities are not so large as in model BH-1, except $\sim 0.07c$ near the rotational axis. At any rate, this model never reproduce the X-ray iron emission lines with the relativistic velocity $0.26c$.

IV. SUMMARY

We developed a two-dimensional hydrodynamic code coupled with radiation transport. The radiation transport is treated by the gray, flux-limited diffusion approximation. The application of the numerical code reproduces well the characteristic features of the mysterious X-ray source SS 433. The radiation hydrodynamic code would be very useful for many problems of two-dimensional accretion disks.

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