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LEFT(RIGHT) FILTERS ON *PO*-SEMIGROUPS

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Abstract. In this paper, we give the characterization of a left (right) filter of *po*-semigroups.

1. Introduction

Kehayopulu([3]) gave the characterization of the filter of S in term of the prime ideals. A *po-semigroup* (: ordered semigroup) is an ordered set S at same time a semigroup such that $a \leq b \Longrightarrow xa \leq xb$ and $ax \leq bx$ for all $x \in S$.

Definition 1([3, 4]). Let S be a *po*-semigroup. A nonempty subset A of S is called an *right* (*left*) *ideal* of S if

1) $AS \subseteq A$(resp. $SA \subseteq A$),
2) $a \in A$ and $b \leq a$ for $b \in S \Longrightarrow b \in A$.

A is called an *ideal* of S if it is a right and left ideal of S.

Definition 2([3, 5]). A subset T of S is called *prime* if $AB \subseteq T \Longrightarrow A \subseteq T$ or $B \subseteq T$ for subsets A, B of S.

T is called a *prime right* (*left*) *ideal* if T is prime as a right (left) ideal. T is called a prime ideal if T is prime as an ideal.

Definition 3. A subsemigroup F of a *po*-semigroup S is called a *left*(resp. *right*) *filter* of S if

1) $ab \in F$ for $a, b \in S \Longrightarrow a \in F$ (resp. $b \in F$),
2) $a \in F$ and $a \leq c$ for $c \in S \Longrightarrow c \in F$.

A subsemigroup F of S is called a *filter*([1, 2]) of S if F is a left and right filter.

In this paper, we give the characterization of a left (right) filter of S in term of the right (left) prime ideals

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2. Main Theorems

THEOREM 1. *Let S be a po-semigroup and F a nonempty subset of S. The following are equivalent:*

1) *F is a left filter of S.*
2) *$S \setminus F = \emptyset$ or $S \setminus F$ is a prime right ideal*

Proof. 1) $\Longrightarrow$ 2). Assume that $S \setminus F \neq \emptyset$. Let $x \in S \setminus F$ and $y \in S$. Then $xy \in S \setminus F$. Indeed: If $xy \notin S \setminus F$, then $xy \in F$. Since F is a left filter, $x \in F$. It is impossible. Thus $xy \in S \setminus F$, and so $(S \setminus F)S \subseteq S \setminus F$.

Let $x \in S \setminus F$ and $y \leq x$ for $y \in S$. Then $y \in S \setminus F$. Indeed: If $y \notin S \setminus F$, then $y \in F$. Since F is a left filter, $x \in F$. It is impossible. Thus $y \in S \setminus F$. Therefore $S \setminus F$ is a right ideal.

Next we shall prove that $S \setminus F$ is prime. Let $xy \in S \setminus F$ for $x, y \in S$. Suppose that $x \notin S \setminus F$ and $y \notin S \setminus F$. Then $x \in F$ and $y \in F$. Since F is a subsemigroup of S, $xy \in F$. It is impossible. Thus $x \in S \setminus F$ or $y \in S \setminus F$. Hence $S \setminus F$ is prime, and so $S \setminus F$ is a prime right ideal.

2) $\Longrightarrow$ 1). If $S \setminus F = \emptyset$, then $F = S$. Thus F is a left filler of S.

Next assume that $S \setminus F$ is a prime right ideal of S. Then F is a subsemigroup of S. Indeed: Suppose that $xy \not{i}nF$ for $x, y \in F$. Then $xy \in S \setminus F$ for $x, y \in F$. Since $S \setminus F$ is prime, $x, y \in S \setminus F$. It is impossible. Thus $xy \in F$, and so F is a subsemigroup of S.

Let $xy \in F$ for $x, y \in S$. Then $x \in F$. Indeed: If $x \notin F$, then $x \in S \setminus F$. Since $S \setminus F$ is a prime right ideal of S, $xy \in (S \setminus F)S \subseteq S \setminus F$. It is impossible. Thus $x \in F$.

Let $x \in F$ and $x \leq y$ for $y \in S$. Then $y \in F$. Indeed: If $y \notin F$, then $y \in S \setminus F$. Since $S \setminus F$ is a right ideal of S, $x \in S \setminus F$. since $S \setminus F$ is a right ideal. It is impossible. Thus $y \in F$.

Therefore F is a left filter of S. □

By the similar method, we have the following theorem 2.

THEOREM 2. *Let S be a po-semigroup and F a nonempty subset of S. The following are equivalent:*

1) *F is a right filter of S.*
2) *$S \setminus F = \emptyset$ or $S \setminus F$ is a prime left ideal*

From Theorem 1 and 2, we get the following.

Corollary ([3]). *Let S be a po-semigroup and nonempty subset of S. The following are equivalent:*

1) *F is a filter of S.*
2) *$S \setminus F = \emptyset$ or $S \setminus F$ is a prime ideal S.*

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