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REMARK ON A SEGAL-LANGEVIN TYPE STOCHASTIC DIFFERENTIAL EQUATION ON INVARIANT NUCLEAR SPACE OF A Γ-OPERATOR

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Abstract. Let $\mathcal{S}'(\mathbb{R})$ be the dual of the Schwartz spaces $\mathcal{S}(\mathbb{R})$, A be a self-adjoint operator in $L^2(\mathbb{R})$ and $\Gamma(\mathrm{A})^*$ be the adjoint operator of $\Gamma(\mathrm{A})$ which is the second quantization operator of A. It is proven that under a suitable condition on A there exists a nuclear subspace $\mathcal{S}$ of a fundamental space $\mathcal{S}_{\mathrm{A}}$ of Hida's type on $\mathcal{S}'(\mathbb{R})$ such that $\Gamma(\mathrm{A})\mathcal{S} \subset \mathcal{S}$ and $e^{-t\Gamma(\mathrm{A})}\mathcal{S} \subset \mathcal{S}$, which enables us to show that a stochastic differential equation:

$$dX(t) = dW(t) - \Gamma(\mathrm{A})^* X(t)dt,$$

arising from the central limit theorem for spatially extended neurons has an unique solution on the dual space $\mathcal{S}'$ of $\mathcal{S}$.

1. Introduction

Two types of fundamental spaces on infinite dimensional topological vector spaces have been studied by [1, 2, 3, 4, 6, 8, 10] in connection with infinite dimensional geometry and analysis. In general, the nuclearity of the fundamental spaces gives us various fruitful results[5]. Until now, it has been known that the fundamental spaces in the Malliavin calculus are not nuclear, while the original Hida space is nuclear.

Let $\mathcal{S}_{\mathrm{A}}$ be a fundamental space of Hida's type and $\Gamma(\mathrm{A})$ the second quantization operator of A. Inspired by the works [8,9], we construct a fundamental space which is invariant under the semi-group $e^{-t\Gamma(\mathrm{A})}$ and is nuclear and smaller than $\mathcal{S}_{\mathrm{A}}$ even if $\mathcal{S}_{\mathrm{A}}$ is not nuclear. This

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enables us to obtain an unique strong solution of the stochastic differential equation

$$dX(t) = dW(t) - \Gamma(\mathrm{A})^* X(t)dt \tag{1.1}$$

which is a special case of the types considered in [7].

First we begin by giving some notations and explanations. Let $\mathcal{E}$ be a real locally convex topological vector space and $\mathcal{E}^{'}$ the topological dual space of $\mathcal{E}$. We denote by $<,>$ the pairing of $\mathcal{E}$ and $\mathcal{E}^{'}$, and by $|\cdot|_{\mathcal{E}}$ the norm of $\mathcal{E}$ if $\mathcal{E}$ is a Hilbert space. Let $\mathcal{H}$ be a separable real Hilbert space densely and continuously embedded in $\mathcal{E}$. Then identifying $\mathcal{H}^{'}$ with $\mathcal{H}$, we have

$$\mathcal{E}^{'} \subset \mathcal{H} \subset \mathcal{E}.$$

Let μ be the countably additive Gaussian measure on $\mathcal{E}$ whose characteristic functional is given by

$$\int_{\mathcal{E}} \exp\left[i < x, \xi >\right] d\mu(x) = \exp\left[-\frac{1}{2}\left|\xi\right|^2_{\mathcal{H}}\right], \quad \xi \in \mathcal{E}^{'}.$$

Let $\mathcal{S}^{'}(\mathbb{R})$ be the dual of the Schwartz spaces $\mathcal{S}(\mathbb{R})$. If we replace $\mathcal{E}$ by $\mathcal{S}^{'}(\mathbb{R})$, $(\mathcal{E}, \mu)$ is called the white noise space [8].

Let A be a self-adjoint operator in Hilbert space $\mathcal{H}$ and $L^2(\mathcal{E}, \mu)$ be the space of square integrable functions with respect to μ. Further we denote by $\mathcal{H}^{\otimes n}$ the n-fold tensor product space of $\mathcal{H}$, by $\mathcal{S}_{\mathrm{A}}$ the Hida space determined by A and by $\Gamma(\mathrm{A})$ the second quantization operator of A, which will be precisely defined later. From now on we denote the domain of a closed linear operator T densely defined in $\mathcal{H}$ by $\mathcal{D}(\mathrm{T})$ and define $\mathcal{C}^\infty(\mathrm{T}) := \bigcap_{n=1}^{\infty} \mathcal{D}(\mathrm{T}^n)$. We always consider $\mathcal{D}(\mathrm{T}^n)$ as a Hilbert space equipped with the inner product $(\mathrm{T}^n\cdot, \mathrm{T}^n\cdot)_{\mathcal{H}}$. Given $\lambda \in \mathbb{R}$, we mean by $\mathrm{A} \geq \lambda$ that $(\mathrm{A}f, f)_{\mathcal{H}} \geq \lambda(f, f)_{\mathcal{H}}$ for all $f \in \mathcal{D}(\mathrm{A})$. Now we state main result.

THEOREM. *Suppose that $A \geq 1 + \epsilon$, for some $\epsilon > 0$ and there exists a self-adjoint operator B in $\mathcal{H}$ and natural numbers p and q, satisfying the following conditions:*

1) $\mathcal{D}(B^p) \subset \mathcal{D}(A)$

2) the identity map of $\mathcal{D}(B^q)$ into $\mathcal{H}$ is a Hilbert Schmidt operator,

3) $A\mathcal{C}^\infty(B) \subset \mathcal{C}^\infty(B)$.

Then $\mathcal{S}_B$ is a nuclear subspace of $L^2(\mathcal{E},\mu)$ such that

$$\Gamma(A)\,\mathcal{S}_B \subset \mathcal{S}_B.$$

Further suppose that

4) for any nonnegative integers m and k , A^m and B^k are commutative. Then

$$e^{-t\Gamma(A)}\mathcal{S}_B \subset \mathcal{S}_B.$$

2. Space of the White Noise

Before defining a fundamental space of Hida's type, we introduce the following notation. Let $\mathcal{H}$ be a separable Hilbert space. For $f_i \in \mathcal{H}, i = 1, 2, \cdots, n$ we denote the tensor product of them by

$$f_1 \otimes f_2 \otimes \cdots \otimes f_n$$

and define the symmetric tensor product of them by

$$f_1 \hat{\otimes} f_2 \hat{\otimes} \cdots \hat{\otimes} f_n := \frac{1}{n!} \sum_{\sigma \in \Xi_n} f_{\sigma(1)} \otimes f_{\sigma(2)} \otimes \cdots \otimes f_{\sigma(n)}, \tag{2.1}$$

where Ξ_n is the symmetric group of degree n.

Let $\mathcal{H}^{\otimes n}$ and $\mathcal{H}^{\hat{\otimes} n}$ be the n-fold tensor product space and the n-fold symmetric tensor product space of $\mathcal{H}$, respectively. For $f_i, g_i \in \mathcal{H}, i = 1, 2, \cdots, n$, the inner product $(\cdot,\cdot)_{\mathcal{H}^{\otimes n}}$ is given by

$$\begin{aligned} &\left(f_1 \hat{\otimes} f_2 \hat{\otimes} \cdots \hat{\otimes} f_n, g_1 \hat{\otimes} g_2 \hat{\otimes} \cdots \hat{\otimes} g_n\right)_{\mathcal{H}^{\hat{\otimes} n}} \\ &= \left(\frac{1}{n!}\right)^2 \sum_{\sigma,\tau \in \Xi_n} \left(f_{\sigma(1)}, g_{\tau(1)}\right)_{\mathcal{H}} \left(f_{\sigma(2)}, g_{\tau(2)}\right)_{\mathcal{H}} \cdots \left(f_{\sigma(n)}, g_{\tau(n)}\right)_{\mathcal{H}}, \end{aligned}$$

Clearly

$$\mathcal{H}^{\hat{\otimes} n} \subset \mathcal{H}^{\otimes n}, \tag{2.2}$$

and

$$\begin{aligned} &\left(f_1 \hat{\otimes} f_2 \hat{\otimes} \cdots \hat{\otimes} f_n, g_1 \hat{\otimes} g_2 \hat{\otimes} \cdots \hat{\otimes} g_n\right)_{\mathcal{H}^{\otimes n}} \\ &= \left(f_1 \hat{\otimes} f_2 \hat{\otimes} \cdots \hat{\otimes} f_n, g_1 \hat{\otimes} g_2 \hat{\otimes} \cdots \hat{\otimes} g_n\right)_{\mathcal{H}^{\hat{\otimes} n}}. \end{aligned} \tag{2.3}$$

We review the relation between the wick ordering $: x^{\otimes n} :$ for $x \in \mathcal{E}$ used in [4,8] and the wick product [9]. Wick product $:< x, \xi_1 >< x, \xi_2 > \cdots < x, \xi_n >:$ of random variables $< x, \xi_k >, x \in \mathcal{E}, \xi_k \in \mathcal{E}^{'}, k = 1, 2, \cdots, n$, with respect to μ is defined by the following recursion relation [9]:

$$:< x, \xi_1 >:=< x, \xi_1 >,$$

$$:< x, \xi_1 >< x, \xi_2 > \cdots < x, \xi_n >:=< x, \xi_1 >:< x, \xi_2 > \cdots < x, \xi_n >:$$
$$- \sum_{k=2}^{n} \int_{\mathcal{E}} < x, \xi_1 >< x, \xi_k > d\mu(x) :< x, \xi_2 > \cdots < x, \check{\xi_k} > \cdots < x, \xi_n >:,$$

where $< x, \check{\xi_k} >$ means the term $< x, \xi_k >$ is deleted in the product. Then we have

$$<: x^{\otimes n} :, \xi_1 \hat{\otimes} \xi_2 \hat{\otimes} \cdots \hat{\otimes} \xi_n >:=:< x, \xi_1 >< x, \xi_2 > \cdots < x, \xi_n >: . \tag{2.4}$$

It is well known by Wiener-Ito theorem that the space $L^2(\mathcal{E}, \mu)$ has the following orthogonal decomposition

$$L^2(\mathcal{E}, \mu) = \bigoplus_{n=0}^{\infty} \mathcal{K}_n, \tag{2.5}$$

where $\mathcal{K}_n$ consists of n-homogeneous chaos, i.e. each φ in $\mathcal{K}_n$ has the formal expression

$$\varphi(x) =<: x^{\otimes n} :, \hat{f}_n >, \quad \hat{f}_n \in \mathcal{H}^{\hat{\otimes} n}. \tag{2.6}$$

Thus each $\psi \in L^2(\mathcal{E}, \mu)$ can be represented uniquely in the following form :

$$\psi(x) = \sum_{n=0}^{\infty} <: x^{\otimes n} :, \hat{f}_n >, \quad \mu - a.e. \quad x \in \mathcal{E}. \tag{2.7}$$

Moreover, we have

$$|\psi|^2_{L^2(\mathcal{E},\mu)} = \sum_{n=0}^{\infty} n! \left| \hat{f}_n \right|^2_{\mathcal{H}^{\hat{\otimes} n}}, \quad [4, 8]. \tag{2.8}$$

Let A be a positive self-adjoint operator in $\mathcal{H}$. Then there exists an unique positive self-adjoint operator $\Gamma(\mathrm{A})$ in $L^2(\mathcal{E},\mu)$ such that

$$\Gamma(\mathrm{A})1 = 1$$

and for $\xi_i \in \mathcal{D}(\mathrm{A}), i = 1, 2, \cdots, n$,

$$\begin{aligned}&\Gamma(\mathrm{A}) :< x, \xi_1 > \cdots < x, \xi_n >:\\ &=:< x, \mathrm{A}\xi_1 > \cdots < x, \mathrm{A}\xi_n >:\\ &=<: x^{\otimes n} :, (\mathrm{A}\otimes\cdots\otimes\mathrm{A})(\xi_1\hat{\otimes}\cdots\hat{\otimes}\xi_n) > .\end{aligned}$$

We denote by $\mathcal{P}_{\mathrm{A}}$ the collection of all polynomials of the form

$$\omega(x) = P(< x, \xi_1 > \cdots < x, \xi_m >), \quad \xi_i \in \mathcal{C}^\infty(\mathrm{A}),$$

where $P(t_1, \cdots, t_m)$ is a polynomial of $(t_1, \cdots, t_m)$. For each $p \in \mathbb{R}$ we define a semi-norm ${}_{\mathrm{A}}\|\cdot\|_{2,p}$ by

$$ {}_{\mathrm{A}}\|\omega\|_{2,p}^2 := \int_{\mathcal{E}} |\Gamma(\mathrm{A})^p\omega(x)|^2 \, d\mu(x). \tag{2.9}$$

It is not difficult to see that $\Gamma(\mathrm{A})^p = \Gamma(\mathrm{A}^p)$. By (2.4), each ω in $\mathcal{P}_{\mathrm{A}}$ has the following expression:

$$\omega(x) = \sum_{n=0}^{\infty} <: x^{\otimes n} :, \hat{g}_n >, \quad \hat{g}_n \in C^\infty(\mathrm{A})^{\hat{\otimes} n},$$

where $C^\infty(\mathrm{A})^{\hat{\otimes} n} = \overbrace{C^\infty(\mathrm{A})\hat{\otimes}\cdots\hat{\otimes}C^\infty(\mathrm{A})}^{n-\text{times}}$ is the set of finite linear combinations of the form $\xi_1\hat{\otimes}\cdots\hat{\otimes}\xi_n$ with $\xi_i \in \mathcal{C}^\infty(\mathrm{A}), i = 1, 2, \cdots n$. In fact we note that there exists a natural number $k(\omega)$ such that $\hat{g}_n = 0$ for $n \geq k(\omega)$. Since

$$\hat{g}_n = \sum_{i=1}^{m(n)} a_i(n)\xi_{i_1}\hat{\otimes}\cdots\hat{\otimes}\xi_{i_n}, \quad \xi_{i_k} \in \mathcal{C}^\infty(\mathrm{A}), k = 1, 2, \cdots n,$$

by (2.8), ${}_{\mathrm{A}}\|\cdot\|_{2,p}^2$ can be also represented as

$$ {}_{\mathrm{A}}\|\omega\|_{2,p}^{2} = \sum_{n=0}^{\infty} n! \left|(\mathrm{A}^{p})^{\otimes n}\, \hat{g}_n\right|_{\mathcal{H}^{\hat{\otimes} n}}^{2}, \tag{2.10} $$

where

$$ (\mathrm{A}^{p})^{\otimes n} = \mathrm{A}^{p} \otimes \cdots \otimes \mathrm{A}^{p}. $$

For $p \geq 0$, $\left(\mathcal{S}_{\mathrm{A}}\right)_p$ is the completion of $\mathcal{P}_{\mathrm{A}}$ with respect to the seminorm ${}_{\mathrm{A}}\| \cdot \|_{2,p}$. We define the fundamental space $\mathcal{S}_{\mathrm{A}}$ of the Hida distributions on $\mathcal{E}$ by

$$ \mathcal{S}_{\mathrm{A}} := \bigcap_{p \geq 0} \left(\mathcal{S}_{\mathrm{A}}\right)_p. \tag{2.11} $$

If we take $\mathcal{E} = \mathcal{S}^{'}(\mathbb{R})$ and $\mathrm{A} = -\left(\frac{d}{dx}\right)^2 + x^2 + 1$, then $\mathcal{S}_{\mathrm{A}}$ becomes a nuclear space and originally it is called the fundamental space of the Hida distributions.

3. Proof of Theorem

Let $\mathbb{N}_0 = \{0, 1, 2, \cdots\}$ and let $\mathbb{I}^n$ be the set of all naturally ordered n-tuples in $\mathbb{N}_0^n$. For $\alpha = (\alpha_1, \cdots \alpha_n) \in \mathbb{I}^n$, define $n_k(\alpha), 0 \leq k < \infty$, and $n(\alpha)!$ as followings:

$$ n_k(\alpha) := \sharp\{\alpha_j : \alpha_j = k\}, \qquad n(\alpha)! := \prod_{k=0}^{\infty} n_k(\alpha)! $$

Let $e_k, k \geq 0$ be the Hermite functions. For each $\alpha \in \mathbb{I}^n$, we define

$$ H_\alpha(x) = (n(\alpha)!)^{-\frac{1}{2}} \prod_{k=0}^{\infty} :< x, e_k >^{n_k(\alpha)}: . $$

It is a fact that the collection $\{H_\alpha : \alpha \in \mathbb{I}^n, n \geq 0\}$ forms an orthonormal basis for the space $L^2(\mathcal{E}, \mu)$[8]. Since operator A and B are commutative, A has only discrete spectrums; i.e.

$$ \mathrm{A}e_k = \nu_k e_k, k = 1, 2, \cdots, $$

where $\{\nu_k\}$ is the eigenvalues of A and $\{e_k\}$ forms a complete orthonormal basis of $\mathcal{H}$. Let $\{\lambda_k\}$ be the eigenvalues of B with respect to $\{e_k\}$.

Consider the fundamental space $\mathcal{S}_{\mathrm{B}}$ of the Hida distributions on $\mathcal{E}$ determined by B. Then the condition (2) of Theorem yields that $\mathcal{S}_{\mathrm{B}}$ becomes a nuclear space[2]. Take any $\omega(x) \in P_{\mathrm{B}}$ then the following expression holds:

$$\begin{aligned}\omega(x) &= \sum_{\alpha} C_\alpha H_\alpha(x) \\ &= \sum_{\alpha} C_\alpha \left(n(\alpha)!\right)^{-\frac{1}{2}} \prod_{k=0}^{\infty} :< x, e_k >^{n_k(\alpha)}: .\end{aligned} \tag{3.1}$$

Therefore, we get

$$\begin{aligned}
&{}_{\mathrm{B}}\|\Gamma(\mathrm{A})\omega(x)\|_{2,p}^2 = |\Gamma(\mathrm{B}^p)\Gamma(\mathrm{A})\omega(x)|_{L^2(\mathcal{E},\mu)}^2 \\
&= \left| \sum_{\alpha} C_\alpha \left(n(\alpha)!\right)^{-\frac{1}{2}} <: x^{\times n(\alpha)} :, \bigotimes_{k=1}^{\infty} \mathrm{B}^p \mathrm{A} e_k^{n_k(\alpha)} > \right|_{L^2(\mathcal{E},\mu)}^2 \\
&= \left| \sum_{\alpha} C_\alpha \left(n(\alpha)!\right)^{-\frac{1}{2}} \prod_{i=1}^{\infty} \lambda_i^{pn_i(\alpha)} \prod_{j=1}^{\infty} \nu_j^{n_j(\alpha)} \prod_{k=1}^{\infty} :< x, e_k >^{n_k(\alpha)}: \right|_{L^2(\mathcal{E},\mu)}^2 \\
&= \sum_{\alpha} \left(\prod_{i=1}^{\infty} \lambda_i^{pn_i(\alpha)} C_\alpha \right)^2 \left(\prod_{j=1}^{\infty} \nu_j^{n_j(\alpha)} \right)^2 \|H_\alpha\|_{L^2(\mathcal{E},\mu)}^2 \\
&= \sum_{\alpha} \left(\prod_{j=1}^{\infty} \nu_j^{n_j(\alpha)} \right)^2 \left(\prod_{i=1}^{\infty} \lambda_i^{pn_i(\alpha)} C_\alpha \right)^2
\end{aligned}$$

$$\begin{aligned}
&= \sum \left[\frac{\prod_{j=1}^{\infty} \nu_j^{n_j(\alpha)}}{\prod_{i=1}^{\infty} \lambda_i^{\delta n_i(\alpha)}} \right]^2 \left(\prod_{i=1}^{\infty} \lambda_i^{(p+\delta)n_i(\alpha)} C_\alpha \right)^2 \\
&\leq \left[\sum \left[\frac{\prod_{j=1}^{\infty} \nu_j^{n_j(\alpha)}}{\prod_{i=1}^{\infty} \lambda_i^{\delta n_i(\alpha)}} \right]^{2t} \right]^{\frac{1}{t}} \left[\left(\prod_{i=1}^{\infty} \lambda_i^{(p+\delta)n_i(\alpha)} C_\alpha \right)^{2s} \right]^{\frac{1}{s}} \\
&\leq MC^{2(1-\frac{1}{s})} {}_{\mathrm{B}}\|\omega(x)\|_{2,q}^2
\end{aligned} \tag{3.2}$$

where $\frac{1}{t}+\frac{1}{s}=1, s>1, \delta>0$ and $q=s(p+s)$. Since $\lambda_i>\nu_i, i=1,2,\cdots$, we know that

$$M=\left[\sum\left[\frac{\prod_{j=1}^{\infty}\nu_j^{n_j(\alpha)}}{\prod_{i=1}^{\infty}\lambda_i^{\delta n_i(\alpha)}}\right]^{2t}\right]^{\frac{1}{t}}$$

is finite.

The last inequality of (3.2) holds, because let $C=\sup_\alpha|C_\alpha|<\infty$, then $C_\alpha^{2s}\leq C_\alpha^2 C^{2(s-1)}$.

Therefore we have $\Gamma(\mathrm{A})\mathcal{S}_{\mathrm{B}}\subset\mathcal{S}_{\mathrm{B}}$. Thus the proof of the first half of Theorem is completed. By the manner similar to that in the first half, the second half of the Theorem is proved.

4. Strong Solution for a Segal-Langevin Type Equation

In [7], they discussed a fluctuation phenomena for interacting, spatially extended neurons and as a limit equation, they found a suitable fundamental space $\mathcal{D}_{\mathcal{E}}$ of functionals on $\mathcal{E}$ and studied Segal-Langevin type stochastic differential equations:

$$dX_F(t)=dW_F(t)-X_{-\Gamma(\mathrm{A})F}(t)dt,\quad F\in\mathcal{D}_{\mathcal{E}},\tag{4.1}$$

including a class of the weak version of (1.1). A stochastic process $X_F(t)$ indexed by elements in $\mathcal{D}_{\mathcal{E}}$ is called a continuous $L(\mathcal{D}_{\mathcal{E}})$-process if for any fixed $F\in\mathcal{D}_{\mathcal{E}}, X_F(t)$ is a real continuous process and

$$X_{\alpha F+\beta G}(t)=\alpha X_F(t)+\beta X_G(t)$$

almost surely for real numbers α,β and elements $F,G\in\mathcal{D}_{\mathcal{E}}$ and further $\mathrm{E}\left[X_F(t)^2\right]$ is continuous on $\mathcal{D}_{\mathcal{E}}$. $W_F(t)$ is an $L(\mathcal{D}_{\mathcal{E}})$-Wiener process such that for any fixed $F\in\mathcal{D}_{\mathcal{E}}$, $W_F(t)$ is a real Wiener process.

Although the above $\mathcal{D}_{\mathcal{E}}$ is not nuclear, appealing to the results in [7], we get an unique continuous $L(\mathcal{D}_{\mathcal{E}})$-process satisfying (4.1).

We consider the case where for the operator A in (4.1), there exists a self-adjoint operator B satisfying all the conditions of Theorem 1.1. In this case, by Theorem 1.1, there is a nuclear space $\mathcal{S}$ invariant under both $\Gamma(\mathrm{A})$ and a strong continuous semigroup $T(t) = e^{-t\Gamma(\mathrm{A})}$. If we replace $\mathcal{D}_{\mathcal{E}}$ by $\mathcal{S}$ in (4.1), then by the regularization theorem [5] there exists an $\mathcal{S}'$-valued Winer process $W(t)$ such that $< W(t), F >= W_F(t)$ almost surely and the strong form of the equation with $\mathcal{D}_{\mathcal{E}}$ replaced by $\mathcal{S}$ in (4.1) is the following stochastic differential equation on $\mathcal{S}'$:

$$dX(t) = dW(t) - \Gamma(\mathrm{A})^* X(t)dt.$$

Let $T(t)^*$ be the adjoint operator of $T(t)$. Since $\mathcal{S}$ is nuclear, again by the regularization theorem, the stochastic integral $\int_0^t T(t-s)^* dW(s)$ is well defined from the weak form such that

$$\left\langle \int_0^t T(t-s)^* dW(s), F \right\rangle = \int_0^t \langle dW(s), T(t-s)F\rangle .$$

Since $T(t-s)F = F + \int_s^t T(\tau - s)(-\Gamma(\mathrm{A}))F d\tau$, we get

$$\int_0^t T(t-s)^* dW(s) = W(t) + \int_0^t (-\Gamma(\mathrm{A})^*) \left(\int_0^\tau T(\tau - s)^* dW(s) \right) d\tau.$$

Noticing that

$$\int_0^t (-\Gamma(\mathrm{A})^*)\, T(\tau)^* X(0) d\tau = T(t)^* X(0) - X(0),$$

we get that

$$X(t) = T(t)^* X(0) + \int_0^t T(t-s)^* dW(s)$$

is an unique strong solution of (1.1) on $\mathcal{S}'$.

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