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Deciding the Optimal Shutdown time of a Nuclear Power Plant

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원자력 발전소의 최적 운행중지 시기 결정 방법

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A methodology that determines the optimal shutdown time of a nuclear power plant is suggested. The shutdown time is decided considering the trade off between the cost of accident and the loss of profit due to the early shutdown. We adopt the bayesian approach in manipulating the model parameter that predicts the accidents. We build decision tree models and apply dynamic programming approach to decide whether to shutdown immediately or operate one more period. The branch parameters in decision trees are updated by bayesian approach. We apply real data to this model and provide the cost of accidents that guarantees the immediate shutdown.

1. Introduction

In operating a nuclear power plant we must consider when is the optimal time to shutdown a nuclear power plant. The shutdown time has to be chosen considering the trade-off between the possible waste of money resulting from the early shutdown and the risk of accident that may be caused by extending the operational period. Two extreme shutdown points exist. One is to shutdown immediately which guarantees no accidental cost. The other extreme is to operate until we meet an accident. Depending on the cost of accidents and the cost of early shutdown we want to find the optimal shutdown time between those two extremes that minimizes the cost. We choose the dynamic programming approach that decide whether to shutdown immediately or to operate one period of time and then periodically repeat the decision process with additional information about future costs and risks. We adopt Bayesian approach in updating the branch probabilities. At each step we consider only two alternatives which are shutdown and continued operation. We assume that costs and probabilities of various levels of risk are updated as we proceed in time through the decision tree. We develop a model to make a optimal shutdown time decision for a one-period problem and then extends it to the general T-period decision problem.

2. Modeling for deciding the optimal shutdown time

2.1 The one-period decision problem

Consider the decision tree in figure 1 which represents the decision and events in a single period. A rectangle denotes the decision node and a circle denotes a random state. At the beginning of the first year we decide whether to shutdown the plant immediately ($D=1$) or to operate one year ($D=0$). If we shutdown the plant immediately, we must pay off the unpaid balance of the total construction cost, C. If we operate the plant for one year there may be a severe accident with probability p_0. If the I_0 denotes the information set (available and relevant data) at the end of year 0 (beginning of year 1) and X denotes the uncertain time to the next severe accident with known density $p(x|I_0)$ then the probability of an accident, p_0, can be determined by integrating $p(x|I_0)$ over one year. In the branch that includes an accident, we add a large cost C_A that includes cleanup costs, possible loss of life, treatment of survivors, damage of equipment, environmental

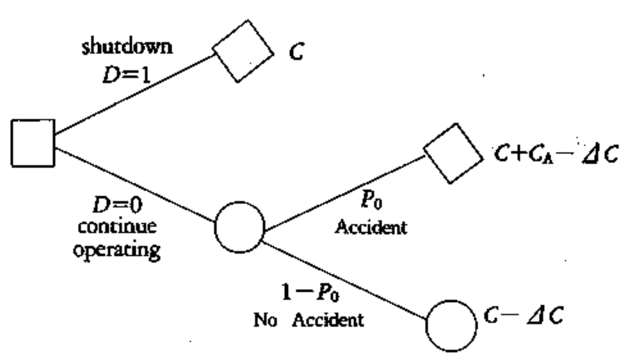

Figure 1. The one-period shutdown or continue decision.

contamination, and so forth. Whether an accident does or does not occur we must also pay off the unpaid balance of the construction cost (plus interest) at the end of each year. We obtain revenues from the sale of electricity, while we use funds to pay the cost of operation and maintenance, C_M. Typically, with a nuclear plant the net revenue obtained by operating a plant for one year is small compared to the large value of C_A. In what follows we will assume that annual revenue compensates for the operation and maintenance cost, and planned amortization of the construction cost, A. These notations are summarized as followings;

C : total construction cost
C_A : cost associated with an accident
C_M : cost of operation and maintenance
r : annual interest rate
A : amortization of the construction cost

It may be very difficult to figure out every cost relevant to the operation of power plant and occurrence of accident. But all costs can be categorized into one of the above classification, and to show the way of finding out the optimal decision through decision tree considering the probability of accident we simplify the cost factors like the above.

The amortization, A, is determined by the total construction cost C, the annual interest rate r, and the number of payment periods T. The cost for a case with no accident is determined by the total expense minus total revenue. The total expense is sum of total construction cost plus interests and cost of operation and maintenance; $C(1+r)+C_M$. The total revenue covers the cost of operation and maintenance and the planned amortization; C_M+A. Hence the cost for a case with no accident is $\{C(1+r)+C_M\}-\{C_M+A\} = C(1+r) - A = C - \Delta C$ where ΔC denotes the reduction in principal due to one year's installment payment of the construction cost plus interest. When the cost associated with shutdown is C, the expected cost associated with continued operation is $p_0(C-\Delta C+C_A)+(1-p_0)(C-\Delta C)$; this number can be compared with the certain cost of shutdown, C, to yield:

shutdown, if $p_0(C+C_A-\Delta C)+(1-p_0)(C-\Delta C) > C$
operate, if $p_0(C+C_A-\Delta C)+(1-p_0)(C-\Delta C) \le C$

which tells us that the probability of a severe accident must be greater than $\Delta C/C_A$ if we are to shutdown the plant. If the reduction in principal is approximately \$500 million and the cost of cleanup is between 1 and 2 billion (say 1.5 billion) then the probability of a severe accident must exceed one third before shutdown is indicated. To put it in a slightly different perspective, only when the expected cost of cleanup, lost lives, decommissioning exceeds \$100 billion will we find that small probabilities of the order of 0.005 warrant the decision to shutdown in one period.

As we see from the above we may have different decision depending on the cost of accident. It is very disputable and difficult problem to assess the related cost, especially the cost of accident that includes the cost of loss of lives. In reality, some country like Australia shutdown the nuclear power plant right after construction because of strong opposition movement of public. In this case we can consider the huge cost of accident that compensates the small probability of accident is taken.

2.2 The T period decision problem

The multi-period problem differs only slightly from the one-period problem formulated above. We must now include an index to denote which time period we are considering and also how the costs and probabilities vary over time. The time dependency that must be explicitly recognized is the changing costs structure which must reflect changing debt levels, operating and maintenance costs, values of life, relocation, decommissioning etc. In the case of the probabilities, they must be correctly updated to reflect the particular branch of the decision tree being used. For example, if we are in the seventh year of operations with no accident history in any of the preceding seven years, then the decisions and probabilities in period 8 should explicitly recognize that no accidents have occurred up to that time. On the other hand, if a history of incidents is likely to influence the occurrence of future accidents then that knowledge should also be brought into the updating process. Another important fact that should be reflected into the updating process is the so called aging effect. The failure rate of an accident in a specific plant is influenced by the time-dependent agents such as repeated demand, or continuous acting of the components in a reactor. The degradation of a component resulting in the loss of function or reduced performance caused by some time-dependent agent or mechanism is called aging. The change in the component failure probability resulting from the degradation due to such aging will be monotonically increasing with time (Meal and Satterwhite, 1988). Therefore as we operate the plant longer with no accident, the probability of severe accident will decrease recognizing that severe accidents are less likely, but at the same time the reactor gets older so that the severe accident is increasingly likely. These two conflicting effects must be appropriately considered in the updating process.

It is customary to require that the construction cost be paid off in a finite number of years, say $T=30$ years with a constant annuity (amortization), A; the unpaid balance (the cumulative amount of debt) decreases slowly early in the life of the design lifetime and more rapidly towards the end. Let C_i denote the balance of the construction cost remaining at the end of year i, with $C_0=C$. The balance of principal and interest owed on the construction cost at the end of year $i+1$ is $C_{i+1}=C_i-\Delta C_i$, $i=0, 1, \ldots, T-1$ where, as we have already mentioned, $\Delta C_0, < \Delta C_1 \ldots < \Delta C_{T-1}$, reflects the fact that early in the life of the loan there is a relatively small reduction in principal balance by comparison with later years. Generally, at the end of year i, we have a decision tree each of whose sub-trees resembles the ones used in Figure 1 and 2.

2.3 Assessment of branch parameters

We allow the arrival rate of severe accident to be a function of the age of a reactor, and assume it increases monotonically as the reactor gets older. In this paper we assume that the aging effect is linear in time. Then the intensity function that represents the arrival rate of severe accident when reactor is t years old, λ_t, $t\geq 0$, can be expressed as $\lambda_t=\lambda(1+bt)$ where b is the deterministic value that denotes the coefficient representing the increasing pattern of the failure rate as the reactor becomes older, and λ is the unobservable random variable that denotes the arrival rate of severe accident when the reactor is new. If b is one, the failure rate becomes twice in a single period, and if b is 0.1, the failure rate becomes twice in ten years.

Let $n(t)$ be the number of severe accidents in time period $(0, t)$. When we assume $n(t)$ given time independent failure rate, λ, is a Poisson process, the counting process $\{n(t), t\geq 0\}$ becomes a nonhomogeneous Poisson process with intensity function λ_t. If we let

$$m_t=\int_0^t \lambda_s ds$$

then, $\{n(t+s)-n(t)\}$ is Poisson distributed with mean $m_{t+s}-m_t$(Ross, S. M., 1983). We assume that λ is Gamma distributed with parameter α, β;

$$p(\lambda)=\frac{\beta(\beta\lambda)^{\alpha-1}e^{-\beta\lambda}}{\Gamma(\alpha)}$$

The gamma assumption can be justified noting that a system provided with many redundancies tends to

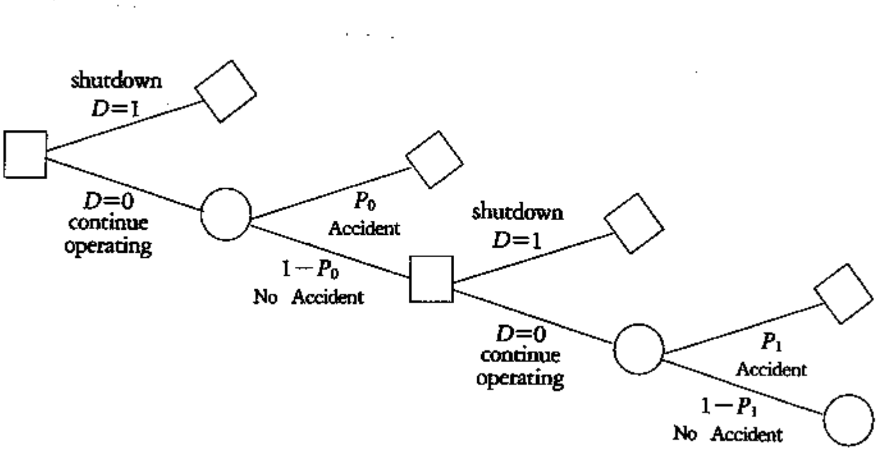

Figure 2. Two-Period Decision Tree.

bunch up toward the low probability of failure. A gamma distribution when the ratio of mean to standard deviation is greater than 1 is guaranteed a bell shaped positive skewed distribution. Then it can be shown that the intensity function λ_t given b and t is also Gamma distributed with parameter α_t, and β_t:

$$\alpha_t = \alpha$$

$$\beta_t = \frac{\beta + (1+bt)t}{1+bt}$$

As we have explained before, the likelihood of the number of accidents in certain time period given the intensity function is Poisson with mean obtained by integrating the intensity function over appropriate periods. The posterior distribution of the intensity function with observed counts of accidents $n(t, t+s)$ in time period $(t, t+s)$ can be approximated by a Gamma distribution with parameters $\alpha_{t+s} = \alpha_t + n(t, t+s)$ and $\beta_{t+s}' = \beta_t + s$ if the time interval s is small. The predictive distribution can be obtained in a closed form by integrating out the unobservable parameter λ_t, and we get

$$\text{Prob.}\{\text{accident in a period} \mid \text{reactor age} = t\}$$
$$= p_t = 1 - \int_1^\infty \int_0^\infty \frac{\lambda_t e^{-\lambda_t x} \beta_t (\beta_t \lambda_t)^{\alpha_t - 1} e^{-\beta_t \lambda_t}}{\Gamma(\alpha_t)} d\lambda_t dx$$
$$= 1 - \left(\frac{\beta_t}{\beta_t + 1} \right)^{\alpha_t} \quad (1)$$

We can see that the parameter updating scheme and the method for obtaining predictive distribution for future risks take care of the aging effects as well as the information provided by operating experiences.

3. Application to real data

Let's assume that the reactor is new at the beginning of the first period so that p_i denotes the probability of an accident in time period $(i, i+1)$ when the reactor is i years old. The optimal policy for nuclear shutdown is obtained by recursively solving the following system of equations where V_i denotes the minimum expected loss with i periods of operation remaining. We choose one decision between shutdown or continued operation whichever offers the smaller cost.

$$V_0 = C_T = 0$$
$$V_i = \text{Min}\,[C_{T-i}\,;\, p_{T-i}(C_{T-i+1} + C_A) + (1 - p_{T-i})V_{i-1}) \quad i = 1, 2, \ldots, T$$

In this formulation we of course assume that both costs and accident probabilities are updated to properly reflect the period in which the calculation applies.

We use EPRI cost data, predictive distribution in (1) and the dynamic programming recursion for the T-period decision tree, when T=30 (corresponding to a 30 year design lifetime). C_{30} is close to zero so that based on our current information shutdown of the plant is guaranteed at the end of 30 years. From the EPRI(1987) TAG report, estimates of C_0, and C_M are \$3.158 $\times 10^9$, and \$3.432 $\times 10^7$/yr, respectively. Assuming 12.5% interest rate compounded yearly, the amount of amortization for the construction cost is \$4.067 $\times 10^8$, and the unpaid balance at the end of each year, C_i, i=1, 2, ..., T, can be calculated. We use a Gamma distribution for the prior distribution of the severe accident rate λ when the reactor is new, with parameters $\alpha_0 = 3.24$, $\beta_0 = 3757.54$ (Yang, 1989). With this data figure 3 shows the minimum cost of accident that warrants the decision of an immediate shutdown for various values of b. We can see that the minimum cost of accident for shutdown option decreases relatively rapidly at the region of small aging effects and it does not change much at the region of large aging effects, and almost saturates around \$13.7 billion.

As a special case, if the aging effect is negligible (b=0), the above formulation always yields one of two extreme solutions, which is either to shutdown immediately or to plan on operating until the end of

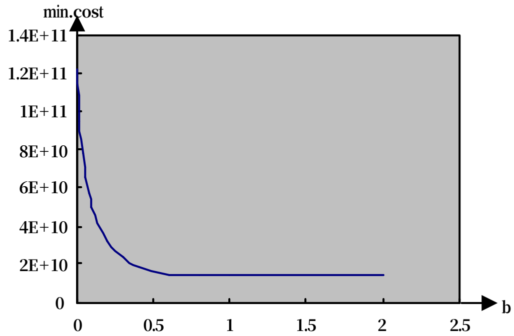

Figure 3. Minimum Cost of Accident for Immediate Shutdown.

the reactor's design lifetime. This result can be explained as follows: first, assume that shutdown will definitely occur on or before the 30th year. Now, assume that the decision at the end of 29th year is to shutdown the plant, in other words $D_{29}=1$. This decision is optimal when

$$C_A > \frac{C_{29}-C_{30}}{p_{29}} = \frac{C_{29}}{p_{29}}$$

since V_0 is assumed to be zero. At the end of 28th year, the optimal decision conditional on $D_{29}=1$ is

$$\text{shutdown } (D_{28}=1), \text{ if } \quad C_A \geq \frac{C_{28}-C_{29}}{p_{28}}$$

$$\text{operate } (D_{28}=0), \quad \text{if } C_A < \frac{C_{28}-C_{29}}{p_{28}}$$

We therefore have the optimal decision $D_{28}=0$ followed by $D_{29}=1$, i.e. a switch in the decision from "continue operations" to "immediate shutdown" when the following inequalities holds:

$$\frac{\Delta C_{29}}{p_{29}} = \frac{C_{29}}{p_{29}} < C_A < \frac{(C_{28}-C_{29})}{p_{28}} = \frac{\Delta C_{28}}{p_{28}}$$

But we know that with a history of no accidents in earlier periods p_i is a decreasing sequence if there is no aging effects; furthermore the normal way in which debt is paid off has ΔC_i as an increasing sequence so that the ratios of debt reduction to probability of an accident are increasing. Thus the inequality as stated implies that the term on the left is smaller than the one on the right, which is a contradiction of our argument for the increasing sequence of debt reduction to accident probability ratios. This argument can be extended to each period in the design life of the reactor so that it can be shown that if $D_j=1$, then $D_k=1$ for all k such that $k < j$. In other words, the optimal policy at the beginning of each period is to plan on never switching the decision. It should be emphasized that in real time the switching decision can actually occur if new information at period i, say I_i, is such that the predictive distribution and cost factors indicate a shutdown decision is recommended even though the plant operated in the previous year.

One should note that the optimal decision for nuclear power plant shutdown is very much dependent on who the decision maker is. If the decision makers are the public, they will assume larger costs for accidents, with subjectively assessed huge (sometimes infinite) cost of loss of lives and fear. On the other hand, if the decision makers are the managerial personnel in a nuclear power plant, the construction cost may be the most concerned cost and the cost of loss of lives will be based on the estimates from the accidental data in other fields. Therefore the assessment of cost of accident is a quite subjective and disputable problem. Anyhow our model and methodology developed in this paper can be applied no matter who is going to make a decision. It provides with the optimal decision together with the expected optimal cost. If we could purchase perfect information for certain amount of money and were informed when the next accident would occur before we made a decision, the optimal decision would be to operate plant until the instant before the accident and incur no cost other than the remaining balance of construction cost. In any case, we are not willing to pay more than the expected optimal cost, since probabilistically we are warranted to cost that amount if we follow the optimal decision. Therefore the expected optimal cost at the beginning of the first year is the upper bound we would spend in an effort to collect more useful information and improve forecasting, etc.

4. Summary

For a given plant design, improved decisions on when to shutdown an existing nuclear plant may be obtained by making better predictions of failure rates (see, for example, Lewis, 1984; Chow and Oliver, 1988; and Yang, 1989 for methods of estimating failure rates), by exerting efforts to collect more relevant information or by improving decision making models which put that information to best use. There always exists a trade-off between the cost of collecting more information and making better decisions. What is the upper limit of resources we should spend in order to reduce our uncertainty about failure rates? It is important that the models include the value of possible loss of life and fear (Owen *et.al.*, 1978) along with cleanup, decommissioning, relocation if the decisions derived from the model are to be useful. What is the cost and the influence of new information from outside sources (other than the plant under consideration) that we have not included? The simple model we have described enables us to make a stochastically optimal decision, which is repeatedly made at the end of each period with updated information, and assess the expected optimal cost that determines the value of perfect information.

We classified the decision into two extreme cases, shutdown or continued operation. For further research, we may add other options such as temporary shutdown for intensive or rough maintenance or overhaul that result in lower probability of accidents. The research that adaptively updates the decision considering the probability of accident is hardly seen. In reality, it is also hard to get sufficient data that makes the above research possible. If statistics can be collected based on the framework of the decision making suggested in this parer, we can extend the research to cover general cases more efficiently. In this paper we suggest one way of making optimal decision incorporating the probability of accident which is updated through the operation of nuclear power plant.

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