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Wavelength Assignment Optimization in Uni-Directional WDM Ring†

Heesang Lee[1] · Ji-Bok Chung[2]
[1]Dept. of IE, Hankuk University of Foreign Studies / [2]Dept. of IE, Seoul National University

단방향 WDM 링의 주파수 할당의 최적화

이희상[1] · 정지복[2]

In this paper, we consider *wavelength assignment problem* (WAP) in *wavelength division multiplexed* (WDM) unidirectional optical telecommunication ring networks. We show that, even though WAP on unidirectional ring belongs to *NP-hard*, WAP can be exactly solvable in real-sized WDM rings for near future demand. To accomplish this, we convert WAP to the vertex coloring problem of the related graph and choose a special integer programming formulation for the vertex coloring problem. We use a column generation technique in a branch-and-price framework for the suggested formulation. We also propose some generic heuristics and do the performance comparison with the suggested optimization algorithm.

1. Introduction

The growth of the Internet, deregulation of the telecommunications industry, and new applications such as electronic commerce, video-on-demand and tele-medicine, have created an ever-increasing demand for greater bandwidth in telecommunication networks. A cost effective way to deliver high speed and broad bandwidth services is to send multiple wavelengths through a single fiber at very high data rates using Wavelength Division Multiplexing (WDM) technologies. WDM transmission systems and related technologies are being developed as a promising building block for the next-generation high speed network infrastructure.

WDM transmission nodes enable the establishment of all-optical WDM channels, called *lightpaths*, between pairs of WDM nodes. Lightpath connection goes through several intrmediate WDM nodes but two lightpaths must not have the same wavelength on a given link(Sato, K., 1996; Ramaswami, R. and Sivarajan, K. N., 1998). This technical requirement is called *wavelength-continuity constraint*.

Taking into account the progress of current WDM technology, 16 wavelength WDM point-to-point terminating multiplexer is currently available and 48 wavelength point-to-point WDM system is just around the corner(Ramaswami, R. and Sivarajan, K. N., 1998; Mukherjee, B., 1997). Point-to-point WDM transmission enables enhancement of transmission capacity comparing TDM transmission, but more significant benefits such as cross-connect throughput and flexible service provisioning can be achieved only by using WDM Add Drop Multiplexer (ADM) and WDM Cross-Connector (XC). Hence the current research for WDM network planning is focused on routing each lightpath that need to be established on the network (*routing problem*: *RP*), and assigning wavelengths to these lightpaths to satisfy the wavelength continuity constraint (*wavelength assignment problem*: *WAP*) using WDM ADM's or WDM XC's.

Although WDM mesh networks can be used extensively in future, at least in the near term, ring topologies are viable because current SONET/SDH

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self-healing architectures are ring oriented and WDM ADM system is expected to be deployed before WDM XC's(Ramaswami, R. and Sivarajan, K. N., 1998). In ADM ring networks, each ADM node has two incident links. Some of the incoming traffic from a link may be intended for the ADM node, in which case it is switched to a local entity through local access ports. The rest of the traffic to that ADM node is forwarded via the other link(Sato, K., 1996).

In this study we assume that each lightpath is established for full duplexing transmission between an ADM demand pair of the WDM ring. Hence in this ring network we consider only one favorable direction (e.g. clockwise in this paper) path for lightpath routing. In this route-fixed WDM ring networks, the main network design problem to consider is WAP in a undirected ring graph without considering RP since the routes for all lightpaths are fixed as one direction. On the contrary, in route-unfixed WDM ring networks, where lightpath for a demand pair can be established using clockwise or counterclockwise routes in a bi-directional ring graph, RP and WAP must be jointly optimized.

In this paper, we focus on minimizing number of the used wavelengths in uni-directional ring. When considering the large bandwidth of WDM lightpaths and the scarcity of wavelength, efficient wavelength assignment is one of the most important network planning schemes to design cost-effective uni-directional WDM ring networks. Main contribution of this paper is that we develop an exact algorithm for WAP in uni-directional ring and perform computational experiments that show the satisfactory performance of the suggested algorithm up to the moderate WDM ADM ring sizes.

The remainder of this paper is organized as follows. In section 2, we study relations between WAP and a coloring problem in a class of graphs so called circular-arc graphs. In section 3, we propose a mathematical formulation for WAP and an exact solution algorithm. In section 4, we study generic heuristics so called first-fit class heuristics and some implementing strategies. In section 5, we show, by computational experiments, that the suggested exact algorithm can find an optimal solution for moderate-sized WDM ring networks. We also do performance comparison between our exact solution algorithm and the heuristics suggested in section 4 and an exact solution algorithm in the literature. In section 6, we give some concluding remarks and discuss further study topics.

2. Wavelength Assignment in Unidirectional WDM Ring

Wavelength assignment on each route-fixed lightpath in a WDM network can be interpreted as the coloring problem of the paths where all paths going trough an edge of the graph should have different colors. When the physical topology of the WDM network is the ring, paths around the ring may be viewed as a collection of arc paths on a circle. Hence we can convert the path-coloring problem for this WDM ADM ring network into a vertex-coloring problem by constructing the following graph so called *coloring graph*(Erlebach, T. and Jansen, K., 1997). For each lightpath of the original WDM ADM ring graph, we define a vertex in the coloring graph, and define an edge between two vertices in the coloring graph if the associated two lightpaths overlap at any link of the WDM ADM ring. This conversion technique can convert the path coloring problem of not only the ring graph but for any arbitrary graph into a vertex coloring problem of the related coloring graph(Erlebach, T. and Jansen, K., 1997; Garey, M. and Johnson, D. S., 1980; Tucker, A., 1975).

A graph $G = (V, E)$ is called an *intersection graph* for a family F of sets if there is an one-to-one correspondence between the vertices of G and the sets of F such that two distinct vertices are adjacent if and only if the associated sets intersect. If F is a family of arcs on a circle, G is called a *circular arc graph* (CAG). When the original WDM graph is a ring, the coloring graph is a CAG. Hence WAP on a uni-directional ring is equivalent to the optimal vertex-coloring problem in the associated CAG. An example of construction of CAG for WAP on a ring is shown in Figure 1. WAP in uni-directional ring is *NP-hard* since the problem to find the minimum number of vertex colors in CAG belongs to *NP-hard* (Garey, M. and Johnson, D. S., 1980). Hence many previous works in the literature have focused on the development of approximation algorithms for minimizing wavelength assignment on WDM uni- directional rings(Sato, K., 1996; Ramaswami, R. and Sivarajan, K. N., 1998; Carpenter, T., 1997; Ramaswami, R. and Sasaki, G. H., 1997; Ellinas, G., 1998).

With a difficulty of *NP-hardness* of the vertex coloring, we encounter another problem for using this conversion: Even for WDM ring networks with small number of ADM's, the coloring graph can be large. For example if WDM ring has 10 ADM's and one lightpath is required for 50% of all ADM pairs,

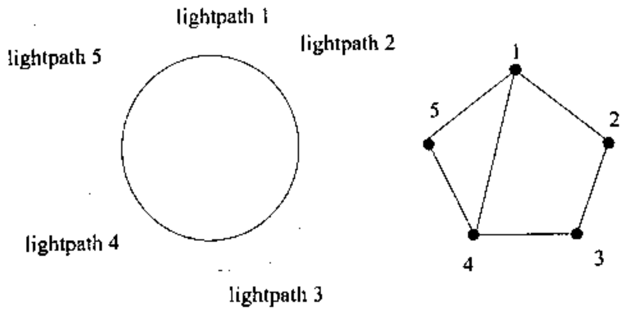

Figure 1. Example of circular-arc graph construction.

we need $1 \times 9 \times 50\% = 45$ light paths in the WDM network. Hence the related coloring graph has 45 vertices and several hundred edges. Hence usual algorithmic techniques for vertex coloring problem may not guarantee to get an optimal solution for WAP for uni-directional ring for moderate size of ADM nodes and lightpath demand.

3. Suggested Optimization Approach

Let $G = (V, E)$ be an undirected graph, with V, the set of vertices, and E, the set of edges. A simple and canonical integer programming (IP) formulation for the vertex coloring problem is possible by defining binary decision variable $y_{ik} = 1$ if vertex i is assigned color k, and $y_{ik} = 0$ otherwise. However, this canonical IP formulation has disadvantages since its linear programming(LP) relaxation is extremely fractional and it has *symmetry property* of the variables. Here the symmetry means that the variables for each k appear in the same way. The symmetry property makes difficult to enforce integrality in one variable without problems showing up in the other variables that can be crucial for using branch-and-bound techniques for the suggested IP formulation. Hence we adopt the following IP formulation that is studied in (Mehrotra, A. and Trick, M. A., 1995).

An *independent set* of G is the set of vertices such that there is no edge in E connecting any pair of vertex of independent set. A *maximal independent set* (MIS) is an independent set that is not included in any other independent set. For example, $S_1 = \{1,3\}$, $S_2 = \{2,4\}$, $S_3 = \{2,5\}$, $S_4 = \{3,5\}$ are all MISs for instance of Figure 1. Let S be the set of all MISs of G and binary variables $x_s = 1$ if MISs will be given a unique color, while $x_s = 0$ otherwise. Then the vertex coloring problem on a CAG (or WAP for uni-directional ring) can be formulated as the following integer programming.

$$
\begin{aligned}
&\text{(MIS IP)} \quad \text{Minimize } \sum_s x_s \\
&\text{subject to} \quad \sum_{\{s:\, i \in s\}} x_s \ge 1, \quad \forall i \in V, \\
&\qquad\qquad\quad x_s \in \{0, 1\}, \quad \forall s \in S.
\end{aligned}
$$

The objective function is to minimize the number of used colors and the constraint implies that each vertex must be assigned at least one color. Note that a feasible solution to this IP may assign multiple colors to a vertex since each constraint has the condition of "not less than 1" instead of the condition of "equal to 1". This multiple color possibility can be corrected by using any one of the multiple possible colors to a vertex. Hence the same color cannot be used for the adjacent vertices since we use the same color only for each vertex in the same independent set.

The MIS based IP formulation has only one constraint for each vertex and without symmetry property for the variables. However the number of variables is huge since the number of all MIS's for a graph is huge. Hence generating all MIS's for the large graph to get the explicit formulation is intractable. We can resolve this difficulty by using only subset of the variables and generating more variables when they are needed (Mehrotra, A. and Trick, M. A., 1995). This technique, called *column generation*, is well known for linear programming with many variables. *Branch-an-price* algorithm, which is a branch-and-bound algorithm with an efficient column generation technique for its LP relaxation, has recently emerged as an effective solution technique for many *NP-hard* integer programming problems (Birge, J. and Murty, K., 1994). The general idea of column generation procedure for LP is as follows. An optimal solution to LP with many columns can be obtained without explicitly including all columns (i.e., MIS in this MIS IP) since only a very small subset of all columns will be in an optimal solution. Column generation techniques in branch-and-price algorithm for IP, has an additional advantage: The column generation formulation of an IP may have a stronger LP relaxation than a canonical compact IP formulation.

In our problem, the column generation technique is as follows. Begin with S', a subset of S, the set of all MIS's. Solve the LP relaxation of (MIS IP) restricted to $s \in S'$. This gives a feasible solution the LP relaxation and a dual value π_i of the dual LP problem for each constraint i of the original LP relaxation. Now, determine if we need more columns.

This can be done by solving the following subproblem, defined as *MWIS Problem*, a problem to find the maximum weighted MIS where the non-negative weights π_i are given for every vertex i(Mehrotra, A. and Trick, M. A., 1995).

(Column Generation Subproblem: MWIS Problem)

$$\text{Maximize} \quad \sum_{i\in V} \pi_i z_i$$
$$\text{Subject to} \quad z_i + z_j \le 1 \text{ for all } (i, j) \in E,$$
$$z_i \in \{0, 1\} \text{ for all } i \in V.$$

If the optimal objective function value to this problem is greater than 1, then the z_i with value 1 corresponds to an independent set that should be added to S'. In this case, the column generation process is repeated until the objective value is not greater than 1. If the optimal value is not greater than 1, then there exist no improving independent sets: Solving the LP relaxation of (MIS IP) over the current S' is the same as solving it over S.

The complexity of this column generation subproblem may greatly affect the solution time of (MIS IP). Fortunately, we can prove MWIS Problem in CAG can be solved in polynomial time due to our following results. An intersection graph $G=(V, E)$ is called an *interval graph* if F is a family of intervals on a line. Note that we can find a related interval graph by cutting a circle of a CAG at one point and removing all arcs that pass through the cut point. MWIS Problem in interval graph can be exactly solved in polynomial ($O(n \log n)$) time, where n is the total number of vertices (Hsiao, J. Y., 1992). Using this algorithm polynomial times and the relationships between CAG's and interval graphs, we can show that MWIS Problem on CAG can be solved in polynomial time by tearing down a CAG as follows:

Consider MWIS problem on the coloring graph that is CAG as the problem to find a maximum weight subset of lightpaths in the original ring graph such that there is no edge in the ring intersecting any pair of chosen lightpaths of the subset. We call such a subset an *independent lightpath set*. Suppose we fix a lightpath k into an independent lightpath set, then we delete all lightpaths that overlapping with lightpath k since they must not be in the independent lightpath set. Since the remaining lightpaths constitute a coloring graph that is an interval graph, we can find maximum weight independent lightpath set in the interval graph and the corresponding maximum weight independent set in the subgraph of the coloring graph in $O(n \log n)$ steps. We augment the maximum weight lightpath set from the interval graph with the path k for a candidate of a maximum weight lightpath set of the circular arc graph. We iterate this procedure for every lightpath and choose the maximum weight independent set among n candidates. Hence the MWIS problem for a CAG can be done $O(n^2 \log n)$ time. We now have the following polynomial time column generation procedure for (MIS IP).

Column generation procedures for (MIS IP): An algorithm for MWIS problem in a CAG

Start.

Do {

Select any lightpath, say lightpath k, among previously unselected ones.

Delete all the lightpaths that overlaps with lightpath k.

Sort the end point of remaining lightpaths in non-increasing order.

Solve the MWIS Problem in the corresponding interval graph.

} until (*all lightpaths considered*).

Compare candidate MWIS from interval graphs and obtain the MWIS for the circular arc graphs.

If the optimal function value of MWIS Problem is more than 1, then an independent set corresponding z_i with value 1 is added to the (MIS IP). Go to Start.

Otherwise stop. No more columns are needed. Stop.

If the resulting solution to LP relaxation of (MIS IP) over S' has an integer solution when the column generation process is finished, then the corresponding solution is an optimal solution for MIP IS over S. When some of variables are not integer, we need to enforce integrality for those variables using branching technique. We adopt "collapsing-into-single- vertex" and "adding-an-edge" branching rule used in (Mehrotra, A. and Trick, M. A., 1995) to branch a fractional solution node in the branch-and-bound tree. One important thing of the column generation within a branch-and-price procedure is that the column generation problem is still not difficult after branching. In our problem, by choosing a pair of "minimal distance non-overlapping" lightpaths for branching, the column generation is maintained as the MWIS problem in a modified coloring graph that is still a CAG.

4. Heuristics

First-Fit Heuristic is a generic class of greedy-type heuristics for assigning a wavelength to a lightpath in WDM rings or mesh networks (Ramaswami, R. and Sivarajan, K. N., 1995). It assumes that the wavelengths are labeled 0, 1, $\cdots$, $W-1$ and assigns to lightpaths a wavelength with the lowest label that is available in each edge of the lightpath for each connection request. In terms of coloring, it sequentially colors the vertex with the lowest available color for each dynamic lightpath. We propose three heuristics by combining the First-Fit idea from the literature with some lightpaths selection rules as follows: longest lightpath first (LPH), shortest lightpath first (SPH), maximum degree in coloring graph first (DEGH). Note that a vertex with maximum degree in coloring graph corresponds to the lightpath overlaps with maximum numbers of other light paths in the original ring graph. Furthermore a lightpath that overlaps with many other lightpaths is likely to have few alternatives to reuse the wavelengths already assigned. The longer the lightpath is, the more it overlaps with other lightpaths, too. Hence the longer lightpath is also hard to have colors when the number of available colors is small since the possible color should be different from the other light path in every edge in the long path. Therefore, LPH and DEGH seem intuitively to perform well than SPH since the more flexible coloring is possible to decrease used colors during heuristics. The generic First-Fit Heuristic is described as the following. The LPH, SPH and DEGH heuristics can be easily described by defining the lighpath labeling rule.

Procedure of First-Fit Heuristic

Do {

Select a first fit lightpath unassigned yet according to the suggested lightpath selection rule.

Assign the lowest label from the set of available wavelengths

} until (*all lightpaths are assigned*)

5. Computational Experiments

The proposed algorithm for WAP has been coded in programming language C and experimented on a SUN Sparc Ultra (167Mhz) workstation using an IP optimization callable library, CPLEX 4.0. By the experiments, we want to prove that the suggested algorithm is computationally feasible to implement in real-sized WDM rings and do the performance comparison with generic heuristics studied in section 4. We experiment five classes of WDM networks that have 5, 10, 15, 20, and 25 WDM ADM ring nodes. To know the effect of demand on the ring to the performance, we set the four demand classes by defining the *demand density*, that is the probability of each demand pair is required, 0.3, 0.5, 0.7, or 0.9. For example, we have $25 \times 24 \times 0.9 = 540$ lightpaths for 25 vertices with demand density of 0.9. By combining of ADM node size and demand density, we have twenty sets for the experiment. We experiment five instances for each of twenty sets, total 100 instances. In Table 1, input parameters of twenty sets are summarized. In <Table 1>, $G(n, d)$ represents that n is the number of ADM's and d is demand density. "Verts" and "Edges" in <Table 1> denotes respectively the number of vertices and

Table 1. Problem inputs

Set	$G(n, d)$	n	Verts	Edges
1	G(5, 0.3)	5	6	10.6
2	G(5, 0.5)	5	10	30.6
3	G(5, 0.7)	5	14	67.2
4	G(5, 0.9)	5	18	113.4
5	G(10, 0.3)	10	27	285.0
6	G(10, 0.5)	10	45	790.8
7	G(10, 0.7)	10	62	1490.0
8	G(10, 0.9)	10	80	2503.0
9	G(15, 0.3)	15	63	1567.8
10	G(15, 0.5)	15	105	4382.4
11	G(15, 0.7)	15	147	8634.8
12	G(15, 0.9)	15	188	14082.0
13	G(20, 0.3)	20	114	5200.0
14	G(20, 0.5)	20	190	14507.8
15	G(20, 0.7)	20	266	28619.6
16	G(20, 0.9)	20	341	47461.8
17	G(25, 0.3)	25	180	13154.6
18	G(25, 0.5)	25	300	36702.0
19	G(25, 0.7)	25	420	72262.8
20	G(25, 0.9)	25	516	118979.2

the average number of edges of the coloring graph. Note that Verts is the same in a given set but Edges of five instances of a given set can be different since the lightpaths can have different overlapping conditions for each instance.

Average performance of five instances for each set is displayed in <Table 2>. In <Table 2> "Col" denotes the average of five instances for the total number of columns generated during the column generation procedure. "BandB" denotes the average of the number of branch-and-bound tree nodes to get the final optimal integer solution from the LP optimal solution. "LB" denotes the average of the maximum load of links that is defined as the maximum number of paths in a link of the original WDM ring network, which is a lower bound of the minimum number of wavelengths for the WDM network. "LP" denotes the average of the optimal objective value of the LP relaxation of (MIS IP), which is another lower bound of the minimum number of wavelengths for the WDM network. "Opt" denotes the average of the minimum number of wavelengths obtained by the suggested algorithm. Heur denotes the average of the number of wavelength obtained by LPH, a heuristic suggested in Section 4. (Comparing the solution qualities among heuristics, LPH is more effective than SPH about 5% and do not show significant difference with DegH, in our computational experiments. Hence we include only the results of LPH for comparison with (MIS IP) in this Table.)

Table 2. Avergae performance

Set	Col	BandB	LB	LP	Opt	Heur
G(5, 0.3)	0.8	0.0	4.0	4.2	4.2	4.2
G(5, 0.5)	1.4	0.0	6.2	6.2	6.2	6.2
G(5, 0.7)	5.6	0.0	8.2	8.2	8.2	8.2
G(5, 0.9)	10.8	0.0	9.6	9.6	9.6	9.6
G(10, 0.3)	8.8	0.6	17.2	17.6	17.6	18.2
G(10, 0.5)	34.2	0.4	25.4	26.2	26.2	27.8
G(10, 0.7)	77.6	0.4	33.2	33.6	33.6	37.2
G(10, 0.9)	127.6	0.0	41.8	41.8	41.8	47.2
G(15, 0.3)	57.6	3.6	36.2	36.8	36.8	39.4
G(15, 0.5)	145.8	1.8	56.6	58.2	58.2	63.2
G(15, 0.7)	269.4	4.8	77.6	78.4	78.4	86.4
G(15, 0.9)	418.2	3.4	96.6	96.6	97.0	100.2
G(20, 0.3)	166.4	0.8	60.8	61.6	61.8	67.4
G(20, 0.5)	324.0	4.8	102.0	103.2	103.4	111.2
G(20, 0.7)	593.8	4.6	139.0	140.8	141.2	155.0
G(20, 0.9)	962.8	12.0	175.0	176.2	176.2	197.8
G(25, 0.3)	293.2	1.8	97.0	99.5	100.0	108.4
G(25, 0.5)	613.6	16.6	160.0	160.7	161.0	174.6
G(25, 0.7)	993.4	19.2	218.8	220.8	220.8	243.4
G25, 0.9)	1798.0	15.4	274.0	276.6	276.6	310.0

As we see in <Table 2>, our column generation procedure does not require generating huge number of columns to get an optimal solution of the LP relaxations of (MIS IP). Moreover, after getting an LP optimal solution on the root node of the branch-and-bound tree, the algorithm can be terminated without traversing many branch-and-bound tree nodes. This can be possible since our column generation formulation has a very strong LP relaxation as we see in <Table 2>. Note that the LP relaxation gap, the difference between the "LP" and "Opt" is only about 0.5% for 100 instances.

In <Table 2>, we can also know that LB, the maximum load of links is a good lower bound for the minimum number of wavelengths when a WDM network has a ring topology. For small size network, this lower bound can be equal to the minimum number of wavelengths while for the large size networks it has a difference from the minimum number of wavelengths up to seven wavelengths (i.e. less than 6.6%) in the worst case of 100 instances.

The LPH heuristic has the solution quality gap between 0% and 13% from the optimum value. <Figure 2> shows the average solution quality gap between (MIS IP) and LPH for four different demand densities of n = 5, 10, 15, 20, and 25. This figure shows that as the number of lightpaths to be established increases, the solution quality gap of the

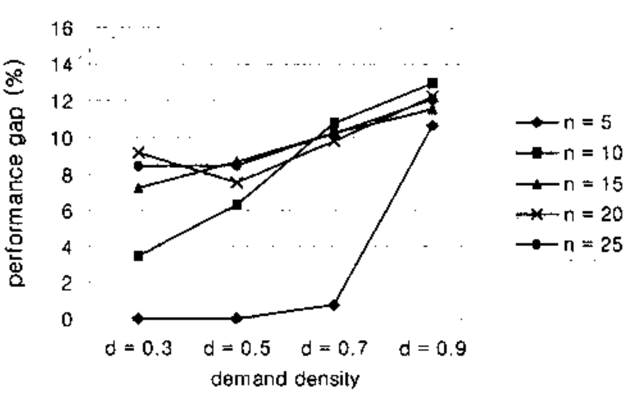

Figure 2. Solution quality gap of LPH heuristic from the optimum value.

heuristic gets larger. It also shows that the suggested optimization algorithm can use significantly fewer wavelengths than the LPH heuristic for large size networks and large demand requirement.

Finally we compare our branch-and-price algorithm with a known branch-and-bound algorithm for the vertex coloring problem, DSATUR algorithm developed in (Brélaz, D., 1979) and improved in (Mehrotra, A. and Trick, M. A., 1995). The results are summarized in <Table 3> and <Figure 3>. In <Table 3> "MIS Time" and "D Time" represents, respectively, the CPU times (in seconds) of the column generation based algorithm for (MIS IP) and DSATUR algorithm. In the experiments, for all five instances of $G(25,0.5)$, $G(25,0,7)$, $G(25,0.9)$, and three instances of $G(20,0.7)$ and two instances of $G(20,0.9)$, DSATUR can not get an optimal solution after one hour's running on the workstaion computer. In this case, which is represented by "*" for the performances of DSATUR in <Table 3>, we have to terminate DSATUR algorithm at one hour since those instances for DSATUR exhaust working memory of the computer, which comes from very fast growth of the branch-and-bound trees of DSATUR algorithm. In <Table 3> "MIS opt" and "D opt" represents, respectively, the minimum number of wavelengths obtained by the suggested algorithm and DSATUR algorithm. Since both algorithms are not heuristics but exact optimization algorithms, the minimum numbers of wavelengths obtained are equal except for five sets where DSTAUR algorithm is terminated after one-hour running. This experiment shows that DSATUR algorithm cannot solve WAP on an engineering workstation computer in one hour for some large size WDM ADM rings. Our MIS based branch-and-price algorithm, however, can solve any instance of 100 instances in less than 35 minutes on the same computer. <Figure 3> shows that the suggested algorithm outperforms DSATUR algorithm in solution time as the problem size increases.

Table 3. Comparison with DSATUR

$G(n,d)$	MIS Time	D Time	MIS Opt	D Opt
G(5, 0.3)	0.01	0.00	4.2	4.2
G(5, 0.5)	0.01	0.00	6.2	6.2
G(5, 0.7)	0.03	0.00	8.2	8.2
G(5, 0.9)	0.05	0.00	9.6	9.6
G(10, 0.3)	0.06	0.00	17.6	17.6
G(10, 0.5)	0.35	0.04	26.2	26.2
G(10, 0.7)	1.02	0.16	33.6	33.6
G(10, 0.9)	2.33	0.66	41.8	41.8
G(15, 0.3)	0.72	0.14	36.8	36.8
G(15, 0.5)	3.54	1.60	58.2	58.2
G(15, 0.7)	10.95	14.60	78.4	78.4
G(15, 0.9)	28.87	77.02	97.0	97.0
G(20, 0.3)	4.59	19.16	61.8	61.8
G(20, 0.5)	18.81	44.44	103.4	103.4
G(20, 0.7)	75.84	*1560.00	141.2	*141.2
G(20, 0.9)	226.59	*1358.50	176.2	*176.6
G(25, 0.3)	16.03	74.26	100.0	100.0
G(25, 0.5)	86.70	*3600.00	161.0	*161.4
G(25, 0.7)	336.94	*3600.00	220.8	*221.0
G(25, 0.9)	1382.18	*3600.00	276.6	*277.2

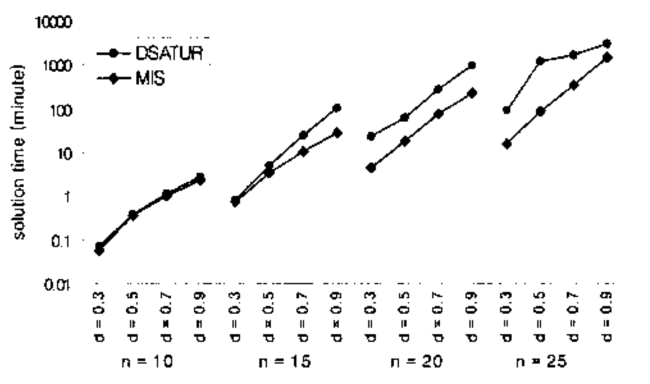

Figure 3. Solution time comparison between (MIS IP) and DSATUR.

We can solve WAP in unidirectional ring not to an approximation solution but an optimal solution for up to the instances of 25 WDM ADM nodes, 540 lightpaths, and 120,077 edges (overlapping conditions of the lightpath pairs) less than 35 minutes. We think 25 WDM ADM nodes size is sufficient large size for WDM ADM rings since 16 is the maximum number of ADM nodes in the current SONET transmission ring technologies. Note also that the current experimental WDM transmission systems can support up to 48 wavelengths and more than 100 wavelengths multiplexing seems to be impossible in near future because of characteristic of used wavelengths (Ramaswami, R. and Sivarajan, K. N., 1998; Mukherjee, B., 1997). As we see in Opt of Table 2, every instance for 25 ADM nodes and most of instances for 20 ADM nodes, we need more than 100 wavelengths. Hence our experiment covers sufficiently large networks for practical implementation of the near future demand that the current developing WDM technology can support.

6. Conclusions

In this paper, we have shown that WAP on uni-directional WDM ADM ring, though belongs to *NP-hard*, can be exactly solvable up to 25 WDM ADM rings that can be enough size for near future demand. To accomplish this, we proposed MIS based IP formulation for the vertex coloring problem on CAG and a branch-and-price solution approach using column generation techniques. We also propose some generic heuristics and do the performance comparison with the suggested optimization algorithm. Vertex coloring approach for wavelength assignment to bi-directional rings or mesh topology and combining these results with path routing is one of the topics of further study.

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이 희 상

서울대학교 산업공학과 학사

서울대학교 산업공학과 석사

Georgia Tech. Industrial & Systems Engineering 박사

현재: 한국외국어대학교 산업정보시스템공학부 부교수

관심분야: 정수계획법, 통신 네트워크

정 지 복

서울대학교 산업공학과 학사

서울대학교 산업공학과 석사

서울대학교 산업공학과 박사

현재: (주) 프라이스 워터하우스 쿠퍼스 컨설턴트

관심분야: 최적화 이론, 통신 네트워크, 물류 시스템