Approximated Solution of Model for Three-Phase Fluidized Bed Biofilm Reactor in Wastewater Treatment

Jeong-Woo Choi[1]*, Junhong Min[1], Won-Hong Lee[1], and Sang Baek Lee[2]

[1] Department of Chemical Engineering, Sogang University, C.P.O.Box 1142, Seoul 100-611, Korea
[2] Department of Chemical Engineering, Cheju National University, Cheju 690-756, Korea

Abstract An approximated analytical solution of mathematical model for the three phase fluidized bed bioreactor (TFBBR) was proposed using the linearization technique to describe oxygen utilization rate in wastewater treatment. The validation of the model was done in comparison with the experimental results. Satisfactory agreement was obtained in the comparison of approximated analytical solution and numerical solution in the oxygen concentration profile of a TFBBR. The approximated solutions for three modes of the liquid phase flow were compared. The proposed model was able to predict the biomass concentration, dissolved oxygen concentration the height of efficient column, and the removal efficiency.

Keywords: three phase fluidized bed biofilm reactor, wastewater treatment, mathematical model, analytical solution, oxygen concentration

Three phase fluidized bed bioreactor (TFBBR) has been developed to increase the height of efficient column (HEC) due to oxygen limitation in the two phase fluidized bed reactor [1,2]. To provide a rational criteria for design and operation of a TFBBR in wastewater treatment, a mathematical model to describe a TFBBR should be developed since the model can identify important system variables and serve a basis for system optimization [3-6]. The mathematical model to describe the oxygen utilization for a TFBBR in wastewater treatment was developed by the authors [7,8], which was proposed to describe the oxygen concentration distribution. This model consisted of the biofilm model that described the oxygen uptake rate and the hydraulic model that presented characteristics of liquid and gas phase. Numerical simulation result of this mathematical model was in good agreement with experimental result [8]. However, numerical solution requires the repeated computation with computer to analyze the effect of operating condition whenever the parameter values was changed. Insteads, the analytical solution possesses the advantages such as the ability to analyze the effects of the conditions on the result without the repeated computation. In this paper, an approximated analytical solution for a TFBBR model is proposed using Taylor series expansion as a linearization technique.

It can be considered that a TFBBR is operated under the steady state condition in which each spherical medium with uniform size is covered with biofilm of a uniform thickness and the wastewater is passing upwards through the reactor in an axial dispersion mode, plug flow mode or completely mixed mode. The mathematical model of a TFBBR was developed based on the biofilm model that describes the oxygen uptake rate and the hydraulic model that presents characteristics of liquid and gas phase. The detail derivation of model was shown in reference 8. The approximated mathematical solution of the model was proposed for the three kinds of liqid flow type.

* Corresponding author
Tel: +82-2-705-8480 Fax: +82-2-711- 0439
e-mail: jwchoi@ccs.sogang.ac.kr

Case I: If liquid flows in an axial dispersion mode, hydrodynamics of fluids are described as an axial dispersion fluids in the liquid phase and a plug flow in the gas phase as follows. The detail derivations of following governing equation and nomenclature are depicted in reference 8.

For liquid phase,

$$E_{z1}\varepsilon_1\frac{d^2C_1}{dz^2} - \varepsilon_1 U_1\frac{dC_1}{dz} + k_1\bar{a}_g\left(\frac{C_g}{M} - C_1\right) - \Phi\varepsilon_s C_1^{0.45} = 0 \quad (1)$$

$$\text{B. C.} \quad C_1 = C_{1i} + E_{z1}\frac{dC}{dz} \quad \text{at} \quad z=0 \quad (2)$$

$$\frac{dC}{dz} = 0 \quad \text{at} \quad z=1 \quad (3)$$

where $\Phi = 3.012(\rho_{bd}\kappa_o)^{0.55} D_{ec}^{\ 0.45} r_p^{\ -0.9}$ and

$\varepsilon_s = (1-\varepsilon_{1g})$.

For gas phase

$$U_g\frac{dC_g}{dz} + k_1\bar{a}_g\left(\frac{C_g}{M} - C_1\right) = 0 \quad (4)$$

$$\text{B. C.} \quad C_g = C_{gi} \quad \text{at} \quad z=0 \quad (5)$$

where z is axial position; E_{zl} is axial dispersion coefficient; ε_l is liquid porosity; k_l is mass transfer coefficient between gas and liquid phase; $\bar{a}_g$ is average interfacial area of gas bubble per unit volume of reactor; M is Henry constant; C_g and C_{li} are concentration of gas phase and liquid phase at inlet position, respectively; U_l and U_g are superficial liquid velocity and superficial gas velocity, respectively; D_{ec} is effective diffusivity of oxygen in the biofilm; κ_o is intrinsic zero order rate constant for oxygen; ρ_{bd} is the density of dry biofilm; ε_s is solid porosity and ε_{lg} is gas and liquid porsity; r_p is the radius of bioparticle.

In dimensionless form, Eqs. (1-5) are reduced to

$$\frac{1}{P_e}\frac{d^2\overline{C}}{d\theta^2}-\frac{d\overline{C}}{d\theta}+\beta(\delta\overline{G}-\overline{C})-\varpi\overline{C}^{0.45}=0 \qquad (6)$$

$$\frac{d\overline{G}}{d\theta}+\lambda(\delta\overline{G}-\overline{C})=0 \qquad (7)$$

$$\text{B.C.} \quad 1=\overline{C}-\frac{1}{P_e}\frac{d\overline{C}}{d\theta} \quad \text{at} \quad \theta=0 \qquad (8)$$

$$\frac{d\overline{C}}{d\theta}=0 \quad \text{at} \quad \theta=1 \qquad (9)$$

$$\overline{G}=1 \quad \text{at} \quad \theta=1 \qquad (10)$$

where

$$\overline{C}=\frac{C}{C_i},\ \beta=(\frac{k_l\bar{a}_g H}{U_l}),\ \lambda=\frac{k_l\bar{a}H}{U_g}\frac{C_{li}}{C_{gi}},$$

$$\theta=\frac{z}{H},\ \delta=(\frac{C_{gi}}{C_{li}M}),\ \varpi=\frac{\Phi\varepsilon_s C_i^{0.45}}{U_l} \qquad (11)$$

To achieve the analytical solution, $\overline{C}^{0.45}$ is linearized by the Taylor series expansion,

$$\overline{C}^{0.45}=0.55+0.45\overline{C} \quad \text{for} \quad -1<\overline{C}<1 \qquad (12)$$

Substituting Eq. (12) into Eq. (6), Eq. (6) becomes

$$\frac{1}{P_e}\frac{d^2\overline{C}}{d\theta^2}-\frac{d\overline{C}}{d\theta}+\beta(\delta\overline{G}-\overline{C})-0.45\,\varpi\overline{C}-0.55\varpi=0 \qquad (13)$$

To obtain analytical solution for Eq. (13), a general solution using basis can be introduced as $\overline{C}=\sum_{i=1}^{3}C_{o_i}e^{m_i\theta}+$ *constant*. Then Eq. (7) can be rearranged to

$$(-m-\lambda\delta)\overline{G}+\lambda\overline{C}=0 \qquad (14)$$

where let $m=d\overline{C}/d\theta$ and m is a polynomial coefficient. And Eq. (13) is written as

$$(\frac{1}{P_e}m^2-m-\beta-0.45\varpi)\overline{C}+\beta\delta\overline{G}-0.55\varpi=0 \qquad (15)$$

Substituting Eq. (14) into Eq. (15),

$$(\frac{1}{P_e}m^2-m-\beta-0.45\varpi+\frac{\lambda\beta\delta}{m+\lambda\delta})\overline{C}-0.55\varpi=0 \qquad (16)$$

For the particular solution,

$$\frac{1}{P_e}m^2-m-\beta-0.45\varpi+\frac{\lambda\beta\delta}{m+\lambda\delta}=0 \qquad (17)$$

$$\frac{1}{P_e}m^3+(\frac{1}{Pe}\lambda\delta-1)m^2-(\lambda\delta+\beta+0.45\varpi)m-0.45\varpi\lambda\delta=0 \qquad (18)$$

Three polynomial coefficients, m, are obtained using Eq. (18).

The particular solution of Eq. (16) is as follows.
Let A be substituted into $\overline{C}$ of Eq. (16) ($\overline{C}=A$) .

$$(-\beta-0.45\varpi+\beta)A=0.55\varpi \qquad (19)$$

$$\text{and then} \quad A=-\frac{11}{9} \qquad (20)$$

The general solution of Eq. (16) becomes

$$\overline{C}=\sum_{i=1}^{3}C_{o_i}e^{m_i\theta}-\frac{11}{9} \qquad (21)$$

The coefficients, C_{o_i}, can be obtained by substituting the boundary conditions (8), (9) and (10) into Eq. (21) as follow.

$$\begin{bmatrix} 1-m_i/P_e \\ m_i e^{m_i} \\ m_i^2/P_e-m_i-\beta-0.45\varpi \end{bmatrix}\begin{bmatrix} C_{o_1} \\ C_{o_2} \\ C_{o_3} \end{bmatrix}=\begin{bmatrix} \frac{20}{9} \\ 0 \\ -\beta(\delta+\frac{11}{9}) \end{bmatrix} \qquad (22)$$

Thus the oxygen concentration of liquid phase along the height of fluidized bed can be obtained with the analytical solution, Eq. (21) and boundary conditions Eq. (22), when liquid phase flow is axial dispersion flow mode.

Case II: If wastewater passes upwards through the reactor in a plug mode, hydrodynamics of liquid phase are described as follows.

$$-U_l\varepsilon_l\frac{dC_l}{dz}+k_l\bar{a}_g(\frac{C_g}{M}-C_l)-\Phi\varepsilon_s C_l^{0.45}=0 \qquad (23)$$

$$\text{B.C.} \quad C_l=C_{li} \quad \text{at} \quad z=0 \qquad (24)$$

Hydrodynamics of gas phase is represented by Eq. (7) with boundary condition Eq. (10).
Using dimensionless variables, Eqs. (23) and (24) are rewritten as

$$-\frac{d\overline{C}}{d\theta}+\beta(\delta\overline{G}-\overline{C})-\varpi\overline{C}^{0.45}=0 \quad (25)$$

$$\text{B.C.} \quad \overline{C}=1 \quad \text{at} \quad \theta=0 \quad (26)$$

By the linearization, Eq. (25) becomes

$$-\frac{d\overline{C}}{d\theta}+\beta(\delta\overline{G}-\overline{C})-0.45\varpi\overline{C}-0.55\varpi=0 \quad (27)$$

Introducing the general solution with basis ($\overline{C}=\sum_{i=1}^{2}\gamma_i e^{m_i\theta}$ + *constant*) into Eq. (26),

$$(-m-\beta-0.45\varpi)\overline{C}+\beta\delta\overline{C}-0.55\varpi=0 \quad (28)$$

where let $m=\frac{d\overline{C}}{d\theta}$

Substituting Eq. (14) in Eq. (28),

$$(-m-\beta-0.45\varpi+\frac{\lambda\beta\delta}{m+\lambda\delta})\overline{C}-0.55\varpi=0 \quad (29)$$

By rearrangement of Eq. (29),

$$m^2+(\lambda\delta+\beta+0.45\varpi)m+0.45\varpi\lambda\delta=0 \quad (30)$$

The roots of Eq. (30) are

$$m_1=\frac{-q_1+\sqrt{q_1^2-4q_2}}{2}, \quad m_2=\frac{-q_1+\sqrt{q_1^2-4q_2}}{2} \quad (31)$$

where $q_1=\lambda\delta+\beta+0.45\varpi$ and $q_2=0.45\lambda\varpi$

For obtain the particular solution, Let constant, B, is substituted in $\overline{C}$ into Eq. (29).
Then,

$$(-\beta-0.45\varpi+\beta)B=0.55\varpi \quad (32)$$

$$\text{and then} \quad B=-\frac{11}{9} \quad (33)$$

The general solution of Eq. (27) is obtained as following.

$$\overline{C}=\sum_{i=1}^{2}\gamma_i e^{m_i\theta}-\frac{11}{9} \quad (34)$$

The parameter, γ_i is obtained by applying the boundary condition, Eq. (26).
Substituting Eq. (34) into boundary condition, Eq. (26),

$$1=\sum_{i=1}^{2}\gamma_i-\frac{11}{9} \quad (35)$$

Substituting boundary conditions, Eq. (10), and Eq. (26), into Eq. (28),

$$(m_1+\beta+0.45\varpi)\gamma_1+(m_2+\beta+0.45\varpi)\gamma_2=\beta(\delta+\frac{11}{9}) \quad (36)$$

Combining Eq. (35) and Eq. (36)

$$\gamma_1=\frac{1}{(m_1-m_2)}[\beta(\delta-1)-\frac{20}{9}(m_2+0.45\varpi)] \quad (37)$$

$$\gamma_2=\frac{20}{9}-\gamma_1 \quad (38)$$

The oxygen concentration of liquid phase along the height of fluidized bed can be obtained using the analytical solution, Eq. (34) with coefficients, Eqs. (37) and (38) when liquid phase flow is in plug flow mode.

Case III: If liquid phase flows in type of completely mixed type, material balance for the liquid phase is decribed as

$$\varepsilon_1\frac{Q}{V_R}(C_{li}-C_{lo})-\varepsilon_s\Phi C_1^{0.45}+\frac{1}{H}\int_0^H k_l\overline{a}_g(\frac{C_g}{M}-C_{lo})dz=0 \quad (39)$$

where Q is volumetric flow rate; C_{li} and C_{lo} are inlet and outlet oxygen concentrations in liquid phase; H is expanded bed height; V_R is total reactor volume.
Hydrodynamics of gas phase is represented as following.

$$U_g\frac{dC_g}{dz}+k_l\overline{a}_g(\frac{C_g}{M}-C_{lo})=0 \quad (40)$$

Eq. (40) has the same boundary condition as Eq. (4).
Using the dimensionless variables and linearization technique, Eq. (39) is rewritten as

$$1-\overline{C}_o-0.55\varpi-0.45\varpi\overline{C}_o+\beta\int_0^1(\delta\overline{G}-\overline{C}_o)d\theta=0 \quad (41)$$

where $\overline{C}_o$ is the ratio of outlet oxygen concentration to inlet concentration, $\frac{C_{lo}}{C_{li}}$.
Introducing dimensionless variables to Eq. (40),

$$\frac{d\overline{G}}{d\theta}+\lambda(\delta\overline{G}-\overline{C}_o)=0 \quad (42)$$

Combining Eq. (42) with boundary condition, Eq. (10),

$$\ln\frac{\delta\overline{G}-\overline{C}_o}{\delta-1}=-\delta\lambda\theta \quad (43)$$

or

$$\overline{G}=\frac{1}{\delta}[\exp(-\delta\lambda\theta)(\delta-1)+\overline{C}_o] \quad (44)$$

Substituting Eq. (44) into the integral term of Eq. (41),

$$\int_0^1(\delta\overline{G}-\overline{C})d\theta=\frac{1-\delta}{\delta\lambda}[\exp(-\delta\gamma\theta)-1] \quad (45)$$

Table 1. Experimental values in simultion (All values except D_{ec} and k_o are in reference 1; D_{ec} and k_o in reference 3 and 10, respectively)

C_{gi}	2.795×10^{-10}	U_g	1.06
C_{li}	1.397×10^{-10}	U_l	0.5036
D_{ec}	1.5×10^{-5}	ρ_{bd}	0.03
D_T	9.4	ρ_l	1.0
H	75	ρ_m	1.7
$k_l\bar{a}_g$	1.87×10^{-2}	κ_o	1.62×10^{-5}
r_m	0.043	μ_l	0.9×10^{-2}

Substituting Eq. (45) into Eq. (41),

$$1-\overline{C}_o-0.55\varpi-0.45\varpi\overline{C}_o+\frac{\beta(1-\delta)}{\delta\lambda}[\exp(-\delta\lambda\theta)-1]=0 \quad (46)$$

Rearranging Equation, the final solution is obtained such as

$$\overline{C}_o=\frac{\delta\lambda(1-0.55\varpi)+\beta(1-\delta)[\exp(-\delta\lambda\theta)-1]}{\delta\lambda(1+0.45\varpi)} \quad (47)$$

The oxygen concentration of liquid phase along the height of fluidized bed is represented as outlet oxygen concentration when liquid phase flow is completely mixed flow mode, which can be obtained using Eq. (47).

These approximated analytical solutions in three types of flow modes were compared with the experimental data [1] as shown in Fig. 1, while the parameters in Table 1 were used. The oxygen concentration was decreased gradually in the inlet part of column and was decreased slightly in the upper part of column. It was due to the fact that the the oxygen reacion rate is slower due to the lower oxygen concentration in the uppper part of column. Fig. 1 showed an axial dispersion model was more suitable to represent a TFBBR than mixed model or plug model for liquid phase except the end of column. The mixed model for the liquid phase was more suitable than the axial dispersion model at the end of column. It was due to the fact that Peclet number was changed by the bioparticle size distribution along the column. But analytical solution by simple approximation is useful in most region of a TFBBR which the back mixing was not dominent by large particles. This results were in agreement with El-Temtany's proposal in which the liquid phase was well described by an axial mixing when the particle sizes were smaller than 3 mm [9].

TFBBR model comprised with two differential equations describing hyrodynamics of liquid phase in a axial dispersion mode and gas phase in the plug flow mode was solved with two methods. One was numerical integration of differential equations including Peclet number that represents bioparticle size distribution using Finite Difference Method [8], and the other was approximated analytical solution by Taylor series expansion as the linerization technique. The error from the linearization was examined by the comparison with the numerical solution as shown in Fig. 2. Analytical solution was not able to represent the completely mix-

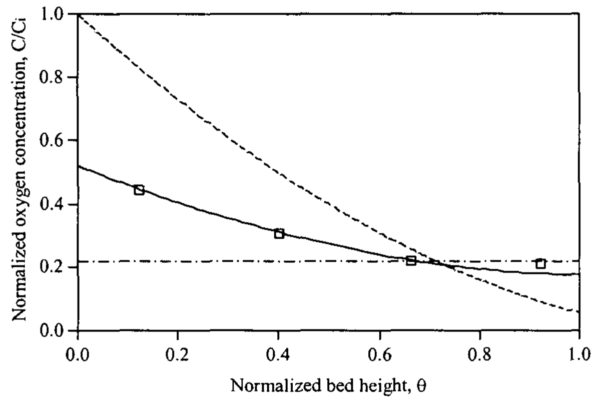

Fig. 1. The comparison of the experimental data and various liquid flow models. (broken line, plug flow mode; dotted line, completely mixed mode; solid line, axial dispersion flow mode; opened rectangule, experimental data [1]).

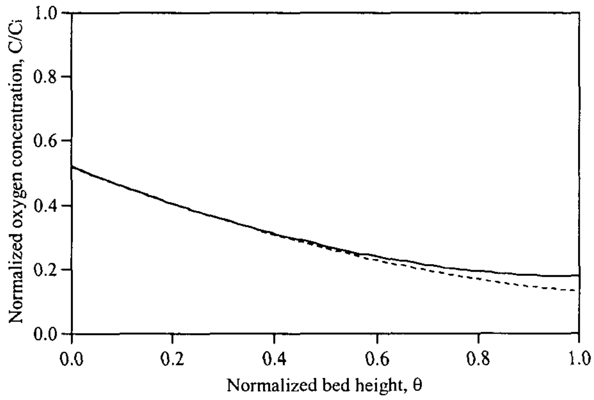

Fig. 2. The comparison of numerical solution and approximated solution on the performance of the TFBBR (solid line, numerical solution by Finite Differential Method; broken line, approximated analytical solution).

ing effect generated at the end of column due to the constant Peclet number while the variable P_e number was used in the numerical solution. But the result showed a good agreement except at the end of column. Thus the proposed approximated technique can be used as the solution technique to understand the performance of TFBBR without computer simulation.

The removal efficiency of organic pollutants can be calculated with total oxygen mass consumed that is obtained from oxygen uptake rate and biomass concentration. The ratio of chemical oxygen demand (COD) removal to oxygen consumption is expressed as [1]

$$\frac{\text{COD eliminated}}{O_2\text{ consumed}}=3.4 \quad (48)$$

The effects of superficial velocities of liquid and gas on removal efficiency are represented at given operating condition as shown in Fig. 3. The removal efficiency is nearly constant or slightly decreases as superficial liquid

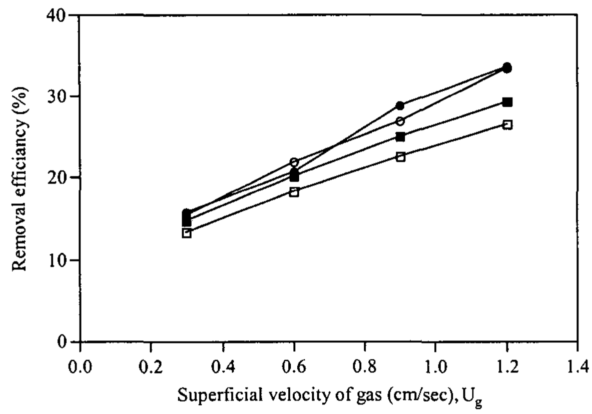

Fig. 3. The effects of superficial velocities of gas and liquid on the removal efficiency of a TFBBR at vaious superficial velocities of liquid (solid line and closed circle, 0.3 cm/sec; dotted line and open circle, 0.6 cm/sec; shortly broken line and closed rectangle, 0.9 cm/sec; long broken line and open rectangle, 1.2 cm/sec).

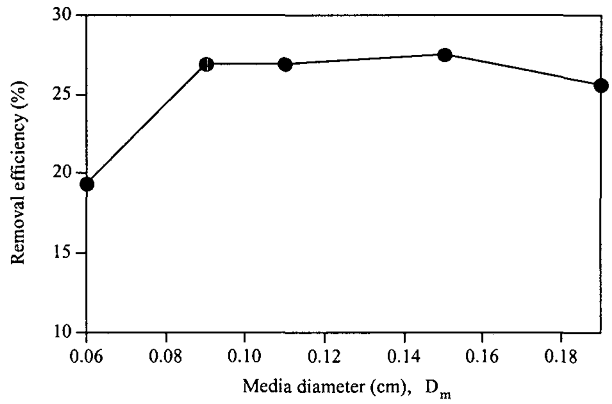

Fig. 4. The effect of media size on the removal efficiency of a TFBBR.

velocity increases. The reason is that dissolved oxygen concentrations increases as superficial velocity of liquid increases, which reduces biomass concentration and contact time between liquid and biofilm. Thus total oxygen consumption amount is nearly constant due to the counteraction between dissolved oxygen concentration and biomass concentration. At high superficial liquid velocity, the dissolved oxygen concentration increases slightly, and biomass concentration and contact time decreases at the same degree, so that removal efficiency is low at all range of superficial gas velocity. The removal efficiency is strongly related to superficial gas velocity as shown in Fig. 3. The efficiency increases as superficial gas velocity increases. The reason is that the amount of oxygen transferred from the gas phase to the liquid phase is increased as superficial gas velocity increases due to the dependence of mass transfer coeeficient to superficial gas velocity, However, the contact time between liquid and biofilm is nearly the same at any superficial gas velocity since the fluidization of bioparticle is mainly done by upflow of liquid.

Fig. 4 represented that the optimal media size existed to enhance removal efficiency at given operating condition based on the proposed approximated solution. The variation of media size enables the control of biomass concentration and dissolved oxygen concentration, so that the maximum removal efficiency that depends on the dissolved oxygen concentration and the biomass concentration exists.

NOMENCLATURE

$\bar{a}$: Average interfacial area per unit volume of reactor, cm^{-1}
C : Oxygen concentration, g/cm^3
C_l : Oxygen concentration in the liquid phase, g/cm^3
$\bar{C}$: Dimensionless oxygen concentration in the liquid phase
$\bar{C}_o$: The ratio of outlet oxygen concentration to inlet concentration, C_{lo}/C_{li}.
D : Diameter, cm
E_{zl} : Axial dispersion coefficient, cm^2/sec
$\bar{G}$: Dimensionless oxygen concentration in the gas phase
H : Expanded bed height, cm
k : Mass transfer coefficient, cm/sec
M : Henry constant
m_i : Polynomial coeffienct
P_e : Peclet number, $\dfrac{U_l D_p}{E_{zl}}$
Q : Volumetric flow rate, cm^3/sec
U : Superficial phase velocity, cm/sec
V_R : Total reactor volume, cm^3

Greek Symbols

β : $\dfrac{k_l \bar{a}_g H}{U_l}$, dimensionless
δ : $\dfrac{C_{gi}}{C_{li} M}$, dimensionless
ε : Porosity, dimensionless
η_o : Effectiveness factor for intrinsic zero order reaction, dimensionless
θ : Dimensionless length
κ_o : Intrinsic zero order rate constant for oxygen, sec^{-1}
λ : $\dfrac{k_l \bar{a}_g H}{U_g} \dfrac{C_{li}}{C_{gi}}$, dimensionless
μ_l : Viscosity of the liquid phase, g/cm · sec
ρ : Density, g/cm
σ : Surface tension, g/cm
Φ : $3.012(r_{bd} k_o)^{0.55} D_{ec}^{0.45} r_p^{-0.9}$
χ : Biomass concentration, g/cm^3
ϖ : $\Phi \dfrac{\varepsilon_s H}{U_l} C_{li}^{-0.55}$, dimensionless

Subscripts

g	: Gas phase
i	: Inlet value
l	: Liquid phase
lg	: Liquid and gas phase
m	: Media
o	: Outlet value
p	: Particle
s	: Solid phase

REFERENCES

[1] Chatib, B., A. Grasmick, S. Elmaleh, and R. B. Aim (1981) Biological wastewater treatment in a three-phase fluidised-bed reactor. pp. 192-204 In: P. F. Cooper and B. Atkinson (eds). *Biological Fluidized Bed Treatment of Water and Wastewater.* Ellis Horwood Limited, Chichester, U.K.

[2] Sutton, P. M., J. Hurvid, and M. Hoeksema (1999) Biological fluidized-bed treatment of wastewater from by-product coking operations: Full-scale case history. *Water Environ. Technol.* 71: 5-9.

[3] Fan, L. S. (1989) *Gas-Liquid-Solid Fluidization Engineer ing,* Butterworths, Boston, MA, U.S.A.

[4] Shieh, W. K., A. M. Asce, and L. T. Mulcahy (1981) Fluidized bed for biological wastewater treatment. *J. Environ. Eng. Division.* EE1: 293-295.

[5] Bailey, J. E. and D. F. Ollis (1986) *Biochemical Engineering Fundamentals,* 2nd ed., McGraw-Hill Book Co., New York, NY, U.S.A.

[6] Schügerl, K. (1997) Three-phase-biofluidization - Application of three-phase fluidization in the biotechnology-A review. *Chem. Eng. Sci.* 52: 3661-3668.

[7] Choi, J. W. (1983) A theoretical study of the oxygen reaction rate in a three-phase fluidized bed biofilm reactor, M.S. Thesis, Sogang University, Seoul, Korea.

[8] Choi, J.-W., J. Min, W. H. Lee, and S. B. Lee (1999) Mathematical model for three-phase fluidized bed biofilm reactor in wastewater treatment, *Biotechnol. Bioprocess Eng.* 4: 51-58.

[9] El-Temtany, S. A., Y. O. El-Sharnoubi, and M. M. El-Halwagi (1979) Liquid dispersion in gas-liquid fluidized Beds. *Chem. Eng. J.* 18: 151-159.

[10] Williamson, K. and P. I. McCarty (1976) A model of substrate utilization by bacterial films. *J. Water Pollut. Control Fed.* 48: 281-296.

[Received November 24, 1999; accepted February 12, 2000]