韓國産業應用學會論文誌 제3권 제1호 53-59, 2000.2
J. of Korea Society of Industrial Application, Vol.3, No.1, 53-59, February. 2000

단섬유 보강 복합재료의 기계적 특성 평가에 관한 연구
Prediction of Tensile Properties for Short-fiber-reinforced Composites

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<요 약>

단섬유 보강 복합재료의 종횡비(aspect ratio)를 변화시키며 기계적 특성(탄성계수, 인장강도)을 평가하였다. 2차원 다중 파이버(multi-fiber) 모델을 이용하여 엇갈린(staggered) 배열과 규칙적(aligned) 배열에 대해 유한요소 해석을 하였다. 단섬유 복합재료의 유효탄성계수 및 인장강도는 섬유와 기지의 탄성계수비, 섬유 배열상태, 그리고 단섬유 종횡비의 함수로 표현되었으며, 해석결과의 탄성계수와 인장강도는 이론 모델의 결과와 사출 성형된 PEEK 복합재료 시험편의 결과와 비교하였다. 시험결과는 낮은 종횡비에서 이론 모델 결과와 일치함을 보였다. 단섬유 보강 복합재료의 배열 및 종횡비 변화에 따른 섬유보강 효과에 따른 계면응력 상태는 기계적 특성 결정에 중요한 영향을 보였다.

Keyword : Aspect Ratio, Cox Model, Tensile Modulus, Array, Composite

1. INTRODUCTION

Recent development in high-performance polymer composites has made it possible to offer advanced composites with superior weight-to-strength ratios compared to the conventional metal alloys. Especially carbon fiber reinforced plastics are being used increasingly in load bearing components such as aircraft components, pressure vessels and pipes. The type of processing used in these materials is also know to affect the mechanical properties owing to fiber length, volume fraction and orientation.

The short-fiber composites, not as strong or as stiff as contiguous fiber composites, do have several attractive characteristics that make them worthy of consideration for other applications such as automotive industry. Those are the possibilities of fabricating components because of complex geometrical shape, fast and inexpensive processing methods. But the existing short - fiber composites have not been fully developed to yield optimum performance due to no clear understanding of processing parameters and their evaluation techniques.

Many micromechanical models are introduced to study the mechanical properties of short-fiber reinforced composites. The key model is Cox model[1]. There are also many studies using unit cell concept with single

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fiber[2,3] In general, there are three models - aligned discontinuous fiber model, off-axis aligned fiber model and randomly oriented discontinuous fiber model - to study the mechanical properties of discontinuous fiber composites for theoretical and numerical analyses. Even though randomly oriented discontinuous fiber model is considered to be a representative type of a typical composites, aligned discontinuous fiber model is the most widely using model since this model is easy and acceptable. Aligned discontinuous fiber model is begin and developed with the Cox model and this also is very important basic tool to understand the randomly oriented composites.[4,7]

Among these factors affecting the properties of composites, few of the key elements having profound effects on the properties are the volume, modulus and geometry of the reinforcing materials. The geometry of reinforcing material can be described as shape, size, aspect ratio(l/d) and distributions.[4] Many micromechanical models show good agreement in low fiber volume fraction with experimental data but poor agreement as V_f increase. The reason is believed to be the interactions between neighboring fibers that can be negligible in low V_f. Therefore, the model with multi-fiber composite is more desirable to study . If the components in the composites are deformed elastically, the shape and distribution are not important factors. But there is a significant plastic deformation in polymer matrix, the distribution of reinforcing fibers is a very important. Tvergaard[5] shows a decreasing of flow stress and strain hardening in the case of staggered array when the perfectly aligned and staggered fiber models are studied using unit cell model.

The objective of this study is to investigate the effective modulus(E_c) and ultimate tensile strength(S_u) of randomly oriented fiber reinforced composites by changing the aspect ratio(l/d), fiber distribution pattern and fiber volume fraction(V_f) using 2-D multi- fiber composites model. And the obtained finite element(FE) results are compared with the results of the theoretical models and the experimental data of injection molded PEEK composite. Also, fracture mechanisms of short-fiber composites are discussed by analyzing the stress/strain distribution in the composite and the stress transferring mechanisms from fiber to matrix as a role on reinforcement efficiency.

2. THEORETICAL MODELS

Among the many models predicting elastic modulus of short-fiber composites from empirical modeling based on the experimental observations to sophisticated analytical treatments based on a microscopic point of view, few models that consider the fiber aspect ratio will be discussed in this study. Hwang and Gibson[6] try to explain longitudinal modulus using finite element analysis by considering the fiber end effect and modified Cox model. The modulus by modified Cox model using inverse rule of mixtures is[6,7]

$$\frac{1}{E_{MCl}} = \frac{V_{cl}}{E_{cl}} + \frac{V_m}{E_m} = \frac{L/(L+e)}{E_{cl}} + \frac{e/(L+e)}{E_m} \quad (1)$$

where E_{MCl} = longitudinal modulus of modified Cox model, E_{cl} = longitudinal modulus of Cox model, V_{cl} = volume fraction of the Cox model in modified Cox mode, L = Length of Cox model, e = distance between fiber ends in modified Cox model and L+e = Length of modified Cox model. Also, Halpin[8] predicted modulus of aligned discontinuous fiber reinforced composites by modifying the Halpin-Tsai equation that used for the longitudinal and transversal moduli of continuous composites. Both moduli of modified Halpin-Tsai equation are [7]

$$\frac{E_L}{E_m} = \frac{1+\xi\eta_L V_f}{1-\eta_L V_f} \quad \text{where } \eta_L = \frac{(E_f/E_m)-1}{(E_f/E_m+\xi}$$

$$\frac{E_T}{E_m} = \frac{1+2\eta_T V_f}{1-\eta_T V_f} \quad \text{where } \eta_T = \frac{(E_f/E_m)-1}{(E_f/E_m+2} \quad (2)$$

where E_L is the longitudinal modulus, E_T is transverse modulus and ζ = 2(l/d). Here, the short- fiber composites are considered as quasi-isotropic composites with randomly oriented fiber in 3-D. Tsai and Pagano[9] have shown that the modulus of short-fiber composites can be predicted approximately using Halpin-Tsai equation(Eqn.2) of E_L and E_T as

$$E_c = \frac{3}{8}E_L + \frac{5}{8}E_T \quad \text{and} \quad G_c = \frac{1}{8}E_L + \frac{1}{4}E_T \quad (3)$$

Christensen and Waals[10] also predict the effective modulus of short-fiber composite using micromechanical equation of Hashin [11,12] and Hill[13] that based on the averaging concept of elastic modulus. When the fiber aspect ratio is very small in the short-fiber composites, the effective modulus of randomly fiber oriented composite with 3-D is[7,10]

$$E_c = \frac{[E_1 + (4\nu_{12}^2 + 8\nu_{12} + 4)K_{23}][E_1 + (4\nu_{12}^2 - 4\nu_{12} + 1)K_{23} + 6(G_{12} + G_{23})]}{3[2E_1 + (8\nu_{12}^2 + 4\nu_{12} + 7)K_{23} + 2(G_{12} + G_{23})]} \tag{4}$$

Where K_{23} is plane strain bulk modulus for dilatation at 2-3 plane. The five independent engineering constants, E_1, ν_{12}, G_{12}, G_{23} and K_{23} are calculated by Hill[13] and Hashin [11,12]. We compared the results of these models with results of the 2-D multi-fiber composites models and experimental data.

3. ANALYTICAL METHODS

The 2-D multi-fiber composite model developed from the RVE(Representative volume element) concept is used to analyze the mechanical properties of PMC. The special code system for modeling and meshing has been developed to accept the changing of aspect ratio at given fiber volume fraction for aligned and staggered fiber geometries. Fig.1 shows the 2-D multi-fiber composite model representing x-direction as the loading direction along with the fibers and y-direction as the cross-section of fiber. There are 4 fiber layers in the model with the constant distance between the neighboring fiber centers for different fiber volume fraction and aspect ratio. Two types of model with different length vs. height ratio (D/L=1:5 and 2:5) are considered to see the effects of aspect ratio and the location of fibers.

The phase of matrix and fiber is a isotropic material and interface between two components has a perfect bonding. Fiber has a linear elastic behavior up to break and matrix has a elastoplastic behavior. Both stress-strain curves, determined by the multi-linear isotropic hardening rates, elastic modulus and yielding stress are given as input data. The FE computations are performed using 4 noded isoparametric elements for 2-D model. The elastic modulus is chosen to be 2.5 GPa for matrix and 250 GPa(E_f/E_m=100) for fiber. The chosen Poisson's ratio is 0.3 for matrix and 0.25 for fiber respectively. The ultimate tensile

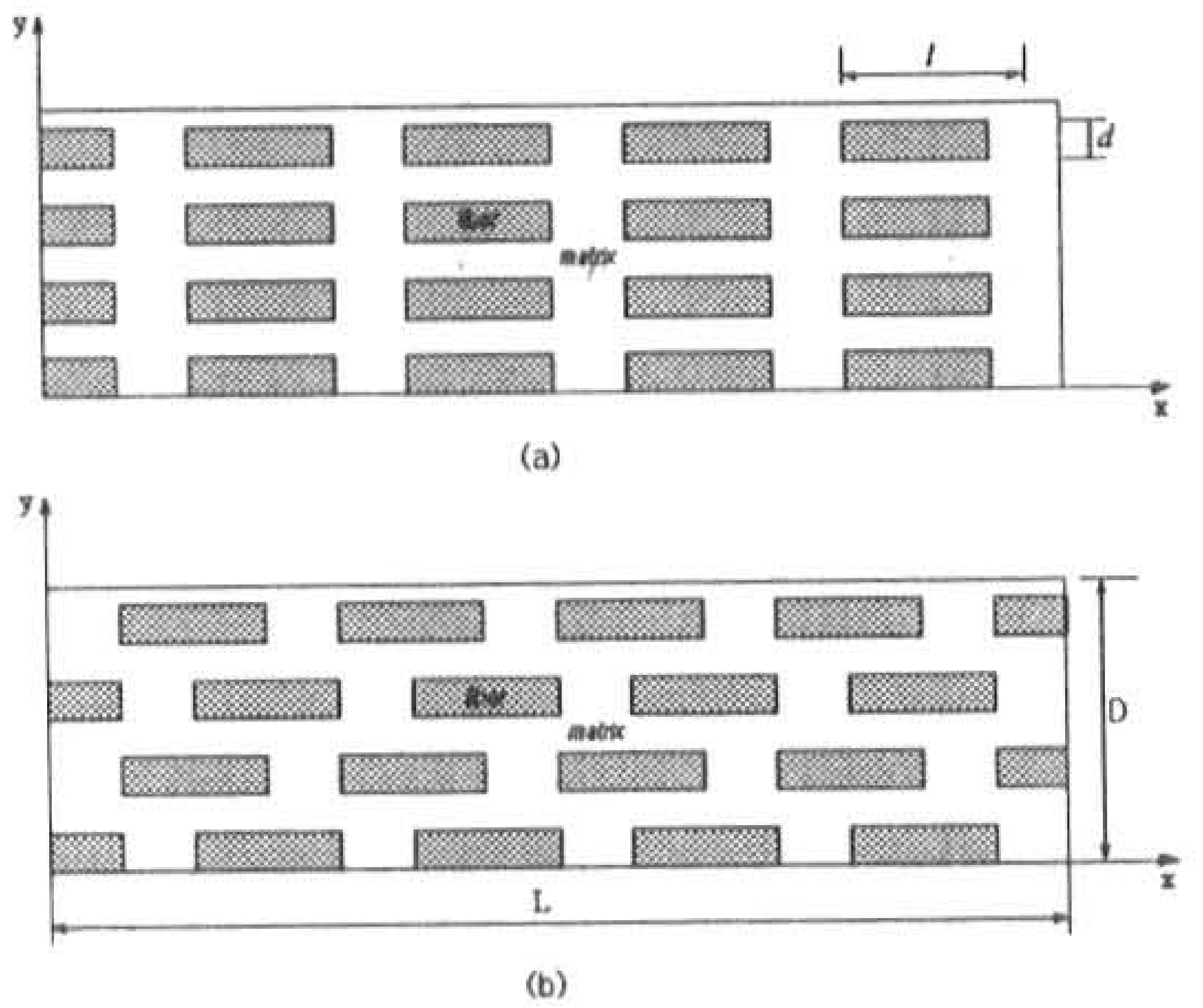

Fig.1 Two-dimensional multi-fiber model for short-fiber reinforced composites. (a)aligned array, (b)staggered array

strength is determined at 5% strain and 85MPa for matrix.

The FE formulations in 2-D multiple fiber model are under the condition of plane strain with boundary conditions of $dx=0$ at $x=0$ and $dy=0$ at $y=0$ as shown in Fig.1.

The displacement controlled computations are performed by the general purpose FE program, ANSYS 5.0. To obtain tensile stress-strain behavior numerically, the applied far field tensile strain(ε_c) is loaded from 0% to 5% stepwise. Each step has 20 small substeps and iteration of substeps are carried out until satisfying the permit error of 10^{-6}. To solve nonlinearity, Newton-Raphsons method has been employed. The model is based on incremental plasticity theory using von Mises yield criterion, Prandtl-Reuss equations and isotropic hardening rule. The strains here are assumed to develop instantaneously, that is, independent of time. The geometrical volume changes has been permitted as increase the strain. Both the effective modulus(E_c) and tensile strength of short-fiber composite can be obtained in this FE analysis as

$$[\sigma]_c = \frac{\int_n [\sigma_c]_n dV_n}{\int_n dV_n} \quad \text{and} \quad [\sigma]_c = [E]_c[\varepsilon]_c \tag{5}$$

where $[\sigma]_c$ represents the average stress for 2-D multi-fiber model, $[\sigma_c]n$ represents the stress in nth element, dv_n is the volume of nth element and $[E]_c$ is the effective modulus of short-fiber reinforced composites. The ultimate tensile strength is assumed as the stress at 5% strain of composite.

4. EXPERIMENTAL METHODS

The PEEK/carbon fiber composite specimens are made of 10, 20, 30 and 40 weight % fiber with fiber length of 1.0 mm manufactured by RTP Corp., Winone, MN. U.S.A. for the present study. The ASTM D638 tensile specimens with 2 mm thickness are used to measure the tensile properties of short-fiber composites. Strain gages are chosen to measure specimen modulus. The open-faced general purpose strain gage and M-bond 200 kit from Micro-measurement Group Inc. are used in this study. The 2100 multichannel system by measurement Group Inc. is used to measure strain with a bridge voltage of 1 volt. Both types of specimen are molded on an ARBURG 221E/150 injection molding machine.

Since the mechanical properties of the semicrystalline matrix material depends on microstructural details such as spherulites size, orientation, degree of crystallinity, and degree of transcrystallinity that depend on the processing conditions, same molding conditions were kept for all samples. All tests are done using an Instron universal testing machine in room temperature and relative humidity of 50%. A testing speed is 1.25 mm/min as a recommended testing speed for many polymeric materials. Typically, five specimens are used within a single evaluation.

5. RESULTS AND DISCUSSIONS

5. 1 Stress-strain Response

Figure 2a and 2b represent the numerically predicted stress-strain curves for multi-fiber models up to 5% of composite strain with fiber volume fraction of 10 % and modulus ratio(E_f/E_m) of 100. As shown in Figs.2, the aligned fiber array shows the higher flow stress and strain hardening than the staggered fiber array. And, the differences between aligned and staggered fiber models increase with the fiber aspect ratio(AR). This is generally agreed with the results of Tvergaard(1990) who studied the tensile properties of the whisker reinforced MMC (Metal Matrix Composites) using a micro-mechanical model.

As shown in figure, tangent modulus of aligned array increases with aspect ratio in the range of 2 - 20. But staggered array do not proportionally increase with aspect ratio. For the composite with 30 % V_f compared with 10 % V_f, the ultimate strength of staggered array do not increase with the V_f as the aligned array does. This means that the reinforcing effects by fiber of staggered array are less than those by aligned array as the V_f increases.

There are two basic differences between two fiber models. First, the fiber ends overlap for

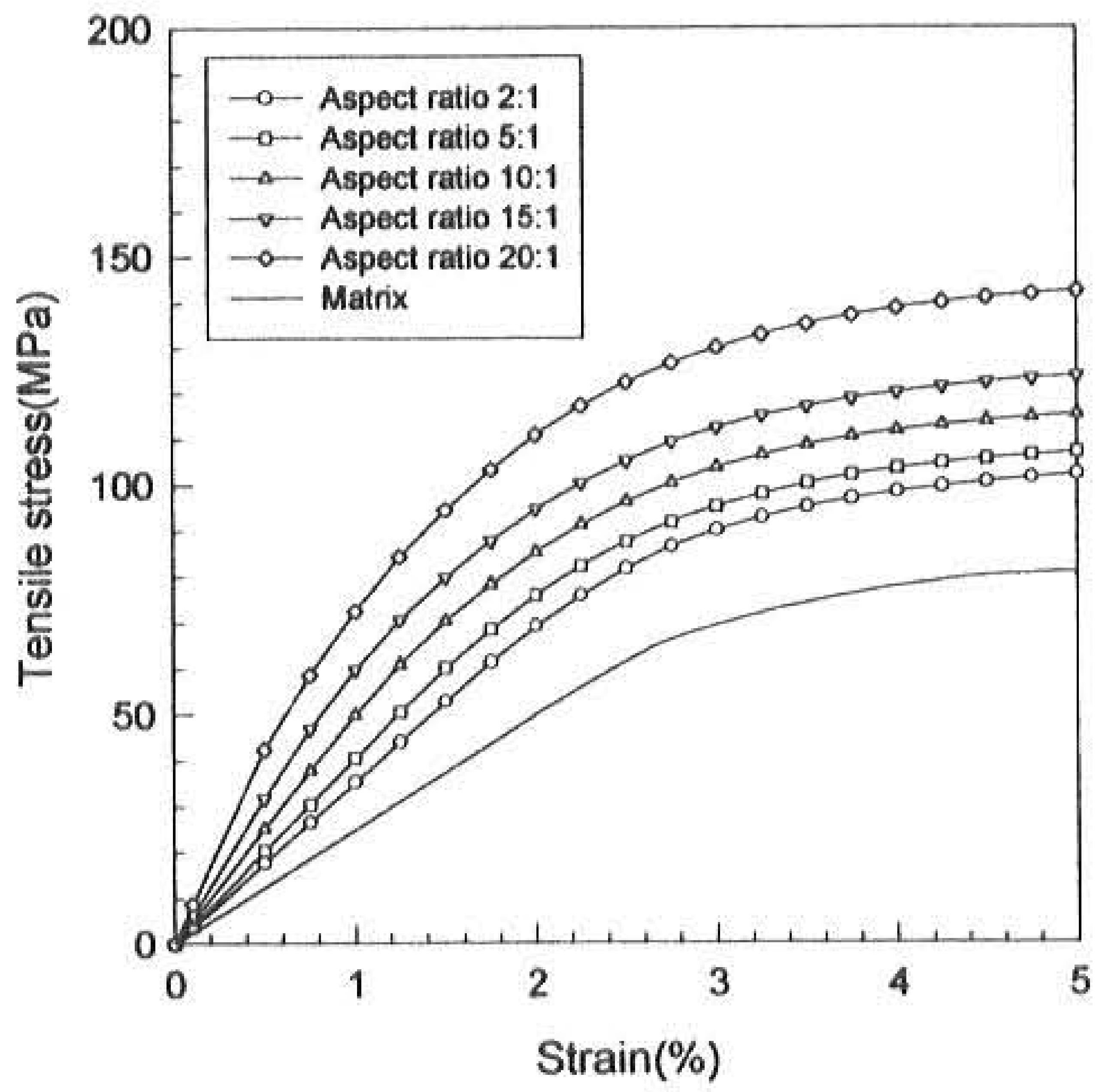

Fig.2a Numerically predicted stress-strain curves for multi-fiber model with aligned array, (V_f=10%, E_f/E_m=100).

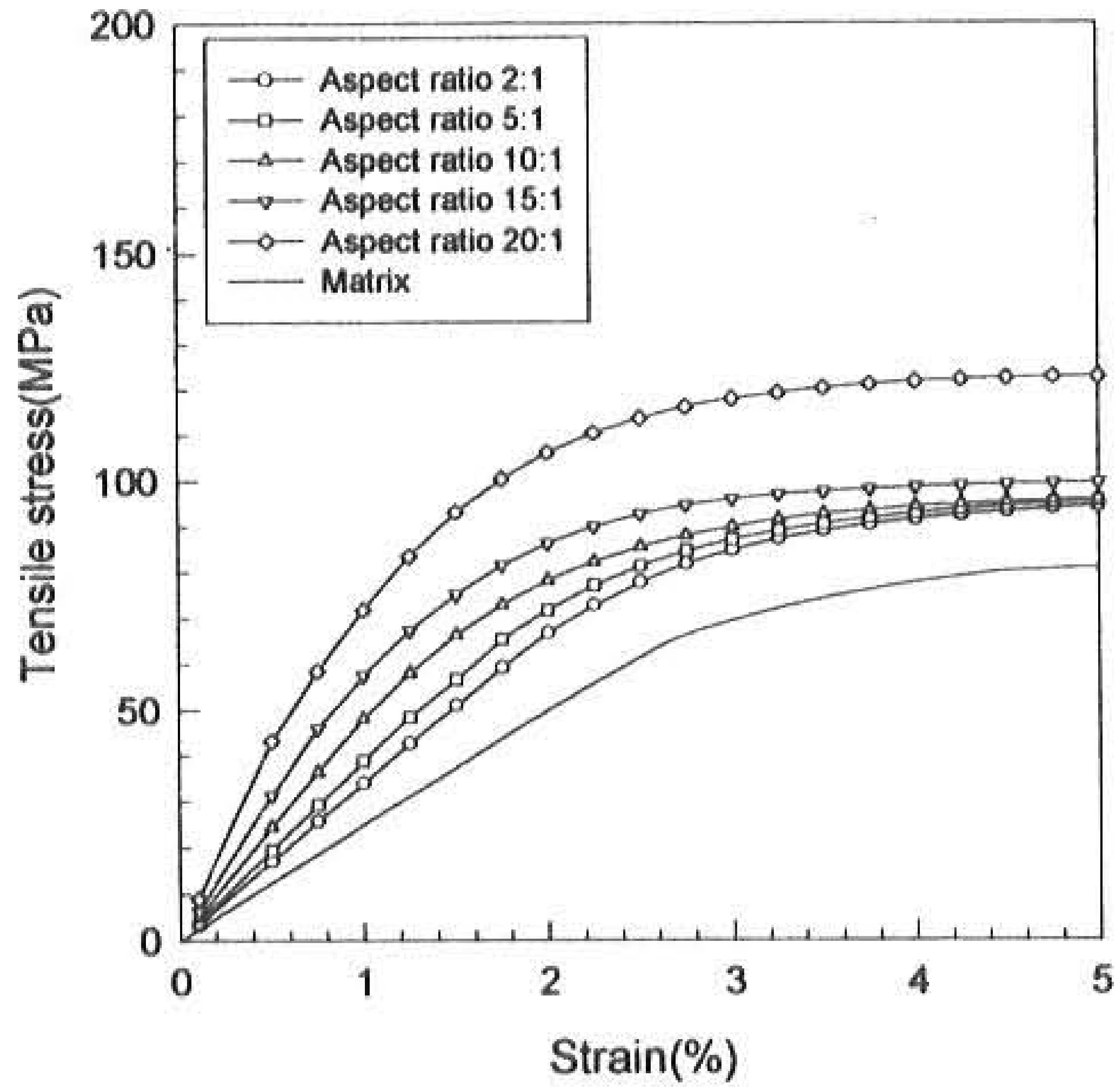

Fig.2b Numerically predicted stress-strain curves for multi-fiber model with staggered array, (V_f=10%, E_f/E_m=100).

higher fiber aspect ratios. Second, the half cell with half fiber is located in the one side of aligned fiber array instead of both sides for staggered array. As can be seen by comparison to figures, differences between the predicted stress-strain curves for two geometries with different fiber arrangements are quite significant in this case. When the load is applied in one side, the fibers can transfer the load from the beginning in the

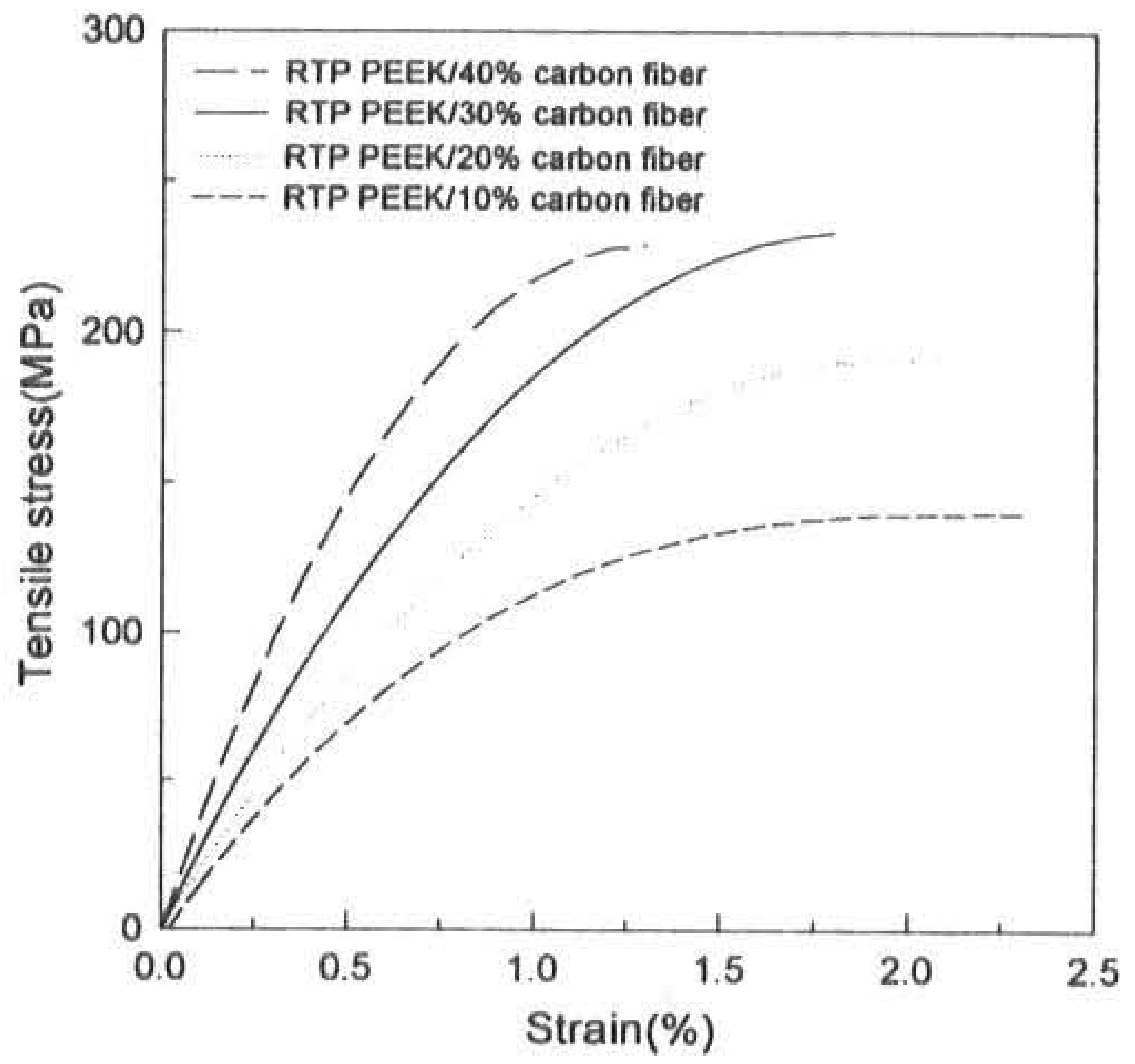

Fig.3 Tensile stress-strain curves of PEEK /carbon fiber composite.

case of staggered fiber array.

The elastic tensile modulus and ultimate tensile strength(S_u) are increase with aspect ratio even though the linear elastic region in stress-strain curves decrease. This can be assumed that plastics deformation in matrix is increasing as aspect ratio increase with the same composites strain(ε_c). Values of elastic modulus and S_u are higher for aligned fiber array (AFA) than staggered fiber array(SFA). Fig.3 shows the stress-strain behavior of PEEK/carbon fiber composites. Fig.4 shows the numerically predicted S_u as a function of aspect ratio with different V_f.

5. 2 Effective Tensile Modulus

Based on the previously discussed models, Fig.5 compares the results of this finite element method(FEM) analysis with the calculated results of Christensen-Waals[10] and Tsai-Pagano [9] as a function of effective modulus ratio(E_c/E_m) and V_f. As shown in figure, FEM results agree well with Christensen-Waals prediction as aspect

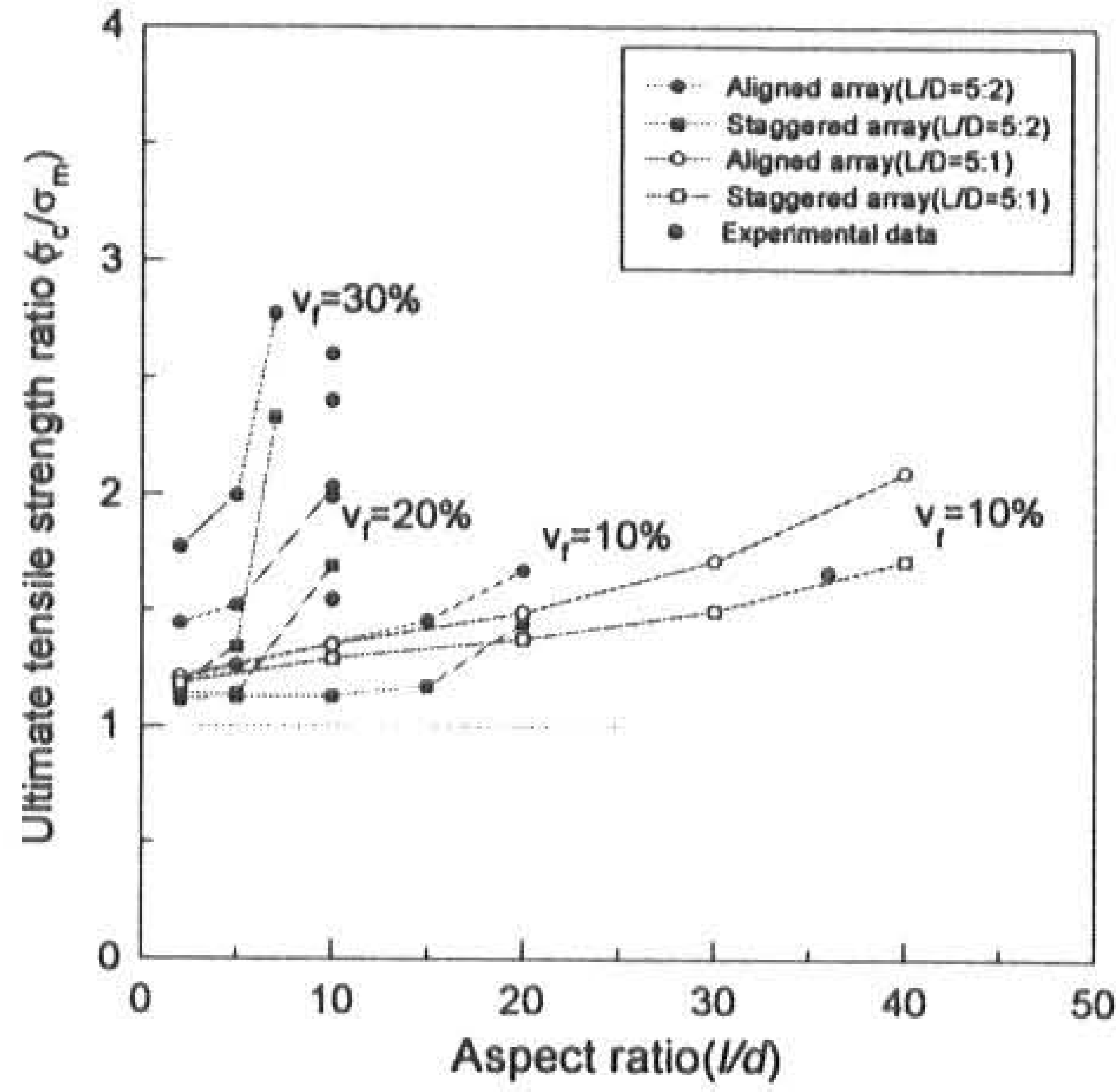

Fig.4 Ultimate tensile strength ratio of short-fiber composites as a function of fiber volume fraction versus aspect ratio(l/d), (E_f/E_m=100).

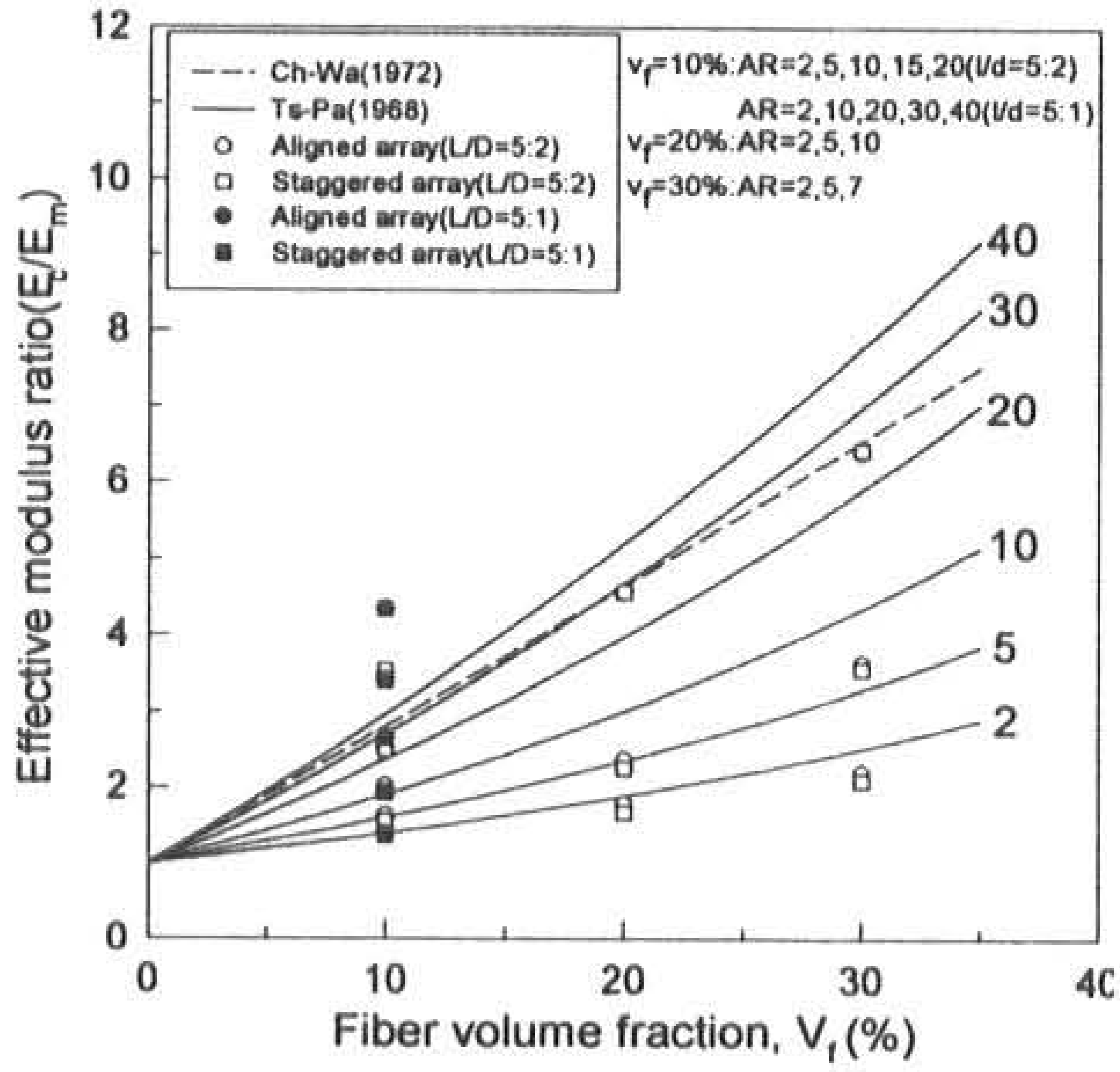

Fig.5 Effective modulus ratio of short-fiber reinforced composites as a function of fiber volume fraction.

ratio increase. Also, FEM results show good agreement with Tsai-Paganos prediction especially at low aspect ratio.

Fig.6 shows the calculated effective modulus according to Hwang and Gibson[6] as a function of aspect ratio with different V_f. Our FEM results agree well with their prediction at lower aspect ratio up to 10. But the difference increase with aspect ratio. Considering the modified Cox model with single fiber, this gap is stemmed from the absence of fiber/matrix interactions of the modified Cox model and can be a limitation. Compared with the experimental results, FEM results show a good agreement at low V_f.

Curtis et al.[14] has measured the stiffness and strength of an injection molded polyamide thermoplastic. Six series of molding compounds reinforced with glass and carbon fibers are used. As shown in Fig.6, FEM results also agree well with their experimental data.

It can be clearly seen that increasing the fiber aspect ratio increases the composite elastic modulus, proportional limit, and tangent modulus.

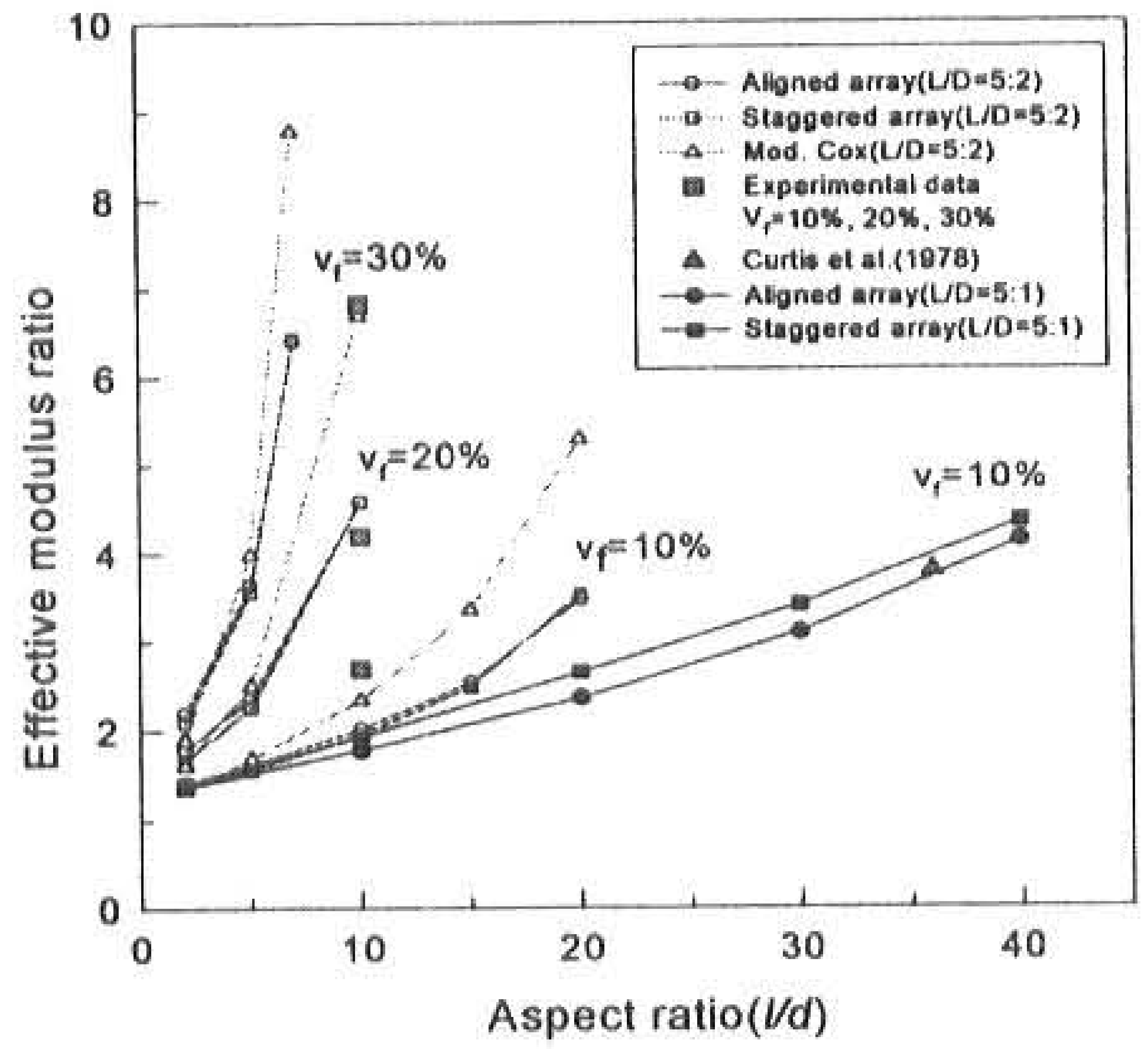

Fig.6 Effective modulus ratio(E_c/E_m) of short-fiber composites as a function of fiber volume fraction versus aspect ratio(l/d), (E_f/E_m=100).

5. 3 Stress Distribution

Fig.7 shows the axial stress distribution for AFA and SFA with aspect ratio(l/d) of 2 and strain at 0.5%(elastic). At low strain (0.5%) with (D/L)=2:5, AFA case has a higher stress in the matrix and interface due to the fiber/matrix interactions. This suggests that AFA case can have a better stress transfer than SFA case. Fig.7 shows the higher stresses and maximum stresses in the matrix is low at high strain(3%). Also, SFA case has much lower stress distribution than AFA case. This gives a lower strain hardening and diminishing tangent modulus. In the elastic region, there is a little difference in the matrix axial stress between SFA and AFA as expected. On the other hand, in the elastoplastic region, the axial matrix stress adjacent of the interface for the AFA case is much lower than that for the SFA case.

Fig.8 shows the distribution of hydrostatic stress with (D/L)=1:5. Both aligned and staggered array models have a higher local hydrostatic stress in the matrix as aspect ratio increase at given composites strain. Here, aligned array also shows ahigher hydrostatic stress than staggered array. In general, the hydrostatic stresses in the matrix adjacent to the fiber/matrix interface is much higher for the SFA whereas, in the end gap region, those are much higher for the AFA. This suggests that failure mechanisms can similar when aspect ratio is low(AR=2), but those can be significantly different as aspect ratio(AR>10) increase.

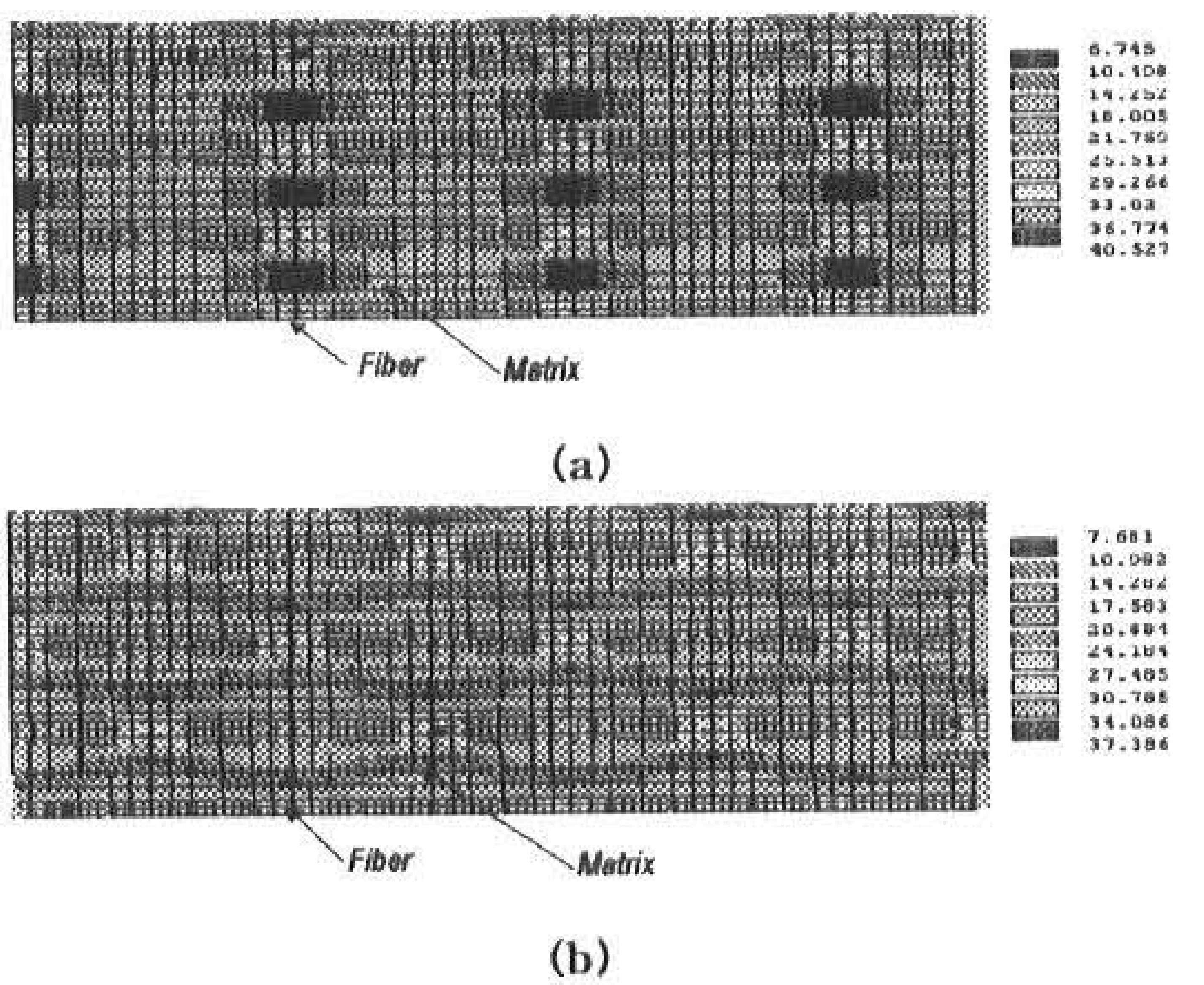

Fig.7 Contour of axial stress for (a)aligned and (b)staggered fiber array with aspect ratio(l/d) of 2:1, (V_f=10%, ε_c=0.5%).

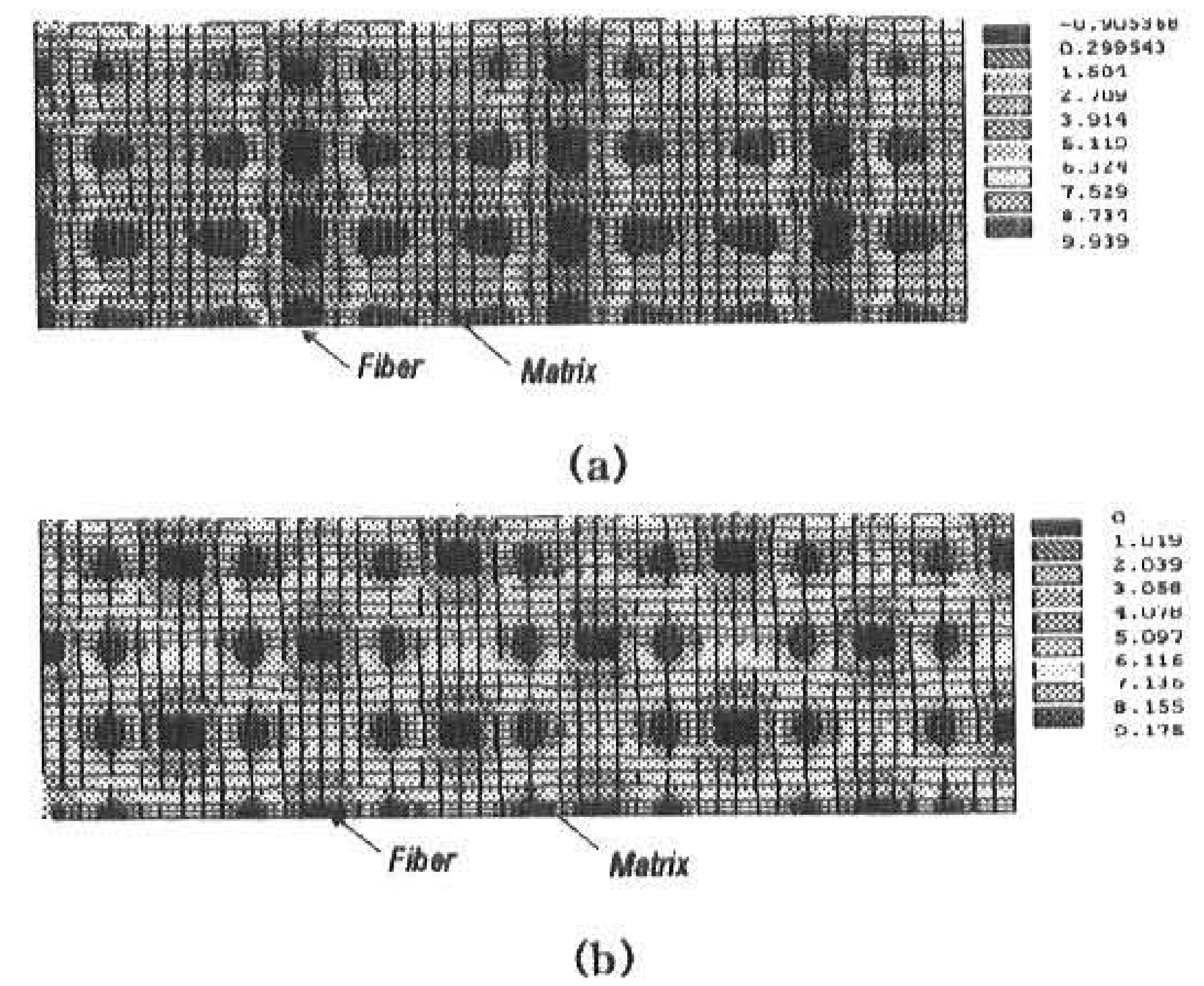

Fig.8 Contour of hydrostatic stress for (a)aligned and (b)staggered fiber array with aspect ratio(l/d) of 2:1, (V_f=10%, ε_c=0.5%).

6. CONCLUSIONS

Both effective tensile modulus and ultimate tensile strength of short-fiber reinforced composite were predicted using 2-D multi-fiber composites with aligned and staggered fiber models. Elastic analysis accurately predicts the composite stiffness, as well as the increase in stiffness with increase fiber volume content and fiber aspect ratio. Elastoplastic analysis generally predicts well the overall nonlinear composite stress-strain response and provides a detailed local history of the initiation and propagation of plastic deformation and of the presence of unique states of stress.

It is found that aligned array shows the better load transfer than staggered array by having higher effective modulus and ultimate strength.

Also, aspect ratio shows a significant reinforcing effects in general, but small influence on strain hardening and flow stress in the case of staggered fiber geometry.

By comparing with various theories and FEM results and by analyzing the stress distribution of composite models, not only the interactions but also stress transfer between fiber and matrix have a significant influence on the composite modulus and strength.

Also, multi - fiber model of short - fiber composite can predict the mechanical properties more accurately than single fiber model. The magnitude and spatial variation of hydrostatic stresses were significantly different between aligned and staggered fiber geometries and their difference increase with aspect ratio.

This suggests that the fracture mechanisms would be quite different type for the different arrays and aspect ratios.

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(1999년 12월20일 접수, 2000년 2월25일 채택)