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Elastic Critical Load and Effective Length Factors of Continuous Compression Member by Beam Analogy Method

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Abstract

The critical load of a continuous compression member was determined by the beam-analogy method. The proposed method utilizes the stress-analysis results of the analogous continuous beam, where imaginary concentrated lateral load changing its direction is applied at each midspan. The proposed method gives a lower bound error of critical load and can predict the span that buckles first. The effective length factors for braced frame columns can be easily determined by the present method, but result in the upper bound errors in all cases, which can lead to a conservative structural design

Keywords : beam analogy method, slope-deflection method, Kinney's fixity factor, modified slope-deflection method, effective length factor, finite element method

1. INTRODUCTION

In the structural design of a beam-column, the effect of axial compressive force, P, is included by multiplying the factor $1/(1-P/P_{cr})$ with the beam moment and deflection. In the case of a single span beam-column, the elastic critical load, P_{cr} is easily determined, whether the sectional property of that member is constant or variable along its axis. For a multi-span beam-column, however, the conventional neutral equilibrium method, or energy principle-based Rayleigh-Ritz method cannot be efficiently applied to the determination of critical load. In this case, modified slope-deflection method or a numerical method for example, the finite difference or the finite element method, becomes a useful tool for the determination of critical load or the stress analysis of a continuous beam-column.

In this paper, the *Beam-analogy method* is proposed to determine the approximate critical loads of continuous compression members. The main idea of the beam analogy method is to replace the continuous compression member by a continuous beam, to which concentrated lateral loads are applied at each mid-span of the multi-span beam. The mid-span concentrated loads are made to change their directions in order to simulate the buckling mode. The results of stress analysis of the beam are used to calculate Kinney's[3] fixity factors. Finally, the critical load is expressed by fixity factors. The *Beam analogy method* may also be applied to the determination of effective length factors of braced frame columns.

2. BEAM ANALOGY METHOD

The *Beam analogy method* can be explained with the following simple example. Fig. 1(a) shows a 2-span continuous compression member whose critical load is to be determined by the proposed method. *The first step* of the proposed method is to replace the given compression member with a continuous beam. Fig. 1(b) shows the analogous beam, where the direction of an imaginary concentrated lateral load, Q, at each mid-span is alternating its direction to simulate the buckling mode.

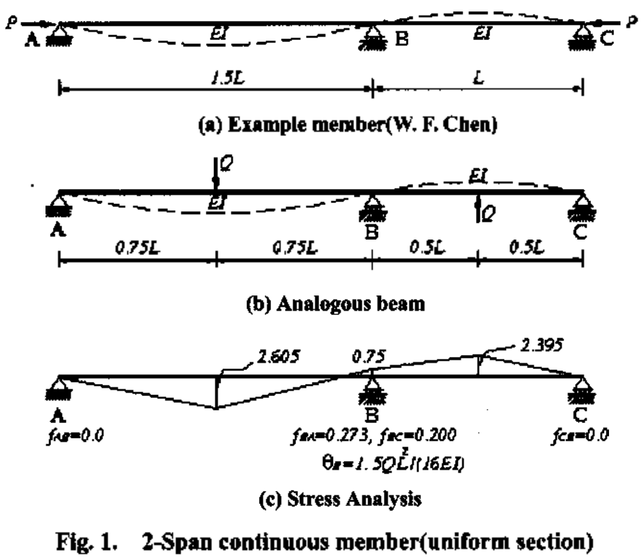


Fig. 1. 2-Span continuous member(uniform section)

The second step is the stress analysis of the analogous beam(Fig.1(b)). In the present example, the usual slope-deflection method may be conveniently applied to obtain end moments and rotation angles at the supports. Fig. 1(c) shows the stress analysis results.

The third step of the proposed method is to find Kinney's fixity factors[3] by utilizing the following relationship.

$$\left|M_{\alpha\beta}\right| = \left(\frac{4EI}{L}\right)_{\alpha\beta} \cdot \frac{f_{\alpha\beta}}{f'_{\alpha\beta}} \cdot \left|\theta_\alpha\right|,$$

$$f'_{\alpha\beta} = 1 - f_{\alpha\beta} \quad (1)$$

Where | | denotes the absolute value. The above relationship was introduced by the author[1] to the eigenvalue problems of the tapered bars. He also applied Eq. 1 to the analysis of single span steel beams[2] with partially fixed ends. In Fig.1(c), $f_{AB} = f_{CB} = 0.0$ by Kinney's definition (simply supported ends). When Eq.1 is applied to the first span ***AB*** and to the second span ***BC*** at the intermediate support ***B***, f_{BA} and f_{BC} are determined in the following way.

$$\frac{0.75QL}{8} = \frac{4EI}{1.5L}\cdot\frac{f_{BA}}{1-f_{BA}}\cdot\frac{1.5QL^2}{16EI}$$

$$\therefore f_{BA} = \frac{0.75}{2.75} \approx 0.273 \quad (2\text{-a})$$

$$\frac{0.75QL}{8} = \frac{4EI}{L}\cdot\frac{f_{BC}}{1-f_{BC}}\cdot\frac{1.5QL^2}{16EI}$$

$$\therefore f_{BC} = \frac{0.75}{3.75} = 0.200 \quad (2\text{-b})$$

The final step is to determine the critical load or effective length factor (***K***-factor) using the following expressions.

$$(P_{cr})_{\alpha\beta} = (1+f_{\alpha\beta})(1+f_{\beta\alpha})\pi^2\left(\frac{EI}{L^2}\right)_{\alpha\beta} \quad (3)$$

$$= \pi^2\cdot\left(\frac{EI}{K^2L^2}\right)_{\alpha\beta}$$

where ***K***-factor is defined by

$$K = \{(1+f_{\alpha\beta})\cdot(1+f_{\beta\alpha})\}^{-0.5} \quad (4)$$

The validity of Eq. 3 can be easily demonstrated ; That is, when a single prismatic member is simply supported at its both ends, then $f_{\alpha\beta} = f_{\beta\alpha} = 0.0$, by definition. With $f_{\alpha\beta} = f_{\beta\alpha} = 0.0$, Eq. 3 yields $P_{cr}=\pi^2EI/L^2$ ($K=1.0$).

In the same way, $P_{cr}=2\pi^2EI/L^2$ is obtained for the member with one end simple supported($f_{\alpha\beta}=0.0$) and the other fixed ($f_{\beta\alpha}=1.0$). Finally, the critical load of the member fixed at both its ends ($f_{\alpha\beta} = f_{\beta\alpha} = 1.0$) is expressed by $4\pi^2EI/L^2$ ($K=0.5$).

With the fixity factors of Fig. 1(c), Eq. 3 and Eq. 4 give the following results :

(Span ***AB***)

$$(P_{cr})_{AB} = (1.0)(1.273)\pi^2\frac{EI}{(1.5L)^2}$$

$$= 5.584\frac{EI}{L^2} \quad (5\text{-a})$$

$$K_{AB} = 1/\sqrt{(1\times1.273)} \approx 0.886 \quad (6\text{-a})$$

(Span ***BC***)

$$(P_{cr})_{BC} = (1.2)(1.00)\pi^2\frac{EI}{(L)^2}$$

$$= 11.844\frac{EI}{L^2} > (P_{cr})_{AB}, \quad (5\text{-b})$$

$$K_{BC} = 1/\sqrt{(1\times1.273)} \approx 0.913 \quad (6\text{-b})$$

As mentioned earlier, Fig. 1(a) is the example member chosen by Chen[4]. He applied the *Neutral equilibrium method* to obtain, $P_{cr}=5.89EI/L^2$, which is approximately 5.2% larger than the result given by Eq. 5(a).

Now above results are to be compared with those by two conventional methods.

(Modified slope-deflection method)

Firstly, the *Modified slope-deflection method* (M.S.D.M)[5] will be briefly described. When the axial force, ***P***, is considered, the end moments and rotation angles of a beam-column are related by the following :

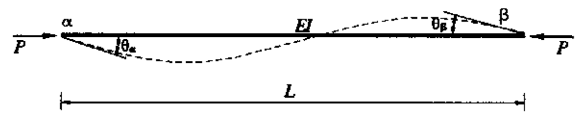


Fig. 2. Deformation of beam-column

$$M_{\alpha\beta} = \left(\frac{EI}{L}\right)_{\alpha\beta}\cdot(\alpha_n\theta_\alpha + \alpha_f\theta_\beta) \quad (7\text{-a})$$

$$M_{\beta\alpha} = \left(\frac{EI}{L}\right)_{\alpha\beta}\cdot(\alpha_f\theta_\alpha + \alpha_n\theta_\beta) \quad (7\text{-b})$$

First, these equations are applied successively to Fig. 1(a). Next, the moment equilibrium conditions at the external and intermediate supports allows one to obtain the following matrix equation:

$$\begin{bmatrix} \alpha_{n1}/1.5 & \alpha_{f1}/1.5 & 0 \\ \alpha_{f1}/1.5 & (\alpha_{n1}/1.5+\alpha_n) & \alpha_f \\ 0 & \alpha_f & \alpha_n \end{bmatrix}\cdot\begin{Bmatrix}\theta_A\\ \theta_B\\ \theta_C\end{Bmatrix} = \begin{Bmatrix}0\\0\\0\end{Bmatrix} \quad (8)$$

Where α_n and α_f are so-called Merchant's stability functions defined by

$$\alpha_n = \frac{\phi_n}{\phi_n^2-\phi_f^2}, \quad \alpha_f = \frac{\phi_f}{\phi_n^2-\phi_f^2} \quad (9\text{-a,b})$$

with

$$\phi_n = \frac{1}{(kL)^2}(1-kL\cot kL) \quad (10\text{-a})$$

$$\phi_f = \frac{1}{(kL)^2}(kL\csc kL - 1) \quad (10\text{-b})$$

$$kL = \sqrt{PL^2/EI} \quad (10\text{-c})$$

α_{n1} and α_{f1} are Eq. 9 with $k_1L=1.5kL$

The characteristic equation for critical load is obtained from Eq. 8. That is :

$$\det\begin{bmatrix} \alpha_{n1}/1.5 & \alpha_{f1}/1.5 & 0 \\ \alpha_{f1}/1.5 & (\alpha_{n1}/1.5+\alpha_n) & \alpha_f \\ 0 & \alpha_f & \alpha_n \end{bmatrix} = 0 \quad (11\text{-a})$$

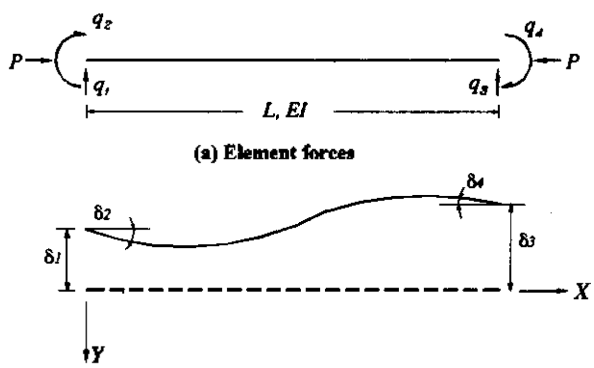
when expanded

$$\frac{\alpha_{n1}}{1.5}\left(\frac{\alpha_{n1}}{1.5}+\alpha_n\right)\alpha_n-\left(\frac{\alpha_{f1}}{1.5}\right)^2\alpha_n-\frac{\alpha_{n1}}{1.5}\alpha_f^2=0 \qquad (11\text{-b})$$

The least root satisfying Eq. 11(b) is found by a *trial and error procedure*, which yields $kL=2.4265$ and $P_{cr}=5.888EI/L^2$ by Eq. 10(c). This critical load coincides with that of above mentioned result of Chen.

(Finite element method)

Frequently used *finite element method*(**F.E.M**)(5)(6)(7) will also be briefly described. Fig. 3 shows an element of a beam-column subjected to constant axial force, P and a set of



(a) Element forces

(b) Element displacements

Fig. 3. Beam-column element

nodal forces $\{q\}$. In Fig. 3, $\{\delta\}$ denotes nodal displacement vector corresponding to $\{q\}$. The element stiffness matrix, $[k]$ combines $\{q\}$ and $\{\delta\}$ in the following form.

$$\{q\}=[k]\{\delta\}, \quad [k]=[k]_b-P[k]_g \qquad (12)$$

in which

$$[k]_b=\frac{EI}{l^3}\begin{bmatrix} 12 & & \text{Symm.} & \\ -6l & 4l^2 & & \\ -12 & 6l & 12 & \\ -6l & 2l^2 & 6l & 8l^2 \end{bmatrix} \qquad (13\text{-a})$$

and

$$[k]_g=\frac{1}{15l}\begin{bmatrix} 18 & & \text{Symm.} & \\ -1.5l & 2l^2 & & \\ -18 & 1.5l & 18 & \\ -1.5l & 0.5l^2 & 1.5l & 2l^2 \end{bmatrix} \qquad (13\text{-b})$$

In the above two equations, $[k]_b$ denotes the flexural(or bending) stiffness matrix and $[k]_g$ is called geometric(or initial stress) stiffness matrix. As one could see in Eq. 12, flexural stiffness is decreased, due to the axial compressive force, P.

The structural stiffness matrices for the entire member are obtained by transforming the individual element matrices from element to structural coordinates and then by assembling the resulting matrices. Finally, boundary conditions should be applied to the assembled matrices. The procedures are easily found from textbooks on the finite element method or structural stability(5)(6)(7) ; hence, a detailed explanation will be omitted.

The external force vector, $\{Q\}$ and corresponding displacement vector, $\{\Delta\}$ are related by

$$\{Q\}=[K]\{\Delta\} \qquad (14)$$

where $[K]$, the structure stiffness matrix obtained after the application of boundary conditions, takes the form

$$[K]=[K]_b-P[K]_g \qquad (15)$$

With external force vector, $\{Q\}=\{0\}$, Eq.(14) and (15) constitute a typical eigenvalue problem.

$$([K]_b-P[K]_g)\{\Delta\}=\{0\} \qquad (16)$$

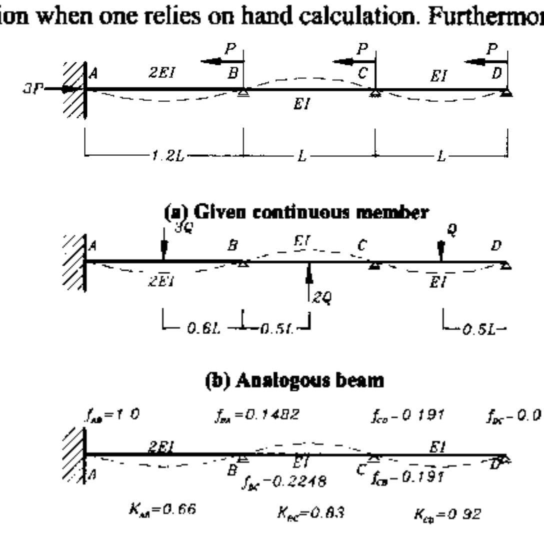
To obtain the least eigenvalue (here, the elastic critical load corresponding to the first mode of buckling) by computer-aided iteration method, Eq.(16) should be transformed into the following form

$$([K]_b^{-1}[K]_g-\frac{1}{P}[I])\{\Delta\}=\{0\} \qquad (17)$$

where $[I]$ is the unit(or identity) matrix.

When the *finite element method* is applied to Fig. 1(a) one can obtain nearly the same critical load as that by the modified slope-deflection method. Here, it is observed that the representative two methods, (the one, an analytical method or modified slope-deflection method and the other, a numerical or finite element method) can result in the same value of the elastic critical load.

The procedures necessary to obtain the final result by above mentioned methods involve complicated calculation when one relies on hand calculation. Furthermore,



(a) Given continuous member

(b) Analogous beam

(c) Fixity factors and K-factors

Fig. 4. 3-Span continuous member

these methods can neither predict the span that buckles first, nor determine the effective length factor, ***K***, of each span.

For a clearer explanation of the proposed method, another example of 3-span continuous compression member is adopted. Fig.4 (b) shows the first step of proposed method. The second step, that is, the stress analysis of the analogous beam shown in Fig.4(b) can proceed in the following ways(the slope-deflection method is adopted).

(End moment equations)

$$M_{AB} = \frac{2EI}{1.2L}(2\theta_B) - \frac{3.6QL}{8} = -5.7792(\frac{QL}{8}) \quad (18\text{-a})$$

$$M_{BC} = \frac{2EI}{L}(2\theta_B + \theta_C) + \frac{2QL}{8} = +0.7585(\frac{QL}{8}) \quad (18\text{-b})$$

$$M_{CB} = \frac{2EI}{L}(\theta_B + 2\theta_C) - \frac{2QL}{8} = -0.5604(\frac{QL}{8}) \quad (18\text{-c})$$

$$M_{CD} = \frac{2EI}{L}(2\theta_C + \theta_D) - \frac{QL}{8} = +0.5604(\frac{QL}{8}) \quad (18\text{-d})$$

$$M_{DC} = \frac{2EI}{L}(\theta_C + 2\theta_D) + \frac{QL}{8} = 0.0 \quad (18\text{-e})$$

(Joint equation)

$$\sum M_B = 0, \quad \frac{6.4}{1.2}\theta_B + \theta_C = -\frac{5.6QL}{8}\cdot\frac{L}{2EI} \quad (19\text{-a})$$

$$\sum M_C = 0, \quad \theta_B + 4\theta_C + \theta_D = \frac{3QL}{8}\cdot\frac{L}{2EI} \quad (19\text{-b})$$

$$\sum M_D = 0, \quad \theta_C + 2\theta_D = -\frac{QL}{8}\cdot\frac{L}{2EI} \quad (19\text{-c})$$

The solution of the simultaneous equations yields the following rotation angles.

$$\theta_B = -69.3(\frac{QL^2}{848EI}) \quad (20\text{-a})$$

$$\theta_C = 72.8(\frac{QL^2}{848EI}) \quad (20\text{-b})$$

$$\theta_D = -69.2(\frac{QL^2}{848EI}) \quad (20\text{-c})$$

(End moment)

Above rotation angles are substituted into slope-deflection equations to obtain the results shown at the right-hand sides of the same equation.

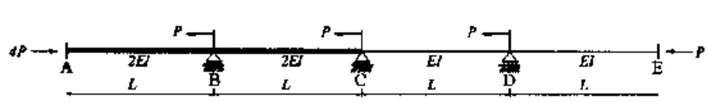
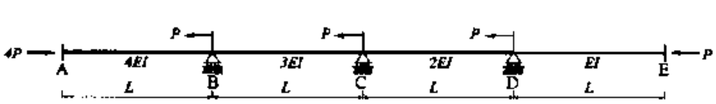
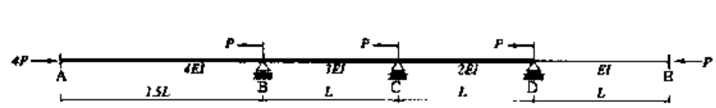
The third step is the determination of fixity factors by using Eq.(1). In the present problem

$$\frac{0.7585QL}{8} = \frac{8EI}{1.2L}\cdot\frac{f_{BA}}{1-f_{BA}}\cdot\frac{69.3QL^3}{848EI}$$

$$\therefore \quad f_{BA} = 0.1482 \quad (21\text{-a})$$

$$\frac{0.7585QL}{8} = \frac{4EI}{L}\cdot\frac{f_{BC}}{1-f_{BC}}\cdot\frac{69.3QL^3}{848EI}$$

Table. 1. Critical load coefficient, $C(P_{cr}=CEI/L^2)$- *Reference values are the M.S.D.M results.*

Loading Conditions	B.C	M.S.D.M	R.E.M	R.A.M	Error (%)
	f_A=0.0 f_E=0.0	9.8696	9.8696	9.8689 (ALL)	0.0
	f_A=0.0 f_E=0.0	10.6221	10.6223	10.6221 (DE)	5.6
	f_A=0.0 f_E=0.0	12.7800	12.7801	12.7801 (BC,CD)	4.9
	f_A=0.0 f_E=0.0	5.6468	5.6469	5.3889 (AB)	4.6
	f_A=0.0 f_E=0.0	5.7209	5.7210	5.4226 (AB)	5.2
	f_A=0.0 f_E=0.0	7.1344	7.1344	6.6068 (CD)	7.4
	f_A=0.0 f_E=0.0	7.5874	7.5877	7.2198 (CD)	4.8
	f_A=0.0 f_E=0.0	9.8696	9.8689	9.8689 (ALL)	0.0
	f_A=0.0 f_E=0.0	10.2130	10.2131	10.0142 (AB)	1.9
	f_A=0.0 f_E=0.0	11.2438	11.2439	10.1393 (DE)	9.8
	f_A=0.0 f_E=0.0	12.6131	12.6132	11.7864 (CD)	6.6
	f_A=0.0 f_E=0.0	5.8586	5.8583	5.4897 (AB)	6.3
	f_A=0.0 f_E=0.0	5.8748	5.8746	5.5110 (AB)	6.2
	f_A=0.0 f_E=0.0	9.4846	9.4843	9.2461 (AB)	2.5
	f_A=0.0 f_E=0.0	9.7375	9.7373	9.3270 (AB)	4.2

$$\therefore \quad f_{BC} = 0.2248 \tag{21-b}$$

$$\frac{0.5604QL}{8} = \frac{4EI}{L} \cdot \frac{f_{CB}}{1-f_{CB}} \cdot \frac{62.9QL^3}{848EI}$$

$$\therefore \quad f_{CB} = f_{CD} = 0.1910 \tag{21-c}$$

The fixity factors are given in Fig. 4(c). The last step is to find the ***K***-factor and the critical local of each span by applying Eq.(3) and (4).

Span ***AB***:

$$K_{AB} = \{(1+1)(1+0.1482)\}^{-0.5} \approx 0.66$$

$$(3P_{cr})_{AB} = 2 \times 1.1482 \frac{\pi^2 (2EI)}{(1.2L)^2}$$

$$\therefore (P_{cr})_{AB} \approx 10.493 \frac{EI}{L^2} \tag{22-a}$$

Span ***BC***:

$$K_{BC} = \{(1.2248)(1.191)\}^{-0.5} \approx 0.83$$

$$(2P_{cr})_{BC} = 1.2248 \times 1.191 \frac{\pi^2 EI}{L^2}$$

$$\therefore (P_{cr})_{BC} \approx 7.198 \frac{EI}{L^2} \tag{22-b}$$

Span ***CD***:

$$K_{CD} = \{(1.191)(1)\}^{-0.5} \approx 0.92$$

$$(P_{cr})_{CD} = 1.191 \frac{\pi^2 EI}{L^2} \approx 11.754 \frac{EI}{L^2} \tag{22-c}$$

Eq. (22) is the final step, where one can easily see that the member of span ***BC*** may buckle under the magnitude of load, $\boldsymbol{7.198EI/L^2}$.

The above-mentioned *modified slope-deflection method* is to be applied to the member of Fig.4(a). When Eq(7-a) and (7-b) are applied to this member, one obtains

$$M_{BC} = \frac{EI}{L}(\alpha_{n2}\theta_B + \alpha_{f2}\theta_c) \tag{23-a}$$

$$M_{BA} = \frac{EI}{L}(\frac{5}{3}\alpha_{n1}\theta_B) \tag{23-b}$$

$$M_{CB} = \frac{EI}{L}(\alpha_{f2}\theta_B + \alpha_{n2}\theta_c) \tag{23-c}$$

$$M_{CD} = \frac{EI}{L}(\alpha_n\theta_C + \alpha_f\theta_D) \tag{23-d}$$

$$M_{DC} = \frac{EI}{L}(\alpha_f\theta_C + \alpha_n\theta_D) \tag{23-e}$$

where α_{n1}, α_{f1}, ⋯ are Merchant's stability functions (see Eq.10) with

$$kL = \sqrt{PL^2/EI}, \quad k_1 L = \sqrt{1.5}kL, \quad k_2 L = \sqrt{2}kL$$

To obtain a characteristic equation, one can start with $M_{DC} = 0$ to obtain $\theta_D = -(\alpha_f/\alpha_n)\theta_C$. Then, the moment equilibrium at the support B,($\Sigma M_B = 0$) and C,($\Sigma M_C = 0$), give the following matrix equation

$$\begin{bmatrix} (5\alpha_{n1}/3 + \alpha_{n2}) & \alpha_{f2} \\ \alpha_{f2} & \alpha_{n2} + (\alpha_n^2 - \alpha_f^2)/\alpha_n \end{bmatrix} \begin{Bmatrix} \theta_B \\ \theta_C \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{24}$$

From the above equation, one gets the characteristic equation

$$(5\alpha_{n1}/3 + \alpha_{n2}) \cdot [\alpha_{n2} + (\alpha_n^2 - \alpha_f^2)/\alpha_n] - (\alpha_{f2})^2 = 0 \tag{25}$$

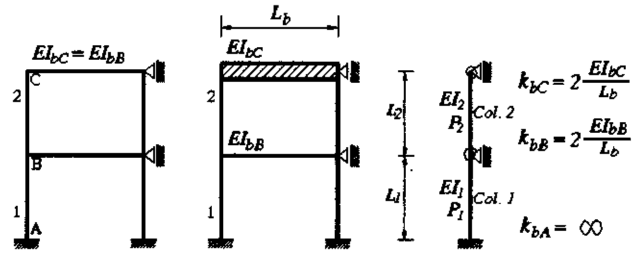
The least root satisfying the above equation is found to be ***kL=2.72787***, gives the elastic critical load

$$P_{cr} = 7.441EI/L^2$$

The proposed beam analogy method (B.A.M.) can be easily applied to multi-span continuous compression members without regard to changes in the sectional properties or the lengths of each span. Some of the critical loads of four-span continuous compression members are shown in Table 1. In this table, the boundary conditions(B.C) are either a frictionless hinge ($f_a = 0.0$) or a fixed end ($f_a = 1.0$). When compared with results by *F.E.M* or *modified slope deflection method*, the beam analogy method (B.A.M) gives lower boundary errors for critical loads. In the column "B.A.M", "**AB**" in the parenthesis denotes that the **AB** span buckles first. For example, ***(DE)*** means that span ***(DE)*** will buckle first under the given conditions.

3. *K*-FACTORS FOR BRACED COLUMNS

In the practical design of framed columns, ***K***-factor concept (rather than elastic critical load) is widely used. Several authors[11][12] have proposed different methods of ***K***-factor determination for framed columns. Until today, however, no unified method is available. Now, the proposed method can be applied to the determination of effective length factors for multi-story braced frame columns. In the braced frames of Fig. 5, columns are not permitted to move horizontally and so they can be modeled as a continuous member of Fig. 5(c).



(a) Type 1 **(b) Type 2** **(c) Idealization**

Fig. 5. Braced frames and modeling

Actually, Fig. 5 is the braced frame and its modeling example was chosen by Hellesland[8] for effective length factors. Hellesland's ***K***-factors for first and second story columns are $\boldsymbol{K_1 = 0.656}$ and $\boldsymbol{K_2 = 0.928}$, respectively, if one assumes $\boldsymbol{L_1 = L_2}$, $\boldsymbol{P_1 = 2P_2}$ in Fig. 5(c). Meanwhile, the beam analogy method applied to Fig. 5(c), gives ***K_1=0.707(error, +7.8%)*** and ***K_2=1.00(error, +7.8%)***.

Fig. 6 shows a two-story steel frame chosen by McCormac[10] for its effective length factor determination. He adopted AISC'S Alignment chart, which is based on the following equation.

$$\frac{G_\alpha \cdot G_\beta}{4}\left(\frac{\pi}{K}\right)^2+\left(\frac{G_\alpha+G_\beta}{2}\right)\left(1-\frac{\pi/K}{\tan(\pi/K)}\right)+\frac{2\tan(0.5\pi/K)}{\pi/K}=1.0 \quad (26)$$

with

$$G_\alpha=\frac{\sum_\alpha\left(\frac{EI}{L}\right)_c}{\sum_\alpha\left(\frac{EI}{L}\right)_b}=\frac{\textit{sum of column stiffness at joint } \alpha}{\textit{sum of beam stiffness at joint } \alpha} \quad (27\text{-a})$$

$$G_\beta=\frac{\sum_\beta\left(\frac{EI}{L}\right)_c}{\sum_\beta\left(\frac{EI}{L}\right)_b}=\frac{\textit{sum of column stiffness at joint } \beta}{\textit{sum of beam stiffness at joint } \beta} \quad (27\text{-b})$$

The G-factors for each joint determined by Eq. 27 are given in the Fig. 6(b) proposed by McCormac.

Finally, the K-factors satisfying Eq. 26 can be found by a *trial and error procedure*. The least roots of ***K*** for the columns are also given in Fig. 6(b).

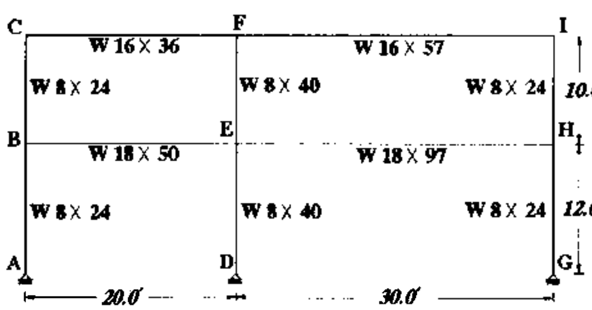


Fig. 6 (a). Two story frame

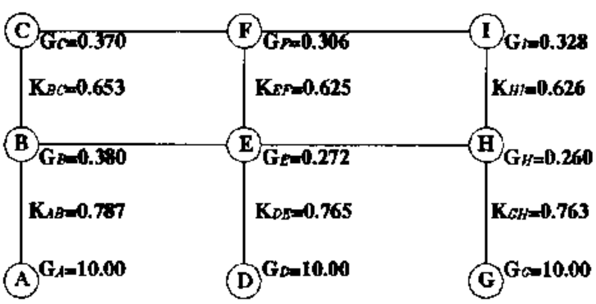
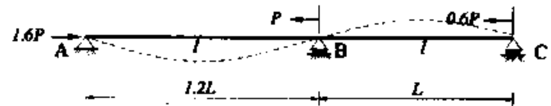
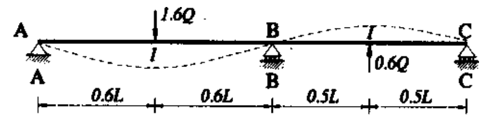


Fig. 6 (b). G-factors & K-factors

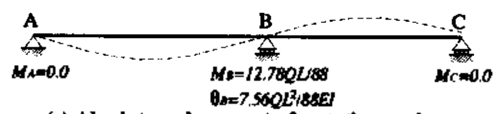
Now, the proposed method is to be applied to this frame with the assumption that the total load of each story changes in the manner of Fig. 7(a). Fig. 7(b) shows the analogous beam. The stress analysis results from the slope-deflection method are given by Fig. 7(c). Finally, the effective length factor for each column determined by Eq. 4 are shown in Fig. 7(d).



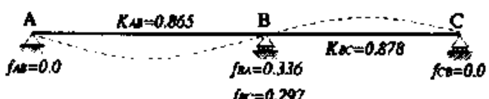
(a) Assumed column load



(b) Analogous beam



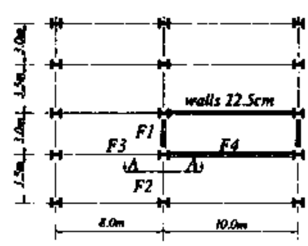
(c) Absolute end moments & rotation angles



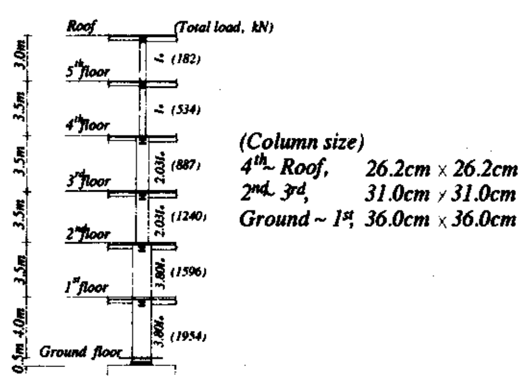
(d) Fixity factors & K-factors

Fig. 7. Analogous Beam

The ***K***-factor for the first story gives 13% error when compared with the result by the Alignment chart. But for the second-story column, the proposed method yields large upper bound errors, which can be acceptable as far as the actual structural design is concerned.



(a) Floor plan



(b) Section A-A

Fig. 8. Design example

Fig. 8 shows a design example of braced columns. This design example is adopted from "*Steel Designer's Manual*"[9]. In the original paper, ***K***-factors assume ***K=0.8*** for the lower two-story columns and ***K=0.7*** for the remaining upper story columns. Fig. 9 shown on the final page of this paper shows an analogous beam, the stress analysis, and the ***K***-factors for the design example columns. As can be seen in Fig. 9(c), the proposed method gives slightly larger values for the ***K***-factors. For this reason, the proposed method, when applied to braced columns, will result in a conservative design.

4. CONCLUSION

Contrary to conventionally used analytical or numerical methods, beam analogy method requires only easy hand calculations for the determinations of elastic critical load and ***K***-factors of a continuous compression member. Even in the cases of continuous compression members with variable span, and sectional dimensions and also with different boundary conditions, the proposed method can predict the span that buckles first.

The beam analogy method gives lower-bound errors of critical loads when compared with those by conventional methods. Meanwhile ***K***-factors for the braced frame columns by proposed method are larger than those by AISC's alignment chart, and upper-bound errors of effective length factors, which lead to a conservative design of either continuous compression members or multi-story braced columns.

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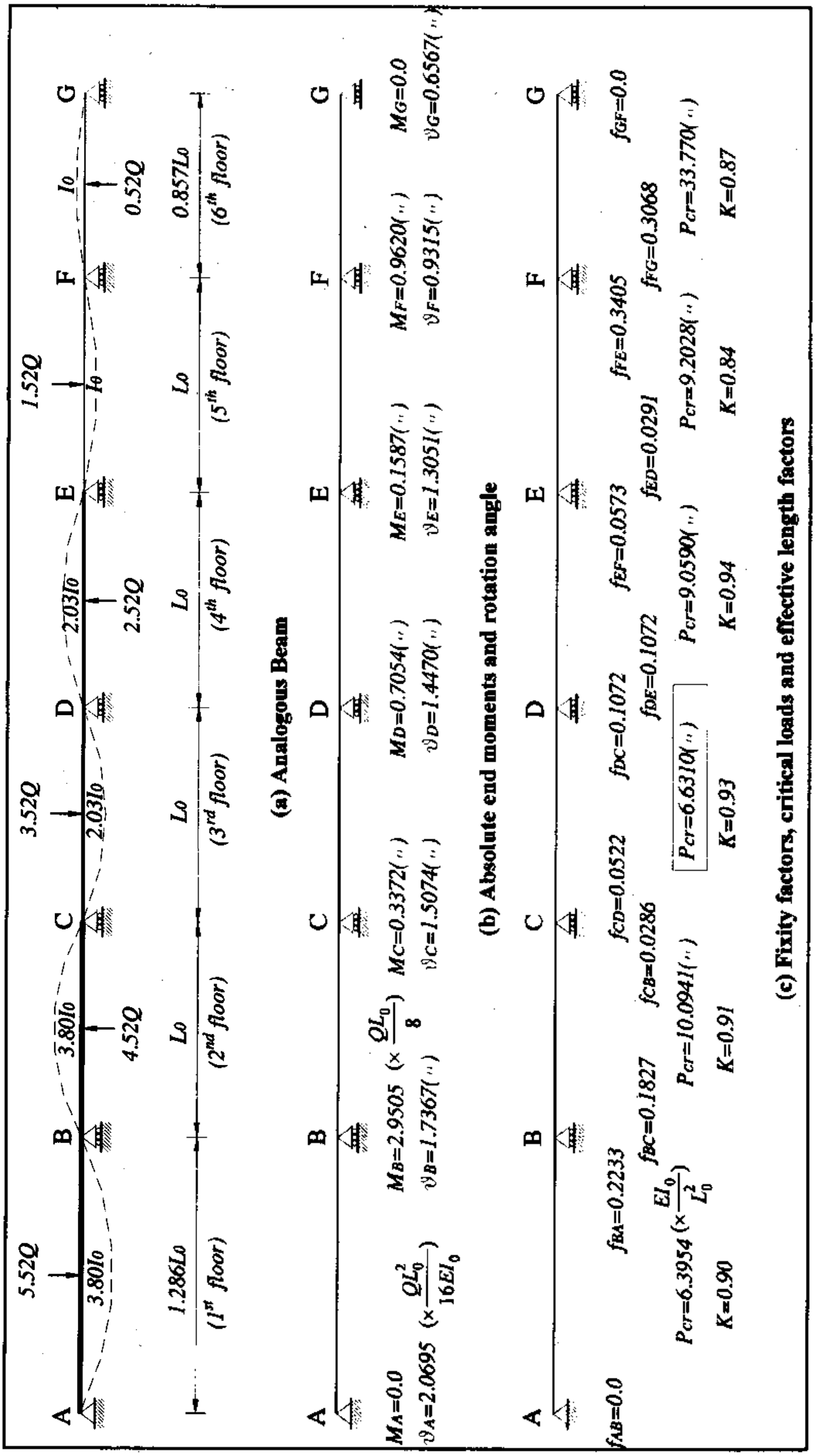

Fig. 9. Multi-story columns modeled as analogous beam