레이더 응용을 위한 다중표적 추적 연구
A Study of Multi-Target tracking for Radar application

이 양 원
Yang-Weon Lee

요 약

본 논문은 레이더 시스템에서 추적 궤적을 갱신하기 위해서 측정값을 할당할 때의 관계를 결합 행렬식으로 표현하여 찾는 최적기법을 연구한다. 이를 위하여 클러터의 분포를 Gibbs의 분포로 가정하고서 마코프 랜덤필드를 이용하여 관계식을 유도하였다. 유도된 식은 MAP 추정기 형태를 갖고 이를 계산하기 위하여 제한사항을 고려한 Lagrangian 방법을 사용하였다.

ABSTRACT

This paper introduced a scheme for finding an optimal association matrix that represents the relationships between the measurements and tracks in multi-target tracking of Radar system. We considered the relationships between targets and measurements as MRF and assumed a priori of the associations as a Gibbs distribution. Based on these assumptions, it was possible to reduce the MAP estimate of the association matrix to the energy minimization problem. After then, we defined an energy function over the measurement space, that may incorporate most of the important natural constraints.

Ⅰ. Introduction

The primary purpose of a multi-target tracking(MTT) system is to provide an accurate estimate of the target position and velocity from the measurement data in a field of view. Naturally, the performance of this system is inherently limited by the measurement inaccuracy and source uncertainty which arises from the presence of *missed detection, false alarm, emergence of* new targets into the surveillance region and disappearance of old targets from the surveillance region. Therefore, it is difficult to determine precisely which target corresponds to each of the closely spaced measurements. Although trajectory estimation problems have been well studied in the past, much of this previous work assumes that the particular target corresponding to each observation is known. Recently, with the proliferation of surveillance systems and their increased sophistication, the tools for designing algorithms for data association have been announced.

In this paper, we derive the new model for data association which reflects the natural constraints of the MTT problem and convert the derived model into the minimization problem of energy function by MAP estimator[1]. The coefficients of energy function is calculated by Lagrange multiplier,[2] and local dual theory[3].

II. Problem Formulation and Energy Function

Fig.1 shows the overall system of multi-target tracking. This system consists of three blocks: acquisition, association, and prediction. The purpose of the acquisition is to determine the initial starting position of the tracking. After this stage, the association and prediction interactively determine the tracks. Our primary concern is the association part that must determine the actual measurement and target pair, given the measurements and the predicted gate centers.

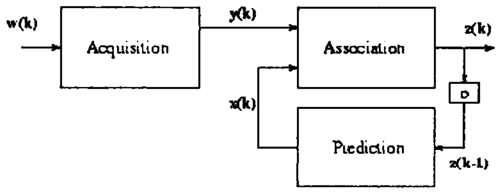

Fig. 1. An overall scheme for target tracking
그림 1. 표적추적 시스템 구성도

Let m and n be the number of measurements and targets, respectively, in a surveillance region. Then the validation matrix Ω [3] is :

$$\Omega = \{\omega_{jt} \mid j \in [1,m], t \in [0,n]\} \qquad (1)$$

where the first column denotes clutter and always $w_{j0} = 1 (j \in [1, m])$. For the other columns, $\omega_{jt} = 1 (j \in [1, m], t \in [1, n])$, if the validation gate of target t contains measurement j and $\omega_{jt} = 0$, otherwise. Based on the validation matrix, we must find hypothesis matrix[4], $\hat{\Omega}$ that must obey the following natural constraints:

$$\begin{cases} \sum_{t=0}^{n} \hat{\omega}_{jt} = 1 \quad for \quad (j \in [1,m]) \\ \sum_{j=1}^{m} \hat{\omega}_{jt} \le 1 \quad for \quad (t \in [1,n]) \end{cases} \qquad (2)$$

Here, $\widehat{\omega_{jt}} = 1$ only if the measurement j is associated with clutter ($t=0$) or target t ($t \neq 0$). Generating hypothesis matrices leads to a combinatorial problem, where the number of data association hypothesis increases exponentially with the number of targets and measurements.

The ultimate goal of this paper is to find the hypothesis matrix given the observation y and x, which must satisfy (2). Let's associate the realizations the gate center x, the measurement y, the validation matrix w, and $\hat{\omega}$- to the random processes X, Y, Ω, and $\hat{\Omega}$. Next, consider that $\hat{\Omega}$ is a parameter space and X, Y, Ω, is an observation space. Then a posteriori can be derived by the Bayes rule:

$$P(\hat{\Omega}|\Omega, y, x) = \frac{P(\Omega, y, x|\hat{\Omega})P(\hat{\Omega})}{P(\Omega, y, x)} \qquad (3)$$

We assume the parameter $\hat{\Omega}, \Omega$ are given and (X,Y) are observed. If the conditional probabilities describing the relationships between the parameter space and the observation spaces are available, one can obtain the MAP estimator:

$$\hat{\Omega}^{*} = \arg\max P(\hat{\Omega}|\Omega, y, x) \qquad (4)$$

As a system model, we also assume that the conditional probabilities are all Gibbs distributions:

$$P(\hat{\Omega}|\Omega, y, x) \equiv \frac{1}{Z} \exp\{-E(\hat{\Omega}|\Omega, y, x)\} \qquad (5\text{-a})$$

$$P(y, x|\hat{\Omega}) \equiv \frac{1}{Z_1} \exp\{-E(y, x|\hat{\Omega})\} \qquad (5\text{-b})$$

$$P(\Omega|\hat{\Omega}) \equiv \frac{1}{Z_2} \exp\{-E(\Omega|\hat{\Omega})\} \qquad (5\text{-c})$$

$$P(\hat{\Omega}) \equiv \frac{1}{Z_3} \exp\{-E(\hat{\Omega})\} \qquad (5\text{-d})$$

$$P(\Omega, y, x) \equiv \frac{1}{Z_4} \exp\{-E(y, x, \Omega)\} \qquad (5\text{-e})$$

where $Z_s(s \in [1,2,3,4])$ is called partition function:

$$Z_s = \int_{\hat{\Omega} \in E} \exp\{-E(\hat{\Omega})\} d\hat{\Omega} \qquad (6)$$

Here, E denotes the energy function. Substituting (5) into (4), (6) becomes

$$\begin{aligned} E(\hat{\Omega}|\Omega, y, x) = \ & E(y, x, |\hat{\Omega}) \\ & + E(\Omega|\hat{\Omega}) \\ & + E(\hat{\Omega}) - E(\Omega, y, x) \end{aligned} \qquad (7)$$

The first term in (7) represents the distance between measurement and target and must be minimized using feasible events. The second term intend to suppress the measurements which are uncorrelated with the valid measurements. The third term denotes constraints of the validation matrix and it can be designed to represent the two restrictions. The energy equations of each term are defined respectively:

$$\begin{cases} E(y,x|\hat{\omega}) & \equiv \displaystyle\sum_{t=1}^{n}\sum_{j=1}^{m} r_{jt}\hat{\omega}_{jt} \\ E(\omega|\hat{\omega}) & \equiv \displaystyle\sum_{t=1}^{n}\sum_{j=1}^{m} (\hat{\omega}_{jt} - \omega_{jt})^2 \\ E(\hat{\omega}) & \equiv \displaystyle\sum_{t=1}^{n}(\sum_{j=1}^{m}\hat{\omega}_{jt} - 1) + \sum_{j=1}^{m}(\sum_{t=0}^{n}\hat{\omega}_{jt} - 1) \end{cases} \qquad (8)$$

(8) into (7), one gets

$$\hat{\omega}^* = \arg\max_{\hat{\omega}}[\alpha\sum_{t=1}^{n}\sum_{j=1}^{m} r_{jt}\hat{\omega}_{jt} + \frac{\beta}{2}\sum_{t=1}^{n}\sum_{j=1}^{m} (\hat{\omega}_{jt} - \omega_{jt})^2 + \sum_{t=1}^{n}(\sum_{j=1}^{m}\hat{\omega}_{jt} - 1) + \sum_{j=1}^{m}(\sum_{t=0}^{n}\hat{\omega}_{jt} - 1)] \qquad (9)$$

where $r_{jt} = (x_j d_{yt} - y_j d_{xt})^2 / (d_{xt}^2 + d_{yt}^2)$, and α and β are parameters of the weighted distance measure and the matching term respectively.

III. Design of Optimal Adaptive Data Association Scheme

The optimal solution for (9) is hard to find by any deterministic method. Instead, one can convert the present constrained optimization problem to an unconstrained problem by introducing Lagrange multipliers ad using the local dual theory[5]. The problem is to find such that where

$$L(\hat{\omega}, \lambda, \varepsilon) = \alpha\sum_{t=1}^{n}\sum_{j=1}^{m} r_{jt}\hat{\omega}_{jt} + \frac{\beta}{2}\sum_{t=1}^{n}\sum_{j=1}^{m}(\hat{\omega}_{jt} - \omega_{jt})^2 + \sum_{t=1}^{n}\lambda_t(\sum_{j=1}^{m}\hat{\omega}_{jt} - 1) + \sum_{j=1}^{m}\varepsilon_j(\sum_{t=0}^{n}\hat{\omega}_{jt} - 1) \qquad (10)$$

Here, λ and ε are just Lagrange multipliers. Note that (10) includes the effect of the first column of the associated matrix, which represents the clutter as well as newly appearing targets. In general setting, we assume m>n, since most of the multitarget problem is characterized by many confusing measurements that exceed far over the number of original targets.

Since (10) is a convex function which guarantes the extrema, using the convex analysis for the local duality, the optimal solution can be obtained by

$$(\hat{\omega}^*, \lambda^*, \varepsilon^*) = \arg\max_{\varepsilon}\max_{\lambda}\min_{\hat{\omega}\geq 0} L(\hat{\omega}, \lambda, \varepsilon) \qquad (11)$$

The necessary condition for achiving extreme in (10) are

$$\begin{cases} \nabla_{\hat{\omega}_{jt}} L(\hat{\omega}, \lambda, \varepsilon) & = 0 \\ \nabla_{\lambda_t} L(\hat{\omega}, \lambda, \varepsilon) & = 0 \\ \nabla_{\varepsilon_j} L(\hat{\omega}, \lambda, \varepsilon) & = 0 \end{cases} \qquad (12)$$

Using (12), one obtains the final representations of the solution:

$$\begin{cases} \hat{\omega}_{it}^{*} & = & \{\beta\omega_{it} - \alpha r_{it}(1-\delta_t) - \lambda_t - \varepsilon_t\}/\beta \\ \lambda_t^{*} & = & -\frac{\beta}{m}(1 + d_{mn}\delta_t) - \frac{1}{m}\sum_{j=1}^{m} f_{it}(\varepsilon_t) \\ \varepsilon_t^{*} & = & \bar{\varepsilon}_t^{\tau} + \mu[\frac{1}{n+1}\sum_{i=0}^{n}\{\beta(\omega_{it} - \overline{\omega}_t) - \alpha(1-\delta_t)(r_{it} - \bar{r}_t)\}] \end{cases} \tag{13}$$

which ε means optimal value of $\bar{\varepsilon}$ at any scan.

(13) contains two parameters α and β. To obtain these parameters, we consider the ML(maximum likelihood) estimation: Given (w, y, x), Θ is estimated as a maximum likelihood estimate such that

$$\Theta = \arg\max_{\Theta} P(\omega, y, x \mid \hat{\omega}, \Theta) \tag{14}$$

where $\Theta \equiv [\alpha \mid \beta]^T$. Unfortunately, although the ML is unique if it exists, the ML estimation is computationally prohibitive due to the calculation of the partition function. Therefore, as an alternative of ML, MPL(Maximum Pseudo Likelihood) is considered. In the MPL estimation, $P(\omega, y, x \mid \hat{\omega}, \Theta)$ is represented as a product of local partition function:

$$P(\omega, y, x \mid \hat{\omega}, \Theta) = \frac{P(y, x \mid \omega, \hat{\omega}, \Theta) P(\hat{\omega} \mid \omega, \Theta) P(\omega \mid \Theta)}{P(\hat{\omega} \mid \Theta)}$$
$$= \prod_{i=0}^{n}\prod_{j=1}^{m} \tfrac{1}{Z_{it}} \exp\{-\Theta^T \Phi(\hat{\omega}_{it})\} \tag{15}$$

where Z_{it} is a local partition function:

$$Z_{it} = \sum_{\hat{\omega}_{it} \in \hat{\omega}} \exp\{-\alpha r_{it}\hat{\omega}_{it}\delta_t - \tfrac{\beta}{2}(\hat{\omega}_{it} - \omega_{it})^2\} \tag{16}$$

and cost function $\Phi(\hat{\omega}_{it})$ is

$$\Theta(\hat{\omega}_{it}) = \begin{bmatrix} r_{it}\hat{\omega}_{it}\delta_t \\ \frac{1}{2}(\hat{\omega}_{it} - \omega_{it})^2 \end{bmatrix} \tag{17}$$

It is proven that (16) is strictly concave with respect to Θ if and only if the parameters that comprise Θ are linearly independent with each other. Therefore, Θ can be found from the gradient search method:

$$\frac{\partial\Theta}{\partial t} = -\mu\nabla_{\Theta} \log P(\omega, y, x \mid \hat{\omega}, \Theta) \tag{18}$$

Putting (15) into (18) arrives

$$\Theta^{\tau+1} = \Theta^{\tau} - \mu\nabla_{\Theta} \ln P(\omega, y, x \mid \hat{\omega}, \Theta)|_{\Theta=\Theta^{\tau}}$$
$$= \Theta^{\tau} - \mu\sum_{i=0}^{N}\sum_{j=1}^{N}[\Phi(\hat{\omega}_{it}) - \tfrac{1}{Z_{it}}\sum_{\hat{\omega}_{it} \in \hat{\omega}} \Phi(\hat{\omega}_{it})\exp(-\Theta^{\tau^T}\Phi(\hat{\omega}_{it}))] \tag{19}$$

where μ and τ are an updating constant and an iteration index, respectively.

In fig. 2, we show the overall computational flow structure. Its structure consists of the two parts: data association and parameter updating. The data association block transforms the input data into the energy equation to obtain the feasible matrix. Inside the block, first is calculated and then . Finally, feasible matrix, will be calculated. The parameter estimation block updates the parameter using the previous input and feasible matrix data.

Fig.2. Overall flow diagram of optimal data association algorithm

According to the overall computational flow structure, the computational complexity analysis is simplified as numbers of multiplications in data association and parameter updating. Suppose that there are n targets and m measurements. And assume that the average iteration number of ε_t and λ_t is $\bar{k}_1$ and $\bar{k}_2$, respectively. In this case, λ and ε's computation in the data association stage require and . The necessary computation of the feasible matrix extractions is multiplications. Therefore the number of multiplications required in the data association part is $O((\bar{k}_1 + \bar{k}_2 + 1)mn)$.

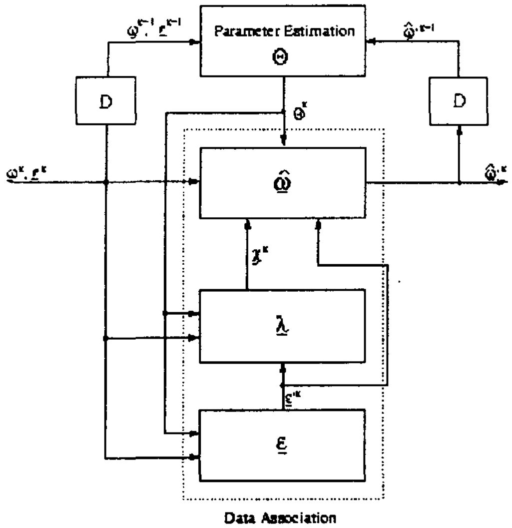

Fig.2. Overall flow diagram of optimal data association algorithm

그림 2. 알고리즘 전체 구성도

IV. Computer simulation

We present some results of the experiments comparing the performance of the proposed MAP estimate adaptive data association(MAPADA) with that of the Hopfield Neural PDA(HNPDA) of Sengupta and Iltis[2]. Though the MAPADA has a good structure for a parallel hardware, currently the algorithm is simulated by a serial computer. The dynamic models for the targets have been digitized using sampling period T normalized to 1s and the state vectors have been represented in 2-dimensional Cartesian coordinates. Furthermore, only position measurements have been assumed to be available. The surveillance region used in the simulation is a 20 km by 20 km square and the initial positions and velocities of 10 targets in 2-dimensional plane are given in Table 1. Since every targets except target 8 and 9 are non maneuvering, the generic target dynamic model has a linear motion characteristics.

Table1. Initial positions/velocity of 10 targets
표 1. 10개의 표적 제원

Target	Position(km)		Velocity(km/s)	
i	x	y	$\dot{x}$	$\dot{y}$
1	-4.0	1.0	0.2	-0.05
2	-4.0	1.0	0.2	0.05
3	-6.0	-5.0	0.0	0.3
4	-5.5	-5.0	0.0	0.3
5	8.0	-7.0	-0.4	0.0
6	-8.0	-8.0	0.4	0.0
7	-5.0	9.0	0.25	0.0
8	-5.0	8.9	0.25	0.0
9	0.5	-3.0	0.1	0.2
10	9.0	-9.0	0.01	0.2

In Fig.3, we shown the result of 10 target tracking using the data of table 1. Table 2. summarizes the rms position and velocity errors for each targets. The performance of the MAPADA is superios to that of HNPDA.

Table 2. RMS Errors in the case of 10 targets
표 2. RMS 오차

target	Position error (km)		Velocity error (km/s)		Track Maintenance (%)	
i	HNPDA	MAPODA	HNPDA	MAPODA	HNPDA	MAPODA
1	0.64	0.42	0.69	0.18	95	100
2	0.64	0.42	0.42	0.17	95	100
3	0.78	0.42	0.22	0.18	100	100
4	0.60	0.43	0.21	0.18	93	100
5	0.59	0.45	0.67	0.18	85	100
6	0.57	0.45	0.20	0.18	100	100
7	0.57	0.42	0.31	0.49	90	100
8	-	2.95	-	1.18	0	53
9	0.62	0.44	0.27	0.21	80	98
10	0.59	0.45	0.21	0.18	100	98

V. Conclusion

In this paper, we have developed the optimal adaptive data association scheme for radar multi-target tracking system. This scheme is designed to determine all parameters automatically and requires multiplications. We have confirmed that all parameters converge to steady states in the data association capability simulation and tracking accuracy is superior to that of HNPDA about 5.2% in view of tracking accuracy under the clutter density of c=0.4. The superiority of this scheme to the HNPDA comes from two important points. At first, this scheme used course weighted distance compared to the averaged weighting distance of HNPDA. Under the heavy clutter ambient, target's direction is more important than the correct position. The second is that the new algorithm can reject the irrespective plots from the validation matrix by incorporating the matching term in the energy equations.

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VI. References

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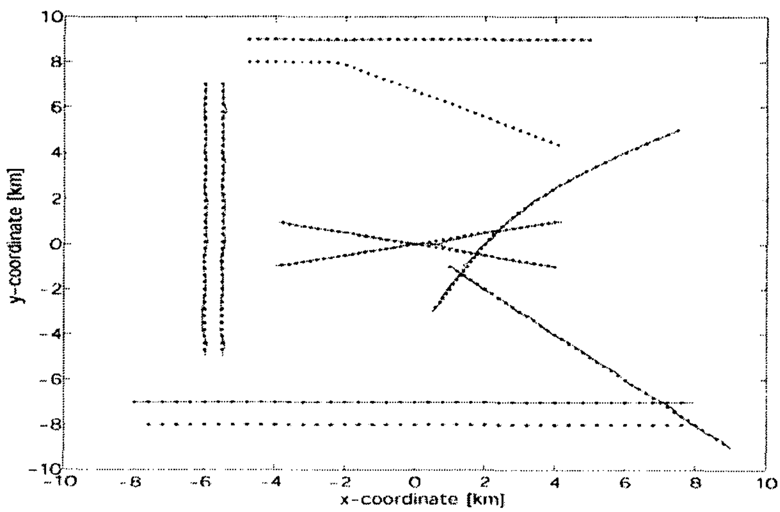

Fig. 3. Tracking 10 targets using MAPODA
그림 3 MAPODA을 이용한 10개의 표적 추적 결과

이양원(Yang Weon Lee)
正會員
1982년 중앙대 전자공학과 졸업.
1991년 서울대대학원 제어계측공학과 졸업(석사).
1996년 포항공대 전자공학과 대학원 졸업(박사).
1982-89년 국방과학연구소 선임연구원.
현재 호남대 공대 정보통신공학과 조교수