Journal of the Korean Society of Oceanography
Vol. 35, No. 1, pp. 11—15, March 2000

A Study on the Generalized Multifractal Dimension and the Spectrum in Seabottom Topography

KYUNGSIK KIM* AND Y.S. KONG[1]
Department of Physics, Pukyong National University, Pusan 608-737, Korea
[1]*School of Ocean Engineering, Pukyong National University, Pusan 608-737, Korea*

The scaling behavior of random fractals and multifractals is investigated numerically on the seabottom depth in the seabottom topography. In the self-affine structure the critical length for the crossover can be found from the value of standard deviations for the seabottom depth. The generalized dimension and the spectrum in the multifractal structure are discussed numerically, as it is assumed that the seabottom depth is located on a two-dimensional square lattice. For this case, the fractal dimension D_0 is respectively calculated as 1.312476, 1.366726, and 1.372243 in our three regions, and our result is compared with other numerical calculations.

INTRODUCTION

Recently, the investigation for the scaling behavior of fractal models has attracted a great deal of interest, since the fractals have been introduced and developed extensively by the pioneering work of Mandelbrot (1983). In general, as is well known, two kinds of the fractals can be distinguished : the deterministic fractals and the random fractals or self-affine fractals. The former has the self-similar structure and the scaling invariance under the scaling transformation in diverse models such as Koch curve and Cantor sets. The latter constitutes the random complicated structure that has been applied to a broader range of problems such as Eden model, the ballistic deposition model (Family *et al.*, 1985; Freche *et al.*, 1985), the mountain heights, the phenomena of cloud drift, and the seashore curve (Vicsek *et al.*, 1988). Among other examples of many fractal models we also can mention the self-avoiding random walk, the percolation clusters, the diffusion-limited aggregation, the random resistor network, the polymer bonds, the turbulence, the chaotic motions (Vicsek *et al.*, 1988; Halsey *et al.*, 1986; Paladin and Vulpiani, 1987; Lee, 1988; Tel, 1988; Farmer, 1982; Benzi *et al.*, 1984), etc.

Furthermore, in the multifractal structure Paladi and Vulpiani (1987) have recently discussed the multifractal concept that the singularities of scaling exponents are essentially related to the random probability. The multifractal structure due to the box-counting method is simply represented by the generalized dimension and the spectrum. Especially, as the multifractals are obtained from the finite value of the random probability distribution divided by the square area on a two-dimensional square lattice, the generalized dimension and the scaling exponents can be calculated from the moments of the probability having the value of singularities. The generalized dimension and the scaling exponents calculated from the scaling behavior of multifractals have widely been developed to apply to the fractal phenomena and the chaotic motions (Halsey *et al.*, 1986; Tel, 1988). Recently, Matsushita and Ouchi (1989) have shown that the self-affinity in Japanese mountain topography is mainly obtained by the numerical method, from which they have discussed the relation between the self-affine exponent and the fractal dimension for the standard deviation of the mountain height. More recently, the numerical values for the generalized dimension and the scaling exponent of the multifractals have been extended to Korean mountain height (Kim and Kong, 1998).

The main purpose of this paper is to investigate numerically on the scaling behavior of both random fractals and multifractals in the seabottom depth on the seabottom topography shown between 129°21′—129°54′*E* and 34°52′—35°19′*N*, where this region is located near to the east of Pusan. The motive that this region is primarily selected is induced to have more data of seabottom depth than any other region around Korean peninsula. For this case, we shall treat for clearity with self-affine exponents on the fractal

*Corresponding author: kskim@dolphin.pknu.ac.kr

structure of the seabottom topography, and with the critical length occurred in the crossover from self-affine exponents for standard deviations of the seabottom depth. From the data for seabottom depths, we present an efficient and convenient method relevant to investigate numerically on the scaling behavior of the generalized dimension and the spectrum. In addition, we introduce well-known relations for the scaling behavior of both random fractals and multifractals. We also carry out the scaling investigations for the standard deviation, the generalized dimension, and the spectrum in our seabottom topography.

SCALING BEHAVIOR OF THE RANDOM FRACTALS AND THE MULTIFRACTALS

First of all, we investigate the scaling behavior of the random fractals in the seabottom topography, as shown in Fig. 1. Because the observed data of seabottom depth in the region of Fig. 1 are existed more than those in any other region, we think that it is certainly convenient to treat with the scaling behavior of the random fractals and the multifractals. The scaling quantities are evaluated after the extrapolation of the data for seabottom depths on a two-dimensional square lattice shown between 129°21′−129°54′*E* and 34°52′− 35°19′*N*. The data are now taken by the value of seabottom depths projected on 100×100 lattice points, where the unit length between two arbitrary lattice points is measured by 500 *m*. In this

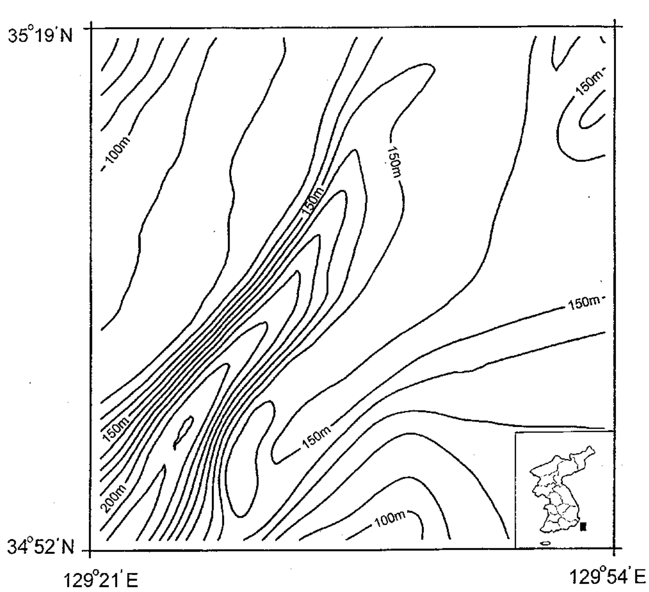

Fig. 1. Seabottom topography taken from 1/312000 scale map between the longitude 129°21′−129°54′*E* and the latitude 34°52′−35°19′*N*.

paper, since we adopt some ideas different from those of Matsushita and Ouchi (1989), we measure the respective positions located at the longitudinal length x_i, the latitudinal length y_i, and the corresponding seabottom depth z_{xi} and z_{yi}, where we focus on lattice points with in the domain of i=1−100. In this way, as we consider two kinds of the measured position in terms of (x_i, z_{xi}) and (y_i, z_{yi}), the standard deviations can be found. In such a case, the standard deviations X, Y, Z_x and Z_y are written as follows:

$$X = \left[\frac{1}{N_x}\sum_{i=1}^{N}(x_i-\bar{x})^2\right]^{1/2} \sim N_x^{\nu_x} \tag{1}$$

$$Y = \left[\frac{1}{N_y}\sum_{i=1}^{N_y}(y_i-\bar{x})^2\right]^{1/2} \sim N_y^{\nu_y} \tag{2}$$

$$Z_x = \left[\frac{1}{N_x}\sum_{i=1}^{N_x}(z_{xi}-\bar{z}_x)^2\right]^{1/2} \sim N_x^{\nu_{zx}} \tag{3}$$

and

$$Z_y = \left[\frac{1}{N_y}\sum_{i=1}^{N_y}(z_{yi}-\bar{z}_y)^2\right]^{1/2} \sim N_y^{\nu_{zy}} \tag{4}$$

Here, $\bar{x}$ is the average value of the longitudinal length, and $\bar{y}$ is the average value of the latitudinal length. The scaling quantities $\bar{z}_x$ and $\bar{z}_y$ are the average values of the seabottom depth, and ν_x, ν_y, ν_{zx} and ν_{zy} are the corresponding self-affine exponents.

Next, let's introduce the generalized dimension D_q, the scaling exponents α_q and f_q on the multifractal structure. It has in general been known that the generalized dimension is represented as the fractal distribution having the infinitely singular values by employing the box-counting method. Accordingly, the generalized dimension and the scaling exponents are known to be given by the formula relation (Halsey *et al.*, 1986; Paladin and Vulpiani, 1987).

$$D_q = \lim_{\varepsilon\to 0}\frac{1}{q-1}\frac{\ln\sum_i n_i p_i^q}{\ln\varepsilon} \tag{5}$$

$$\alpha_q = \lim_{\varepsilon\to 0}\frac{1}{\ln\varepsilon}\frac{\sum_i p_i^q \ln p_i}{\sum_i n_i p_i^q} \tag{6}$$

and

$$f_q = \lim_{\varepsilon \to 0} \frac{1}{\ln \varepsilon} \left[\frac{\sum_i p_i^q \ln p_i}{\sum_i n_i p_i^q} - \ln \sum_i n_i p_i^q \right] \quad (7)$$

where p_i is the probability of the seabottom depth existing on the ith box with the square area $\varepsilon \times \varepsilon$, and the scaling quantity n_i is the number of the box having the probability p_i. By introducing the above expressions, the spectrum f_q and α_q are simply calculated in terms of D_q from the relations (Paladin and Vulpiani, 1987).

$$f_q = \alpha_q - (q-1)D_q \quad (8)$$

and

$$\alpha_q = \frac{d}{dq}[(q-1)D_q] \quad (9)$$

where eqs. (8) and (9) can be obtained by Legendre transformation.

CALCULATION AND SUMMARY

We now present numerical results for the self-affinity in our model of the random fractal structure. The estimates for the self-affine exponent from our measured positions are compared in Figs. 2 and 3. For simplicity, we find from eqs. (1) and (3) that the self-affine exponents, as shown in Fig.2, are obtained as ν_x=1.0 and ν_{zx}=0.867 at $N<N_c$, and ν_x=1.0 and ν_{zx}= 0.625 at $N \geq N_c$, where N_c is the critical length. As shown in Fig. 3, one also estimates scaling exponents as ν_y=1.0 and ν_{zy}=0.957 at $N<N_c$, and ν_y=1.0 and ν_{zy}= 0.718 at $N \geq N_c$, where these exponents are found from eqs. (2) and (4). The value of self-affine exponents in our result is slightly different from that of Kim and Kong (1998) that is obtained numerically for Korean mountain heights. We also can observe directly the critical length for the crossover from the slope of standard deviations. It is really found that the crossover are exhibited clearly around the critical length N_c=11 km on our random fractal structure.

In order to clarify the scaling behavior of the multi-

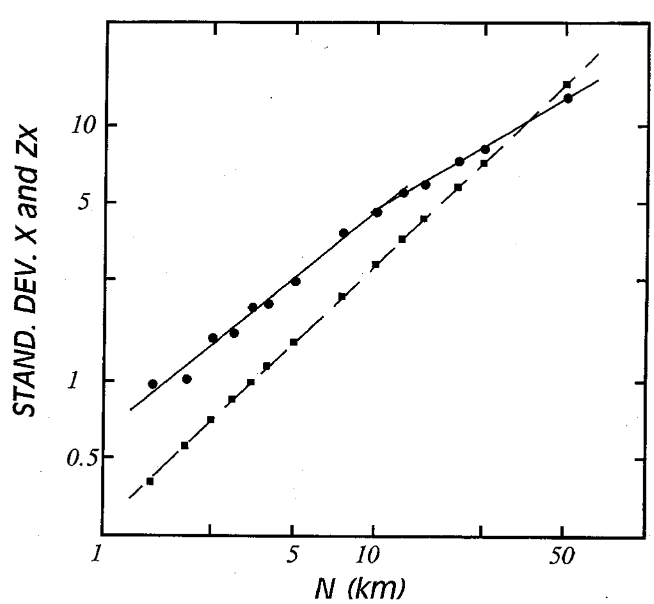

Fig. 2. Plot of the standard deviations of both X (dashed line) and Z_x (solid line) as a function of the zonal length. The crossover for Z_x is exhibited around N_c=11 km. The slopes of standard deviations yield the self-affine exponents with ν_x=1.0 and ν_{zx}=0.867 at $N<N_c$, and ν_y=1.0 and ν_{zy}=0.625 at $N \geq N_c$.

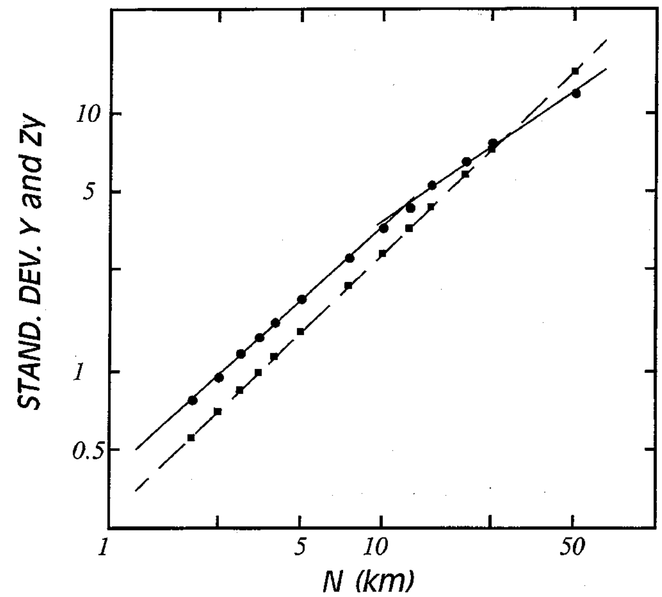

Fig. 3. Plot of the standard deviations of both Y (dashed line) and Z_y (solid line) as a function of the meridional length. The crossover for Z_y is exhibited around N_c=11 km. It is found from the extrapolation method that the slopes of the standard deviations yield the self-affine exponents with ν_y =1.0 and ν_{zy}=0.957 at $N<N_c$, and ν_y=1.0 and ν_{zy}=0.718 at $N \geq N_c$.

Table 1. Number of the seabottom depth in our five regions *A-E* between 110 m and 135 m on the seabottom topography

Region	Seabottom depth	Number of depth
A	100 m—115 m	670
B	115 m—120 m	693
C	120 m—125 m	694
D	125 m—130 m	780
E	180 m—135 m	641

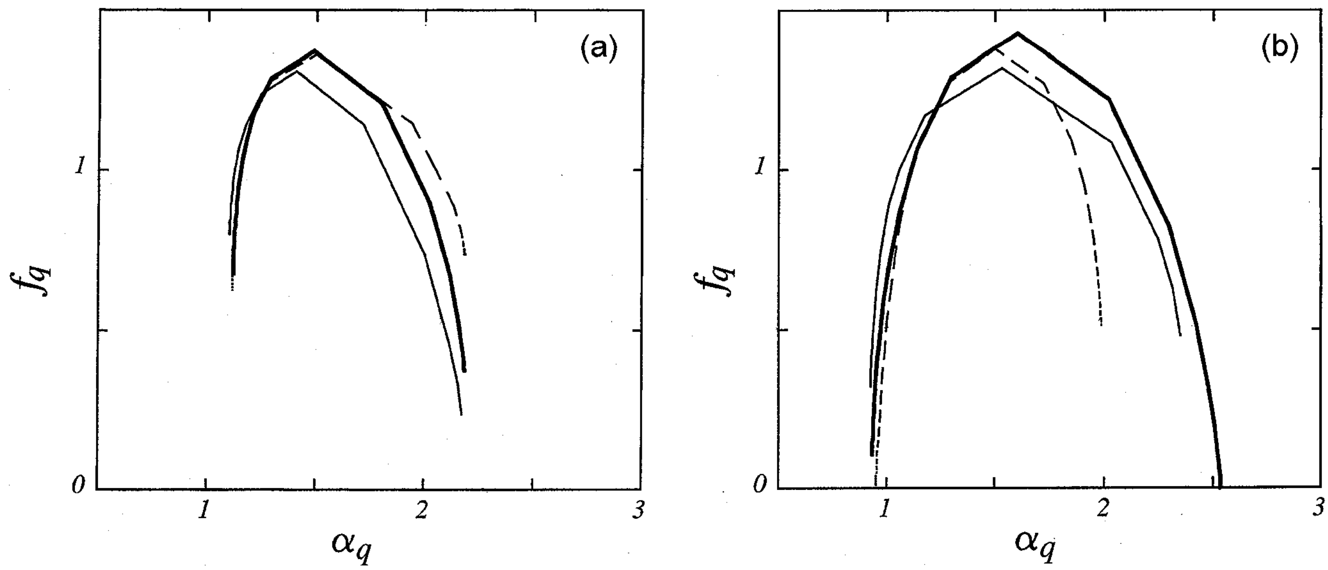

Fig. 4. Plots of f_q versus α_q by using the box-counting method for two cases of the square area (a) 2.5×2.5 km^2 and (b) 5×5 km^2. The values of the seabottom depths in three regions *A*, *B*, and *C* are respectively given by the thin solid, dashed, and thick solid lines.

fractals for the seabottom depth, we assume that the seabottom depths divided by the intervals 5 m are located on each region in two–dimensional lattice. As listed in Table 1, we take into account the data of five regions *A*–*E* in seabottom depths between 110 m and 135 m. The box-counting method is used for these regions of the seabottom depth, where the square areas are 2.5 × 2.5 km^2 and 5 × 5 km^2 for two kinds of box with ε =1/20, 1/10. To find numerically the generalized dimension and the scaling exponents, we only restrict ourselves to the data of three regions *A*, *B*, and *C* in Table 1. However, these values that are ultimately based on the theoretical expressions of eqs. (5)-(7) are calculated numerically in our three regions, and Fig. 5(a)-5(b) show the scaling behavior of the generalized dimension D_q, the scaling expo-

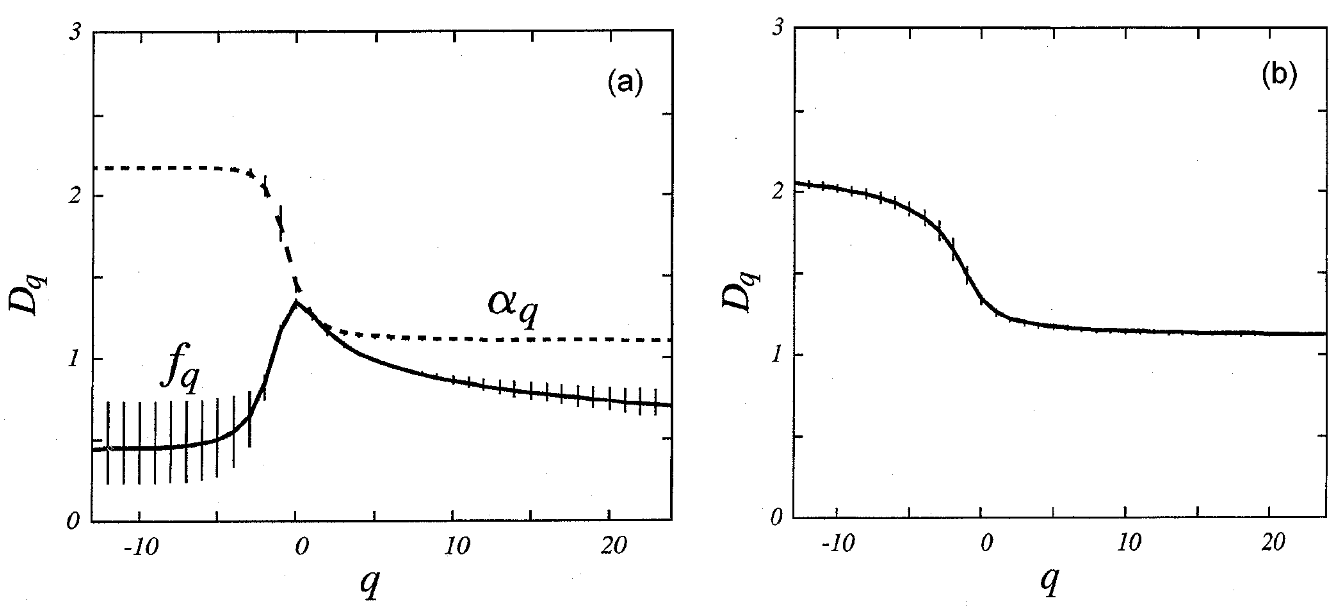

Fig. 5. Plots of (a) α_q, f_q, and (b) D_q as a function of q on the seabottom depths in three regions *A*, *B*, and *C*. These scaling quantities are estimated in the case of the square area 2.5×2.5 km^2, where the vertical bars are the error bars averaged over the square areas of three regions *A*, *B*, and *C*.

Table 2. Value of D_q, α_q, and f_q calculated from the data of the region A in Table 1, where the square area is 2.5×2.5 km^2

Region	D_0=f_0	$D_{+\infty}$	$\alpha_{-\infty}$	$f_{-\infty}$
A	1.312476	1.097696	2.172183	0.231378
B	1.366726	1.108962	2.183449	0.733451
C	1.372243	1.109444	2.183931	0.366725

nents α_q and f_q on the multifractal structure. From Table 2, we find numerically that the scaling exponents are estimated as $\alpha_{+\infty}$=1.097696 and $\alpha_{-\infty}$= 2.172183 only in the case of the region A. In particular, as shown schematically in Fig. 5(b), it is obtained that the maximum value of the generalized dimension, i.e., the fractal dimension D_0 is respectively calculated as 1.312476, 1.366726 and 1.372243 in our three regions A, B and C. Thus as the seabottom depth increases, the fractal dimension is nearly equal to that compared to the numerical result of Matsushita *et al.* (1989).

In conclusion, we have studied on the scaling behavior of both random fractals and multifractals in the seabottom topography between 129°21′–129°54′*E* and 34°52′–35°19′*N*. From the scaling behavior of random fractals in the seabottom topography, the self-affine exponents for standard deviations of the seabottom depth have been found numerically, as plotted in Figs. 2 and 3, and it has been obtained from these exponents that the crossover is occurred at the critical length N_c=11 km. Specifically, by employing the box-counting method on a two-dimensional square lattice, the scaling behavior of multifractals has been investigated numerically. In future, we expect that a further analytical and numerical progress for multifractals may be possible. Futhermore, it will be useful to apply and to reinvestigate the above formalism in other realistic fields (Barnsley, 1988; Kruhl, 1994) such as the structure of the ocean floor, the quantum disorder systems, and the fractured surfaces of minerals.

ACKNOWLEDGEMENTS

This work was supported in part by the academic research fund (KIOS-97-M-08) of Ministry of Education of Korea.

REFERENCES

Barnsley, M., 1988. Fractals everywhere. Academic Press Inc. 394 pp.

Benzi, R., G. Paladin, G. Parisi and A. Vulpiani, 1985. Characterisation of intermittency in chaotic system. *J. Phys. A.*, **17**: 2157−2165.

Family, F. and T. Vicsek, 1985. Scaling of the active zone in the Eden process on the perolation network and the ballistic deposition model. *J. Phys. A.*, **18**: L75−L81.

Farmer, J. D.,1982. Information dimension and the probabilistic structure of chaos. *Z. Naturforsch,* **37a**: 1304−1325.

Freche, P., D. Jullien and H. E. Stanley,1985. Surface structure and anisotropy of Eden clusters. *J. Phys. A.*, **18**: L1163.

Halsey, T. C., M. H. Jensen, L. P. Kanadanoff, I. Procaccia and B. I. Shraiman, 1986. Fractal measures and singularities. *Phys. Rev. B.*, **33**: 1141−1151.

Jullien, R and R. Botet,1985. Scaling properties of the surface of the Eden model in d=2,3,4. *J. Phys. A.*, **18**: 2279−2287.

Kim, K. and Y. S. Kong, 1998. Random fractals and multifractality in Korean topography. *Current Topics in Physics,* **1**: 421−439.

Kruhl, J. H., 1994. Fractals and dynamic systems in geoscience. Springer - Verlag, Berlin, 421 pp.

Lee, J. and H. E. Stanley, 1988. The phase transition in multifractal spectrum of diffusion-limited aggregation. *Phys. Rev. Lett.,* **61**: 2945−2948.

Mandelbrot, B. B., 1983. The Fractal geometry of nature. Freeman, San Francisco, 365 pp.

Matsushita, M. and S. Ouchi, 1989. On the self-affinity of various curves. *Physica D.*, **38**: 246−251.

Paladin, G. and A. Vulpiani, 1987. Anomaluos scaling laws in multifractal objects. *Phys. Rep.,* **156**: 147−225.

Tel, T., 1988. Fractals, multifractals, and thermodynamics. *Z. Naturforsch,* **43a**: 1154−1174.

Turkevich, L. A., 1985. *Phys. Rev. Lett.,* **55**: 1026.

Vicsek, T., 1988. Fractal growth phenomena. World Scientific, Singapore, 488 pp.

Manuscript received May 3, 1999
Revision accepted December 1, 1999