⊠응용논문

다구찌 손실함수 하에서 최적 공정평균 및 스크리닝 한계선의 결정+

홍성훈
전북대학교 산업공학과

Determination of Optimum Process Mean and Screening Limits under a Taguchi's Loss Function

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Abstract

The problem of jointly determining the optimum process mean and screening limits for each market is considered in situations where there are several markets with different price/cost structures. Two inspection procedures are considered; an inspection based on the quality characteristic of interest, and an inspection based on a surrogate variable which is highly correlated with the quality characteristic. The quality characteristic is assumed to be a normal distribution with unknown mean and known variance. A Taguchi's quadratic loss function is utilized for developing the economic model for determining the optimum process mean and screening limits. A numerical example is given.

+ 이 논문은 1999년도 전북대학교의 지원 연구비에 의하여 연구되었음.

1. Introduction

Let us consider a manufacturing process in which items are produced continuously. Suppose there is a lower specification limit L for the quality characteristic of interest Y. The items with $Y < L$ are reprocessed or sold at a discount. Typical quality characteristics under consideration are weights, volume, number and concentration. A process parameter μ for the mean is to be selected so that the expected cost per item is minimized. This problem has been studied by several researchers. Hunter and Kartha(1977) solved the problem of selecting the optimum process mean when the nonconforming items are sold at a reduced price. Bisgaard et al.(1984) extended Hunter and Kartha's model to a situation where the nonconforming items are sold at a price proportional to the amount of ingredient used. Golhar(1987) considered the problem of selecting the optimum process mean in a canning process. Boucher and Jafari(1991), and Al-Sultan(1994) discussed situations in which the items are subjected to lot-by-lot acceptance sampling rather than complete inspections. Arcelus and Rahim(1994) developed a model for jointly selecting optimum target means for both a variables and an attributes quality characteristics, and Chen and Chung(1996) considered an economic model for determining the most profitable target value and the optimum measuring precision level for a production process. Tang and Lo(1993), Lee and Jang(1997), and Hong et al.(1999) determined the optimum process mean and the screening limits when a surrogate variable is used in inspection. Hong and Elsayed(1999) studied the effect of the measurement error on the determination of the optimum process mean for a filling process.

In all the previous work, the inspected items are classified into two quality grades only; conforming items are accepted and nonconforming items are rejected. But, it is common practice to grade the outgoing items on the basis of the quality and then sell them in different markets. This practice has been used for chemical materials and primary materials such as lumber, wheat, cotton, and butter (England and Leenders(1975)). Different grades of a product may be sold at different selling prices under different names or marketed in different chain stores or areas under the same brand name(Tang(1990)). Economic inspection procedures under similar situations are considered by several authors; Tang(1990), Bai and Hong(1990), Kim et al.(1994), Lee and Jang(1997), and Hong et al.(1999). In this paper, economic models are developed for jointly determining the optimum process mean and the screening limits for each market in situations where there are several markets with different price/cost structures. A Taguchi's quadratic loss function is utilized for developing the expected profit function model. The loss caused by imperfect quality may include loss of goodwill, warranty, replacement cost, and handling cost. Classical concept in the field of optimum target value determination assumes that this loss is a constant when an item does not conform to product specifications and is zero otherwise. However, Taguchi(1984) argued that this cost concept was incorrect. Instead, he suggested that a quadratic function of the deviation from the product target value could better measure the true loss. This function has received widespread attention and used by several researchers.

All items are inspected prior to shipment and the decision for disposing the product is made depending on their quality and price/cost structures. Two inspection procedures are considered; an inspection based on the quality characteristic of interest (performance variable), and an inspection based on a surrogate variable which is highly correlated with the quality characteristic. When the performance variable is used as the screening variable, all the nonconforming items can be excluded from shipment. That is, the inspection based on the performance variable may be the best plan in the sense that nonconforming items will not be shipped to the customer. It is often very expensive, however, to inspect all items with the performance variable. In such situations, the use of a variable correlated with the performance variable as the screening variable is attractive when measuring the surrogate variable is relatively cheaper than the performance variable.

This paper is organized as follows: The economic models are formulated and the procedures for obtaining the optimum solution are derived in Section 2 and 3. An example is used to illustrate the solution procedure, and sensitivity analyses are performed in Section 4.

2. Model I: An Inspection Based on the Performance Variable

Let Y be a performance variable representing the quality characteristic of interest and τ denote the target value of Y. We assume that Y is a "larger is better" variable and normally distributed with unknown mean value μ_y and known variance σ_y^2. Suppose that a product can be sold to several different markets. When products are sold to market i, the selling price is A_i, and the item with $y < \tau$ causes a loss of $C_i(y, \tau) = a_i\,(y-\tau)^2$ which is a quadratic function. a_i is a positive constant, and y is the observed value of Y. This function was strongly advocated by Taguchi(1984) and has received widespread attention. If $y \geq \tau$, $C_i(y, \tau) = 0$. Now consider the case where $A_i > A_j$ and $C_i(y, \tau) \leq C_j(y, \tau)$, or $A_i = A_j$ and $C_i(y, \tau) < C_j(y, \tau)$. It is easy to verify that market j is dominated by market i and thus a product should be sold to market i rather than market j. Therefore only the markets which are not dominated need to be considered. Assume that there are m markets which are not dominated. It is not profitable to ship the low quality products to an ordinary market because of the penalty cost $C_i(y, \tau)$. Therefore, market m is considered as an alternative with one of following modes; sell the products at a discount, scrap the products, etc. Without loss of generality, it is assumed that $A_i > A_j$ and $C_i(y, \tau) > C_j(y, \tau)$ for all $i < j$. The condition $C_i(y, \tau) > C_j(y, \tau)$ is equivalent to the condition $a_i > a_j$.

Since a smaller index market i requires higher quality products than a larger index market j, an appropriate inspection procedure is as follows.

i) Take measurement y for each incoming item.

ii) Let δ_i, $i = 1, 2, \cdots, m$, be real numbers such that $\delta_1 \geq \delta_2 \geq \cdots \geq \delta_m = -\infty$ and $\delta_0 = \infty$. If $\delta_i \leq y < \delta_{i-1}$, $i = 1, 2, \ldots, m$, ship the item to market i. Note that if $\delta_i = \delta_{i-1}$, the item will not be shipped to market i.

The item is shipped to market i whenever $\delta_i \leq y \leq \delta_{i-1}$, $i = 1, 2, \cdots, m$. Therefore, the expected revenue for market i is

$$A_i \int_{\delta_i}^{\delta_{i-1}} g(y)dy\ , \tag{1}$$

where $g(y)$ is the probability density function of Y which is a normal density function with mean μ_y and variance σ_y^2. The production cost per item is $c_0 + c_1 y$ which is proportional to the quantity $y \cdot c_0$ and c_1 are positive constants(Bisgaard et al.(1984)). The expected production cost per item thus becomes $\int_{-\infty}^{\infty}(c_0 + c_1 y)g(y)dy = c_0 + c_1\mu_y$. The expected penalty cost for market i caused by imperfect quality is

$$\int_{\delta_i}^{\delta_{i-1}} C_i(y, \tau) g(y) dy \ . \tag{2}$$

Therefore, the expected profit per item is given by

$$EP = -s_y - c_0 - c_1 \mu_y + \sum_{i=1}^{m} \int_{\delta_i}^{\delta_{i-1}} k_i(y) g(y) dy, \tag{3}$$

where s_y is the inspection cost of Y and $k_i(y) = A_i - C_i(y, \tau)$.

The optimum values of $(\mu_y, \delta_1, \delta_2, \cdots, \delta_m)$ can be obtained by maximizing equation (3). We first determine the optimum screening limits $\delta_i^* = \delta_i^* \ (\mu_y)$, $i = 1, 2, \cdots, m$, for given μ_y and then determine μ_y^* maximizing the expected profit. For a given value of μ_y, the expected profit is maximized by choosing the values of $\delta_1, \delta_2, \cdots, \delta_m$ that maximize the fourth term in equation (3). An upper bound of the fourth term for given μ_y is $\int_{-\infty}^{\infty} \{ \max_i k_i(y) \} g(y) dy$. This value is clearly attained by shipping the item to market i whenever $y \in I_i$, $i = 1, 2, \cdots, m$, where I_i is the set of real numbers y satisfying the inequalities $k_i(y) \geq k_j(y)$ for all $j \neq i$, simultaneously. Since $k_i(y) - k_j(y)$ for $j > 1$ is a nondecreasing function of y, it is clear that I_1 is given by the interval $I_1 = [\delta_1^*, \infty)$ where δ_i^*, $i = 1, 2, \cdots, m-1$, are the smallest real numbers satisfying the inequalities $k_i(y) \geq k_j(y)$ for all $j > i$, simultaneously. Similarly, if I_2 is not empty, it is of the form $I_2 = [\delta_2^*, \ \delta_1^*)$. If I_2 is empty, we let $\delta_2^* = \delta_1^*$. By the same reasoning, if I_i, $i = 3, 4, \cdots, m-1$, is not empty, it is of the form $I_i = [\delta_i^*, \ \delta_{i-1}^*)$. If I_i is empty, we let $\delta_i^* = \delta_{i-1}^*$.

Since $k_i(y)$ is a function of the parameters (A_i, a_i, τ), it is clear that the optimum values of $(\delta_1^*, \delta_2^*, \cdots, \delta_m^*)$ also depend on the same parameters and do not depend on the value of μ_y. Inserting the optimum values of $(\delta_1^*, \delta_2^*, \cdots, \delta_m^*)$ into equation (3), we obtain

$$EP = -s_y - c_0 - c_1 \mu_y + \int_{-\infty}^{\infty} \{ \max_i \ k_i(y) \} g(y) dy \tag{4}$$

Setting the partial derivative of equation (4) with respect to μ_y to zero, the following equation is obtained

$$\int_{-\infty}^{\infty} \{\max_i k_i(y)\}(y-\mu_y)g(y)dy - c_1\sigma_y^2 = 0 \tag{5}$$

It is difficult to find closed form expressions for the solution of equation (5). Numerical studies over a wide range of parameter values of $(\tau, A_i, a_i, \sigma_y, c_1)$ indicate that equation (5) has unique solution and it represents a maximum point. Search algorithms such as Newton-Rapson or bisection method can be used for finding the value of μ_y^*.

3. Model II: An Inspection Based on the Surrogate Variable

Let X be a surrogate variable which is relatively inexpensive to measure and highly correlated with the performance variable Y. We assume that X for given $Y=y$ is normally distributed with mean $\lambda_1+\lambda_2 y$ and variance σ^2 where λ_1 and λ_2 are known constants. λ_2 is assumed to be positive so that X and Y have a positive relationship. It can be easily shown that (X, Y) follow bivariate normal density function with means $(\mu_x=\lambda_1+\lambda_2\mu_y, \mu_y)$, and variances $(\sigma_x^2=\lambda_2^2\sigma_y^2+\sigma^2, \sigma_y^2)$ and correlation coefficient $\rho=\{\lambda_2^2\sigma_y^2 / (\lambda_2^2\sigma_y^2+\sigma^2)\}^{\frac{1}{2}}$. Since X and Y are positively correlated, and $A_i > A_j$ and $C_i(y, L) > C_j(y, L)$ for all $i < j$, an appropriate inspection procedure using a surrogate variable is as follows:

i) Take measurement x for each incoming item.

ii) Let ω_i, $i=1,2,\cdots,m$, be real numbers such that $\omega_1 \geq \omega_2 \geq \cdots \geq \omega_m = -\infty$ and $\omega_0=\infty$. If $\omega_i \leq x \leq \omega_{i-1}$, $i=1,2,\cdots,m$, ship the item to market i. Note that if $\omega_i=\omega_{i-1}$, the item will not be shipped to market i.

An item is shipped to market i whenever $\omega_i \leq x \leq \omega_{i-1}$, $i=1,2,\cdots,m$. Therefore, the expected revenue for market i is

$$A_i \int_{\omega_i}^{\omega_{i-1}} \int_{-\infty}^{\infty} h(x, y)dxdy, \tag{6}$$

where $h(x, y)$ is the joint density function of X and Y. We note that

$$h(x, y) = t(y|x)f(x), \tag{7}$$

where $f(x)$ is the marginal density function of X, and $t(y|x)$ is a conditional density function of Y, given $X=x$. It can be shown that $t(y|x)$ is a normal density function with mean $\mu_y + \rho(\sigma_y/\sigma_x)(x-\mu_x)$ and variance $\sigma_y^2(1-\rho^2)$.

The expected penalty cost for market i caused by imperfect quality is

$$\int_{\omega_i}^{\omega_{i-1}} \int_{-\infty}^{\tau} a_i(\tau-y)^2 h(x, y)dydx \tag{8}$$

Since the expected production cost per item is $c_0 + c_1\mu_y$, the expected profit per item is given by

$$EP = -s_x - c_0 - c_1\mu_y + \sum_{i=1}^{m} \int_{\omega_i}^{\omega_{i-1}} \int_{-\infty}^{\infty} A_i h(x, y)dydx$$

$$- \sum_{i=1}^{m} \int_{\omega_i}^{\omega_{i-1}} \int_{-\infty}^{\tau} a_i(\tau-y)^2 h(x, y)dydx, \tag{9}$$

where s_x is the inspection cost of X. After some derivations, the expected profit per item becomes (see Appendix)

$$EP = -s_x - c_0 - c_1\mu_y + \sum_{i=1}^{m} \int_{K_{i-1}}^{K_i} p_i(z)\lambda(z)dz, \tag{10}$$

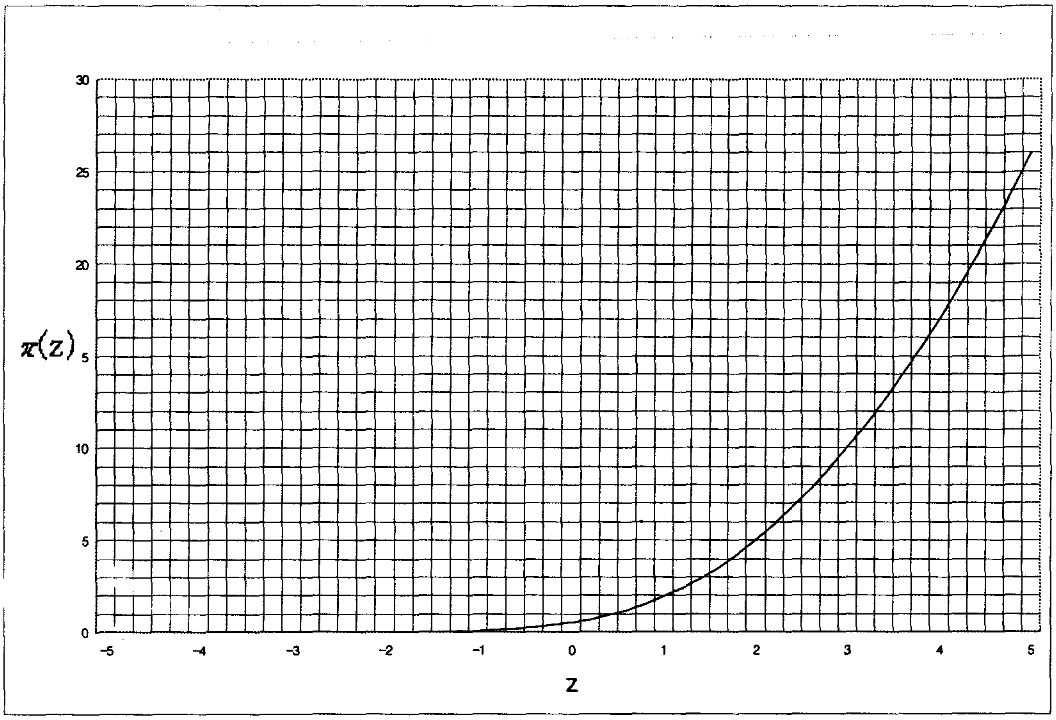
where

$$K_i = \frac{\tau - \{\mu_y + \rho(\sigma_y / \sigma_x)(\omega_i - \mu_x)\}}{\sigma_y\sqrt{1-\rho^2}}, \tag{10a}$$

$$p_i(z) = A_i - a_i\sigma_y^2(1-\rho^2) \cdot \pi(z) \tag{10b}$$

and $\lambda(z)$ is a normal density function with mean $\frac{\tau-\mu_y}{\sigma_y\sqrt{1-\rho^2}}$ and variance $\frac{\rho^2}{1-\rho^2}$. In equation (10b), $\pi(z) = (1+z^2) \cdot \Phi(z) + z \cdot \phi(z)$. A graph of $\pi(z)$ is given in

Figure (1). It shows that $\pi(z)$ is an increasing function of z.



< Figure 1 > Graph of $\pi(z)$

For a given value of μ_y, the expected profit is maximized by choosing the values of $(K_1, K_2, \cdots, K_m)$ that maximize the fourth term in equation (10). An upper bound of the fourth term is $\int_{-\infty}^{\infty}\{\max_i p_i(z)\}\lambda(z)dz$. This value is clearly attained by shipping the item to market i whenever $z \in J_i,\ i=1,2,\cdots,m$, where J_i is the set of real numbers z satisfying the inequalities $p_i(z) \ge p_j(z)$ for all $j \ne i$, simultaneously. Since $\pi(z)$ is an increasing function of z and $a_i > a_j$ for $j > i$, $p_i(z) - p_j(z)$ is a decreasing function of z. By the same reasoning as in the inspection based on the performance variable, J_i is given by the interval $J_i = (K_{i-1}^*, K_i^*]$ where $K_0^* = -\infty$, $K_m^* = \infty$, and K_i^* for $i = 1, 2, \cdots, m-1$ are the largest real numbers satisfying the inequalities $p_i(z) \ge p_j(z)$ for all $j > i$, simultaneously. If J_i is empty, we let $K_i^* = K_{i-1}^*$.

The range of z such that $p_i(z) \ge p_j(z)$ is

$$z \le z_1(i,j), \text{ for } i<j, \qquad z \ge z_1(i,j), \text{ for } i>j, \tag{11}$$

where $z_1(i,j) = \pi^{-1}\left\{\frac{A_i - A_j}{(a_i - a_j)\,\sigma_y^2\,(1-\rho^2)}\right\}$,

which is the real number z satisfying the equation $p_i(z) = p_j(z)$. <Figure 1> can be used for finding the value of $z_1(i,j)$.

Notice that the optimum values of $(K_1^*, K_2^*, \cdots, K_m^*)$ do not depend on μ_y. Inserting $(K_1^*, K_2^*, \ldots, K_m^*)$ into equation (10), we obtain

$$EP = -s_x - c_0 - c_1\mu_y + \int_{-\infty}^{\infty}\{\max_i p_i(z)\}\lambda(z)dz, \tag{12}$$

Setting the partial derivative of equation (12) with respect to μ_y to zero, the following equation is obtained

$$\int_{-\infty}^{\infty}\{\max_i p_i(z)\}\left(z - \frac{\tau-\mu_y}{\sigma_y\sqrt{1-\rho^2}}\right)\lambda(z)dz + c_1\sigma_y\rho^2/\sqrt{1-\rho^2} = 0. \tag{13}$$

It is difficult to find closed form expressions for the solution of equation (13). Numerical studies over a wide range of parameter values of $(\tau, A_i, a_i, \sigma_y, \rho, c_1)$ indicate that equation (13) has unique solution and it represents a maximum point. Search algorithms such as Newton-Rapson or bisection method can also be used for finding the value of μ_y^*. The optimum values of ω_i^*, $i=1,2,\cdots,m-1$ are obtained by

$$\omega_i^* = \lambda_0 + \lambda_1\mu_y^* + \frac{\sigma_x}{\rho\sigma_y}\{\tau - \mu_y^* - \sigma_y\sqrt{1-\rho^2}K_i^*\} \tag{14}$$

4. A Numerical Example

Consider a packing plant of cement industry. The plant consists of two processes; a filling process and an inspection process. Each cement bag processed by the filling machine is moved to the loading and dispatching phases on a conveyor belt.

Inspection is performed by continuous weighting feeders (CWFs). A CWF measures the mA (milli ampere) X of the load cell of the cement bag, which is positively correlated with the weight Y of the cement bag. From theoretical considerations and past experience, it is known that the variance of Y, $\sigma_y^2=(1.25\text{kg})^2$, and that X for given $Y=y$ is normally distributed with mean $4.0+0.08y$ and variance $(0.05\text{mA})^2$. That is, X and Y are jointly normally distributed with unknown means (μ_x, μ_y), known variances $\sigma_x^2=(0.112\text{mA})^2$, $\sigma_y^2=(1.25\text{kg})^2$, and correlation coefficient $\rho=0.894$. The weight marked on each bag is 40kg, and it is the target value τ .

The cement bag can be sold to foreign, domestic, or discount markets. The low quality product is scraped because of the penalty cost. The selling price in the foreign market is higher than that of the domestic market. The cost caused by imperfect quality in the foreign market is also higher than that of the domestic market, because of differences in the costs of identifying and handling a nonconforming item, labor cost, transportation cost, etc. We will consider the foreign market as market 1, the domestic market as market 2, the discount as market 3, and the scrap as market 4. The selling prices and the estimated penalty cost coefficients in dollars are as follows:

	Foreign market	Domestic market	Discount	Scrap
Price (A_i)	40	39	24	0
Penalty cost coefficient (a_i)	10.5	6.5	0.75	0

The production cost in dollars is $6.0+0.6y$ which is proportional to the quantity y, and the inspection costs are $s_x=\$0.2$ and $s_y=\$1.3$.

For Model I, we obtain $(\mu_y^*, \delta_1^*, \delta_2^*, \delta_3^*)$ = (41.74, 39.50, 38.38, 34.34). Hence the cement bags are sold to the foreign market if $y\geq 39.50$, to the domestic market if $38.38\leq y<39.50$, or to the discount market if $34.34\leq y<38.38$. If $y<34.34$, they are scraped. In this case the expected profit per item is \$7.333. The optimum values of μ_y^* and their expected profits are given in <Table 1> for selected values of σ_y for 0.50 (0.25) 2.50. It shows that μ_y^* can be set significantly lower as σ_y decreases. These results agree with our intuition that μ_y^* can be set τ if σ_y is zero. <Table 1> also shows that the expected profit increases as σ_y decreases.

< Table 1 > Optimum Values of μ_y^* and Expected Profits as a Function of σ_y

σ_y	μ_y^*	δ_1^*	δ_2^*	δ_3^*	Expected Profit
0.50	40.56	39.5	38.38	34.34	8.229
0.75	40.95	39.5	38.38	34.34	7.937
1.00	41.35	39.5	38.38	34.34	7.636
1.25	41.74	39.5	38.38	34.34	7.333
1.50	42.13	39.5	38.38	34.34	7.034
1.75	42.51	39.5	38.38	34.34	6.738
2.0	42.88	39.5	38.38	34.34	6.447
2.25	43.24	39.5	38.38	34.34	6.158
2.50	43.60	39.5	38.38	34.34	5.873

For Model II, we obtain $(\mu_y^*, \omega_1^*, \omega_2^*, \omega_3^*)$=(41.84, 7.145, 7.011, 6.599) with expected profit $8.384. In this example the expected profit of Model II is larger than that of Model I. <Table 2> gives the results of varying σ from 0.03 to 0.15. The expected profit tends to increase and μ_y^* tends to decrease as σ decreases. For a detailed discussion on the effect of σ in the optimal target value determination, see Tang and Lo(1993).

< Table 2 > Effects of σ

σ	ρ	μ_y^*	ω_1^*	ω_2^*	ω_3^*	Expected Profit
0.03	0.958	41.79	7.156	7.051	6.696	8.410
0.05	0.894	41.84	7.145	7.011	6.599	8.384
0.07	0.819	41.88	7.223	6.954	6.457	8.361
0.09	0.743	41.92	7.099	6.873	6.265	8.345
0.11	0.673	41.94	7.064	6.776	6.027	8.333
0.13	0.610	41.96	7.019	6.658	5.738	8.325
0.15	0.555	41.98	6.966	6.519	5.399	8.320

While we can obtain accurate values of the selling price for each market, it is sometimes difficult to obtain accurate penalty cost information. To study the sensitivity of this model to the changes of cost parameters, the percentage errors (PE) are given in Table 3 for selected combinations of $a_1, a_2,$ and a_3 with remaining parameters fixed. PE is expressed as

$$PE = \frac{EP^* - EP'}{EP^*} \times 100(\%) \qquad (15)$$

where EP^* and EP' are the expected profit obtained by using the actual cost parameters and the expected profit obtained by using the improper cost parameters, respectively. The values of a_i, $i=1,2,3$, are within $\pm 10\%$ or $\pm 20\%$ to represent overestimated or underestimated values of the actual value. For example, the 10^{th} cost parameters in <Table 3> are a_1=11.55, a_2=7.8, and a_3=0.6 which are obtained by 10% overestimation of a_1, 20% underestimation of a_2, and 20% underestimation of a_3. In this case, PE is 0.050%. <Table 3> shows that the model is very robust to the changes of cost parameters.

< Table 3 > Optimum solutions and percentage errors obtained by using improper cost parameters

No	a_1	a_2	a_3	μ_y^*	ω_1^*	ω_2^*	ω_3^*	PE(%)
1	8.4	5.2	0.6	41.72	7.139	6.993	6.532	0.115
2	8.4	5.85	0.675	41.77	7.128	7.003	6.568	0.073
3	8.4	6.5	0.75	41.80	7.114	7.011	6.598	0.113
4	9.45	7.15	0.825	41.85	7.122	7.018	6.624	0.058
5	9.45	7.8	0.9	41.88	7.105	7.025	6.647	0.149
6	9.45	5.2	0.675	41.74	7.150	6.991	6.569	0.080
7	10.5	5.85	0.75	41.80	7.152	7.001	6.598	0.020
8	10.5	6.5	0.825	41.84	7.145	7.009	6.624	0.004
9	10.5	7.15	0.9	41.87	7.138	7.017	6.647	0.017
10	11.55	7.8	0.6	41.91	7.141	7.028	6.529	0.050
11	11.55	5.2	0.75	41.77	7.164	6.989	6.599	0.086
12	11.55	5.85	0.825	41.82	7.159	6.999	6.625	0.031
13	12.6	6.5	0.9	41.86	7.161	7.008	6.647	0.034
14	12.6	7.15	0.6	41.89	7.156	7.021	6.529	0.039
15	12.6	7.8	0.675	41.92	7.151	7.027	6.565	0.061

5. Conclusions

We have considered the problem of jointly determining the optimum process mean and screening limits for each market in situations where there are several markets with different price/cost structures. A Taguchi's quadratic loss function is utilized for developing profit models. Two inspection procedures are considered; an inspection based on the quality characteristic of interest, and an inspection based on a surrogate variable which is highly correlated with the quality characteristic. It is difficult to find closed form expressions for the optimum process mean as well as to show analytically that the solution is optimum. Numerical analyses over a wide range of parameter values, however, indicate that the expected profit functions are indeed unimodal for the process mean. A numerical search such as Muller's method is used to find the optimum process mean. Extensive sensitivity analyses show that the optimum process mean and screening limits are very insensitive to the changes of cost parameters. Numerical results also show that the optimum process mean tends to increase and the expected profit tends to decrease as the process variation σ_y increases. Numerical studies are performed by using FORTRAN and IMSL (International Mathematical and Statistical Libraries) subroutines on a Pentium II PC. In most cases the results can be obtained within a few minutes. A possible area of further investigation would be the extension of the model to the cases where the parameters $(\sigma_y, \sigma, \lambda_1, \lambda_2)$ are unknown.

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Appendix : Derivation of equation (10)

Expected profit function (9) can be rewritten as

$$EP = -s_x - c_0 - c_1\mu_y + \sum_{i=1}^{m}\int_{\omega_i}^{\omega_{i-1}} A_i f(x)dx$$

$$-\sum_{i=1}^{m}\int_{\omega_i}^{\omega_{i-1}}\left[\int_{-\infty}^{\tau} a_i(\tau-y)^2 t(y|x)dy\right]f(x)dx. \quad \text{(A1)}$$

In equation (A1), it can be easily shown that

$$\int_{-\infty}^{\tau}(\tau-y)^2 t(y|x)dy=\sigma_y^2(1-\rho^2)\cdot\{(1+z^2)\cdot\Phi(z)+z\cdot\phi(z)\}, \tag{A2}$$

where $z=\dfrac{\tau-\{\mu_y+\rho(\sigma_y/\sigma_x)(x-\mu_x)\}}{\sigma_y\sqrt{1-\rho^2}}$, $\phi(\cdot)$ and $\Phi(\cdot)$ are the density and the cumulative distribution functions of the standard normal distribution, respectively. By transforming $Z=\dfrac{\tau-\{\mu_y+\rho(\sigma_y/\sigma_x)(X-\mu_x)\}}{\sigma_y\sqrt{1-\rho^2}}$, and inserting equation (A2) into equation (A1), we obtain equation (10).