▨연구논문

생산공정의 최적공정평균 및 검사기준값의 결정기법 연구

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Determination of Optimal Process Mean and Screening Specification Limits for a Production Process

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Abstract

This paper considers the problem of determining the optimal process mean and screening specification limits of a surrogate variable associated with product quality under two-stage screening procedure. In two-stage screening, the surrogate variable is inspected first to decide whether an item should be accepted, rejected or additional observations should be taken. If additional observations are required, the performance variable of interest is then observed to classify the undecided items. Assuming that the performance variable and the surrogate variable are jointly normally distributed, the optimal process mean and the screening limits are obtained by maximizing the expected profit which includes selling price, production, reprocessing, inspection and penalty costs. A numerical example is presented and numerical studies are performed to compare the proposed two-stage screening procedure with single-stage screening procedures.

Key Words: Process Mean; Screening Specification Limits; Surrogate Variable; Performance Variable; Screening Procedure

1. Introduction

As a result of advances in automated manufacturing systems, sensoring technology and automatic inspection equipment, complete inspections are increasingly being used in industry in order to improve outgoing product quality. Many products are inspected to determine whether its quality characteristic satisfies predetermined acceptance limits. Conforming products are sold at regular price, whereas all others are reprocessed or sold at discount price. Typical quality characteristics under consideration are weights, volume, and geometric dimensions. Products produced by a production process may deviate from the process mean because of variations in raw material, labor and operation conditions. The process mean may be adjusted to a higher value in order to reduce the proportion of the nonconforming products. Using a higher process mean, however, may result in a higher production cost. Consequently, the decision of selecting a process mean should be based on the tradeoff among production cost, payoff of conforming items, and the cost incurred due to nonconforming items.

Several researchers have studied this problem. Bettes(1962), Golhar(1987), and Golhar and Pollack(1988) consider a filling process in which underfilled or overfilled products are reprocessed at a fixed cost. Hunter and Kartha(1977), Bisgaard et al.(1984), and Carlsson(1984) study several sales conditions for products in which the quality characteristic is smaller than the specification limit. Boucher and Jafari(1991) and Al-Sultan(1994) discuss situations in which the items are subjected to lot-by-lot acceptance sampling rather than complete inspections. Elsayed and Chen(1993) and Arcelus and Rahim(1994) determine optimum levels of process parameters for products with multiple characteristics. Lee and Jang(1997) and Hong et al.(1999) consider the problem of jointly determining optimum target values in situations where there are several markets with different price/cost.

In all of these studies, inspection is performed on the quality characteristic of interest (performance variable). In some situations, it is impossible or not economical to directly inspect the performance variable. In such cases, the use of a surrogate variable which is highly correlated with performance variable is an attractive alternative, especially when inspecting the surrogate variable is relatively less expensive than inspecting the performance variable. In cement plants, for example, a performance measure of interest may be the weight of a cement bag, which is difficult to measure directly due to the high-speed of the packing line. The mil-ampere (mA) of the load cell is strongly correlated with the weight of a cement bag and does not require special effort to measure. Hence, it can be used as the surrogate variable(Bai and Lee [1993]). The idea of selecting the cutoff

value on surrogate variable has been studied by many researchers. Bai and Lee (1993) and Tang and Lo(1993) present economic models that determine the process mean and the screening limit on the surrogate variable when inspection is based on surrogate variable instead of performance variable.

In applications where quality assurance is critical, the outgoing quality improvement may be more important than the reduction in the inspection cost. Since a surrogate variable may not perfectly correlated with performance variable, some conforming items may be rejected and excluded from shipment while some nonconforming items may be accepted for shipment. These decision errors are likely to occur when the value of the surrogate variable is close to the screening limits. Consequently, in this situation, there may be an economic advantage to reduce the errors by observing the performance variable even though the inspection may be expensive. Of course, this can only be done when inspection of the performance variable is nondestructive. Accordingly, Tang(1988) and Bai et al.(1995) propose economic two-stage screenings where the surrogate variable is used in the first stage and the performance variable is used in the second stage.

This paper consider the problem of finding the optimal process mean and screening limits of a surrogate variable for a filling process under two-stage screening procedure. In single-stage screening, inspection is performed on the performance variable of interest or a surrogate variable that is correlated with the performance variable. In two-stage screening, a surrogate variable is inspected first to decide whether an item should be accepted, rejected, or additional observations should be taken. If additional observations are required, the performance variable is then observed to classify the undecided items. Assuming that the performance variable and the surrogate variable are jointly normally distributed, the optimal process mean and screening limits of the surrogate variable are jointly determined by maximizing the profit function, which involves selling price, and production, inspection, and penalty costs. In Section 2, two single-stage-screening procedures are reviewed. A two-stage screening procedure for a production process is presented and developed methods for finding the optimal process mean and screening specification limits on the surrogate variable in Section3. A numerical example and analysis of results are given in Section 4.

2. Single -Stage Screening Procedure

In this section, two models are reviewed, denoted Models I and II, for single-stage screening: in Model I, inspection is performed on the performance variable

Y; in Model II, inspection is performed on the surrogate variable X. Models I and II are considered to explain single-stage screening and compare the performance of single- and two-stage screenings.

2.1 Model I: Inspection is performed on Y

Let Y be a performance variable representing the quality characteristic of interest and L be the lower specification limit of Y. Suppose that Y is normally distributed with an unknown process mean μ_y and known variance σ_y^2. All items are inspected prior to shipment to determine whether they meet a lower specification limit L on Y. Let a and r denote the selling price and reprocessing cost, respectively, where items with $Y \geq L$ are sold at a fixed price a to the primary market, and items with $Y < L$ are refilled at a reprocessing cost $r(< a)$. The production cost per item is linear in Y, that is, $b+cy$ where b and c are constants. Let c_y denote the performance inspection cost per item. The profit function $P_{S,I}$ per item is

$$P_{S,I} = \begin{cases} a-b-cy-c_y, & Y \geq L, \\ E(P_{S,I})-r-c_y, & Y < L. \end{cases} \tag{1}$$

The profit function given equation (1) is the same as that of Golhar(1987). See Golhar(1987) for detailed derivations of the expected profit and the optimal process mean.

2.2 Model II : Inspection is performed on the correlated variable X

Let X be a variable that is positively correlated with Y. If X is negatively correlated with Y, we then use $-X$ as the screening variable rather than X. We assume that, for given $Y=y$, X is normally distributed with mean $\lambda_1+\lambda_2 y$ and variance σ^2 where λ_1 and λ_2 are known constants. The constant λ_2 is assumed to be positive so that X and Y have a positive relationship. It can be easily shown that (X, Y) follows a bivariate normal distribution with mean vector, covariance matrix, and correlation coefficient

$$(\mu_x=\lambda_1+\lambda_2\mu_y, \mu_y), \begin{bmatrix} \lambda_2\sigma_y^2+\sigma^2 & \rho\sigma_y\sqrt{\lambda_2^2\sigma_y^2+\sigma^2} \\ \rho\sigma_y\sqrt{\lambda_2^2\sigma_y^2+\sigma^2} & \sigma_y^2 \end{bmatrix}, \rho=\{\lambda_2^2\sigma_y^2/(\lambda_2^2\sigma_y^2+\sigma^2)\}^{1/2}$$

(see Tang and Lo[1993]). Let χ be the screening limit on the decision variable X. If $X \geq \chi$ we conclude that $Y \geq L$, and the item is sold to the primary market at a fixed price a. Since X is not perfectly correlated with Y, some items with $Y < L$ may be sold to the primary market. The errors of accepting items with $Y < L$ incur penalty cost d which includes costs of identifying and handling the nonconforming items, and service and replacement costs. If $X < \chi$, the item is refilled at a reprocessing cost r. The production cost per item is the same as on the previous model, and c_x denotes the inspection cost per item for X. Then, the profit function $P_{S,II}$ per item is

$$P_{S,II} = \begin{cases} a - b - cy - c_x, & X \geq \chi, \quad Y \geq L, \\ a - b - cy - c_x - d, & X \geq \chi, \quad Y < L, \\ E(P_{S,II}) - r - c_x, & X < \chi. \end{cases} \tag{2}$$

The profit function given equation (2) is the same as that of Bai and Lee(1993). See Bai and Lee(1993) for detailed derivations of the expected profit and the optimal process mean and the screening specification limit.

3. Model III : Two-Stage Screening Procedure

Since X is not perfectly correlated with Y, decision errors that reject conforming items or accept nonconforming items may occur. To minimize these errors, we present a two-stage screening in which a correlated variable X is used in the first stage and a performance variable Y is used in the second stage. The two-stage screening procedure is as follows:

First stage: Take a measurement χ *of* X *for each incoming item. The item is (i) accepted if* $\chi \geq \omega_1$, *(ii) undecided if* $\omega_2 \leq \chi < \omega_1$, *and (c) rejected if* $\chi < \omega_2$, *where* $\omega_1 \geq \omega_2$.
Second stage: Observe y *of* Y *for the undecided item and (i) accept if* $y \geq L$, *and (ii) reject if* $y < L$.

The screening limits for X are ω_1 and ω_2. Note that there are no misclassification errors at the second stage because all the undecided items are

inspected with the performance variable. The profit function $P_{T,III}$ is

$$P_{T,III}=\begin{cases} a-b-cy-c_x, & X\geq \omega_1, \quad Y\geq L, \\ a-b-cy-c_x-d, & X\geq \omega_1, \quad Y<L, \\ a-b-cy-c_x-c_y, & \omega_2\leq X<\omega_1, \quad Y\geq L, \\ E(P_{T,III})-r-c_x-c_y, & \omega_2\leq X<\omega_1, \quad Y<L, \\ E(P_{T,III})-r-c_x, & X<\omega_2. \end{cases} \tag{3}$$

Then the expected profit per item is given by

$$\begin{aligned} E(P)=a-b+r-\frac{1}{\alpha}\Big[\, & c_x+d\Psi(\eta_1,-\delta;-\rho)+r+c_y(\Phi(\eta_2)-\Phi(\eta_1)) \\ & +c\Big((L+\delta\sigma_y)\Phi(\eta_1)+\rho\sigma_y\phi(-\eta_1)+\int_{-\eta_2}^{-\eta_1}\phi(z)\beta(z)dz\Big)\Big], \end{aligned} \tag{4}$$

where $\delta=(\mu_y-L)/\sigma_y$ and $\eta_i=(\mu_y-\omega_i)/\sigma_x$ i=1, 2, $\alpha=\Psi(\eta_1-\delta;-\rho)+\Psi(\eta_2,\delta,\rho)$,

$\beta(z)=(L+\delta\sigma_y+\rho\sigma_y z)\Phi((\delta+\rho z)/\sqrt{1-\rho^2})+\sigma_y\sqrt{1-\rho^2}\phi((-\delta-\rho z)/\sqrt{1-\rho^2})$,

$\phi(\cdot)$ and $\Phi(\cdot)$ are the standard normal density and distribution functions, and $\Psi(\cdot,\cdot;\rho)$ is the standardized bivariate normal distribution function with correlation coefficient ρ, respectively. See Appendix for detailed derivation.

The optimal values δ^*, η_1^* and η_2^* can be obtained by maximizing $E(P_{T,III})$. If $E(P_{T,III})$ is a unimodal function of δ and η_i, then the optimal values δ^* and η_i^* can be obtained by equating the derivatives of $E(P_{T,III})$ with respect δ and η_i (i=1, 2) to zero and solving the resulting equations (5)-(7) simultaneously.

$$\begin{aligned} & -d\Phi(\eta_1^*-\delta^*\rho)/\sqrt{1-\rho^2})\phi(\delta^*)+c\sigma_y\Phi(\eta_1^*) \\ & -\frac{(c_y-c\beta(-\eta_2^*))\{\Phi((\eta_2^*-\delta^*\rho)/\sqrt{1-\rho^2})-\Phi((\eta_1^*-\delta^*\rho))/\sqrt{1-\rho^2})\}\phi(\delta^*)}{\Phi((\delta^*-\eta_2^*\rho)/\sqrt{1-\rho^2})} \\ & +c\int_{-\eta_2^*}^{-\eta_1^*}\{\sigma_y\Phi((\delta^*+\rho z)/\sqrt{1-\rho^2})+L\Phi((\delta^*+\rho z)/\sqrt{1-\rho^2})/\sqrt{1-\rho^2}\}\phi(z)dz=0, \end{aligned} \tag{5}$$

$$d\Phi((-\delta^* + \eta_1^*\rho)/\sqrt{1-\rho^2}) - c_y + c\{(L+\delta^*\sigma_y) - \rho\sigma_y\eta_1^* - \beta(-\eta_1^*)\}$$

$$-\frac{(c_y - c\beta(-\eta_2^*))\Phi((-\delta^* + \eta_1^*\rho)/\sqrt{1-\rho^2})}{\Phi((\delta^* - \eta_2^*\rho)/\sqrt{1-\rho^2})} = 0, \tag{6}$$

$$a(c_y - c\beta(-\eta_2^*)) - \{c_\chi + d\Psi(\eta_1^*, -\delta^*; -\rho) + r + c_y(\Phi(\eta_2^*) - \Phi(\eta_1^*))$$

$$+ c(L+\delta^*\sigma_y)\Phi(\eta_1^*) + \rho\sigma_y\phi(\eta_1^*) + \int_{-\eta_2^*}^{\eta_1^*}\phi(z)\beta(z)dz\Big\}\Phi((\delta^* - \eta_2^*\rho)/\sqrt{1-\rho^2}) = 0. \tag{7}$$

It is difficult to show analytically that equations (5)-(7) have a unique solution or to find closed-form solution since the left-hand sides of equations (5)-(7) have $\phi(\cdot)$, $\Phi(\cdot)$, $\Psi(\cdot,\cdot;\rho)$, and integral equation. Numerical studies over a wide range of parameter values of $(\rho, \sigma_y, c, c_\chi, c_y, d, r)$, however, indicate that equations (5)-(7) have a unique solution, the Hessian matrix at $(\delta^*, \eta_1^*, \eta_2^*)$ is negative definite and $(\delta^*, \eta_1^*, \eta_2^*)$ represents a maximum point. Therefore, the optimal values δ^*, η_1^* and η_2^* can be obtained by solving equations (5)-(7) simultaneously and a computational approach such as Gauss-Siedels iterative method can be used to obtain δ^*, η_1^* and η_2^*. The optimal process mean μ_y^* and the screening specification limits ω_1^* and ω_2^* of X are obtained by

$$\mu_y^* = L + \delta^*\sigma_y, \tag{8}$$

$$\omega_1^* = \mu_\chi - \eta_1^*\sigma_\chi, \tag{9}$$

$$\omega_2^* = \mu_\chi - \eta_2^*\sigma_\chi. \tag{10}$$

4. An Illustrative Example

In this section, an illustrative example that originally appeared in Bai and Lee (1993) is presented to illustrate the optimal solution procedures. Numerical studies are also performed to investigate the effects of σ_y, ρ and cost parameters. IMSL(1987) subroutines such as DNORDF, DBNRDF and DQDAG are used to evaluate the standard univariate and bivariate normal distribution functions and

integration respectively.

4.1 Description of the example

Consider a packing plant of cement factory. The plant consists of two processes; a filling process and an inspection process. Each cement bag processed by the filling machine is moved to the loading and dispatching phases on a conveyor belt. Continuous weighing feeders (CWFs) perform inspection. A CWF measures the mA (mil-ampere) X of the load cell of the cement bag. that is positively correlated with the weight Y of the cement bag. From theoretical considerations and past experience, it is known that the variance of Y, $\sigma_y^2=(1.25\,\mathrm{kg})^2$, and that X for given $Y=y$ is normally distributed with mean $4.0+0.08y$ and variance $(0.05\,\mathrm{mA})^2$. That is, X and Y are jointly normally distributed, with unknown means (μ_x, μ_y), known variances $\sigma_x^2=(0.112\,\mathrm{mA})^2$, $\sigma_y^2=(1.25\,\mathrm{kg})^2$ and correlation coefficient $\rho=0.894$. The lower specification limit of the weight is marked on every bag as 40kg. Suppose that the cost components are $a=\$3.0$, $r=\$0.18$, $b=\$0.1$, $c=\$0.06$, $c_y=\$0.04$, $c_x=\$0.004$, and $d=\$6.0$.

For the two-stage screening procedure, we obtain $\delta^*=1.330$, $\eta_1^*=-0.257$ and $\eta_2^*=2.700$ are from equations (5)-(7). Therefore the optimal process mean and screening specification limits for X are

$$\mu_y^*=L+\delta^*\sigma_y=40.0+(1.330\times1.25)=41.662(\mathrm{kg}),$$

$$\omega_1^*=\mu_x-\eta_1^*\sigma_x=4.0+0.08\times41.662-(-0.257\times0.112)=7.304(\mathrm{mA}),$$

$$\omega_2^*=\mu_x-\eta_1^*\sigma_x=4.0+0.08\times41.662-(2.700\times0.112)=7.031(\mathrm{mA}),$$

and $E(P)=\$0.3438$.

4.2 Numerical studies

In the same example, we obtain the following result for Models I and II,

$$\mu_{I,y}^*=41.726(\mathrm{kg}),\ \text{and}\ E(P_I)=\$0.3019,$$

$$\mu_{II,y}^*=41.726(\mathrm{kg}),\ \chi^*=7.239(\mathrm{mA})\ \text{and}\ E(P_{II})=\$0.3067,$$

where χ^* is optimal screening specification limit of Model II.

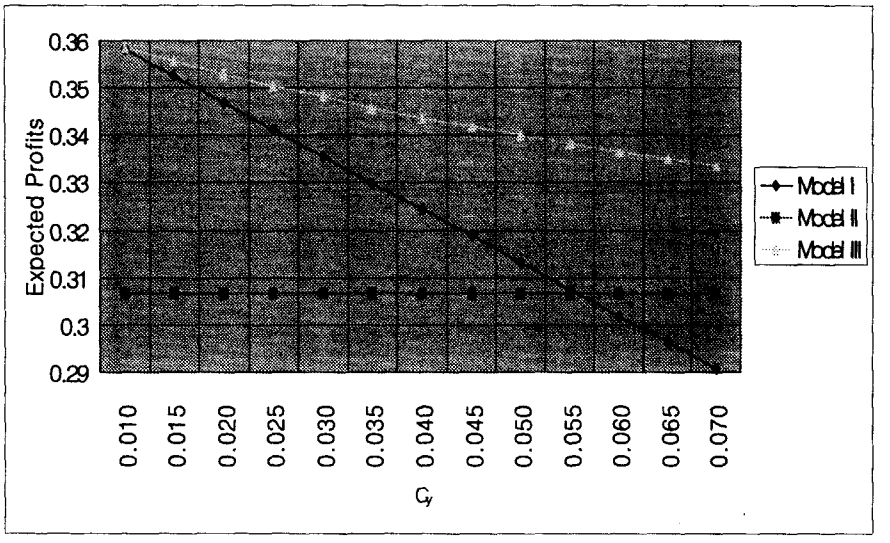
These results agree with our intuition that the expected profits for Models II and III are higher than the single-stage screening procedures. In applications where quality assurance is critical, the outgoing quality improvement may be more important than the reduction in the inspection cost. Since a surrogate variable is not perfectly correlated with quality characteristic, some conforming items may be rejected and excluded from shipment while some nonconforming items may be accepted for shipment. These decision errors are likely to occur when the value of a surrogate variable is close to the screening specification limits. Consequently, in this situation, there may be an economic advantage to reduce the errors by using two-stage screening procedure. However, two-stage screening is somewhat complex to implement. In some cases, it is impossible or not economical to directly inspect the performance variable. In these situations, Model II can be effectively used. We conduct numerical studies to investigate the effects of the parameters $(\rho, \sigma_y, c, c_y, d, r)$.

(i) Effects of σ_y: $E(P_I), E(P_{II})$ and $E(P_{III})$ for the above example are shown in <Table 1> for selected values of σ_y for 0.25 (0.25) 3.50, the expected profit decreases as σ_y increases. The computational results agree with our intuition that expected profit $E(P_{III})$ for the two-stage screening is somewhat higher than that of the single-stage screenings. $E(P_{II})$ is higher than $E(P_I)$ if σ_y takes small values, but $E(P_{II})$ is lower than $E(P_I)$ if σ_y takes large values. It is also shown that μ_y^* increases as σ_y increases.

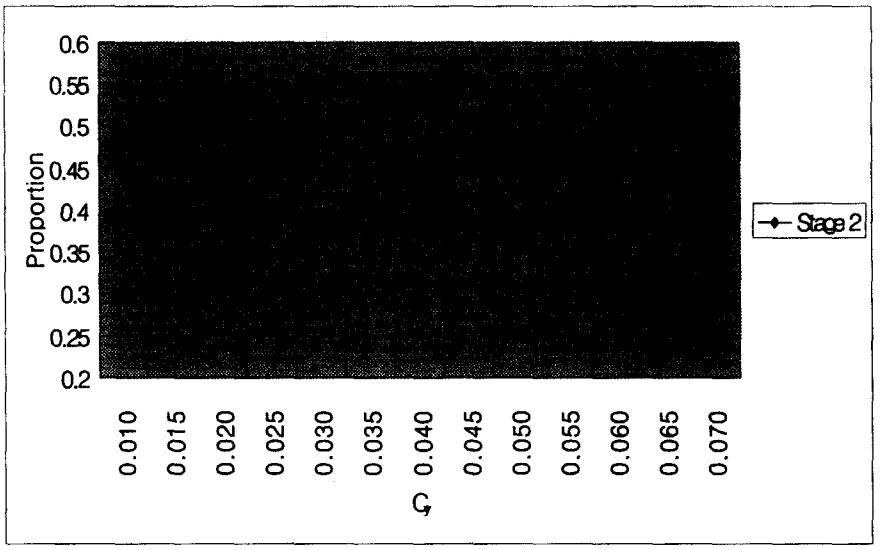
(ii) Effects of c_y: $E(P_I), E(P_{II})$ and $E(P_{III})$ for the above example are shown in <Figure 1> for selected values of c_y for 0.01 (0.005) 0.07. The expected profit $E(P_{III})$ for the two-stage screening is somewhat higher than that of the single-stage screening. The difference increases as c_y increases. $E(P_I)$ is higher than $E(P_{II})$ if c_y takes small values, but $E(P_I)$ is lower than $E(P_{II})$ if c_y takes large values. The inspection proportion of stage 2 is given in <Figure 2> for selected values of c_y or 0.01 (0.005) 0.07. The computational results agree with our intuition that the inspection proportion of stage 2 tends to decrease as c_y increases.

< Table 1> Effects of σ_y

σ_y	Model I		Model II		Model III	
	μ_y^*	$E(P_I)$	μ_y^*	$E(P_{II})$	μ_y^*	$E(P_{III})$
0.25	40.329	0.4147	40.680	0.4492	40.572	0.4541
0.50	40.662	0.3923	41.207	0.4102	40.945	0.4218
0.75	40.997	0.3697	41.667	0.3742	41.231	0.3936
1.00	41.334	0.3471	42.081	0.3404	41.469	0.3678
1.25	41.674	0.3243	42.461	0.3080	41.665	0.3438
1.50	42.017	0.3014	42.813	0.2769	41.836	0.3212
1.75	42.362	0.2784	43.142	0.2467	41.983	0.2997
2.00	42.709	0.2553	43.451	0.2174	42.106	0.2790
2.25	43.059	0.2321	43.742	0.1888	42.218	0.2590
2.50	43.410	0.2088	44.016	0.1609	42.315	0.2397
2.75	43.764	0.1854	44.277	0.1335	42.405	0.2209
3.00	44.120	0.1619	44.525	0.1067	42.479	0.2026
3.25	44.477	0.1383	44.760	0.0803	42.533	0.1847
3.50	44.837	0.1146	44.984	0.0544	43.596	0.1672

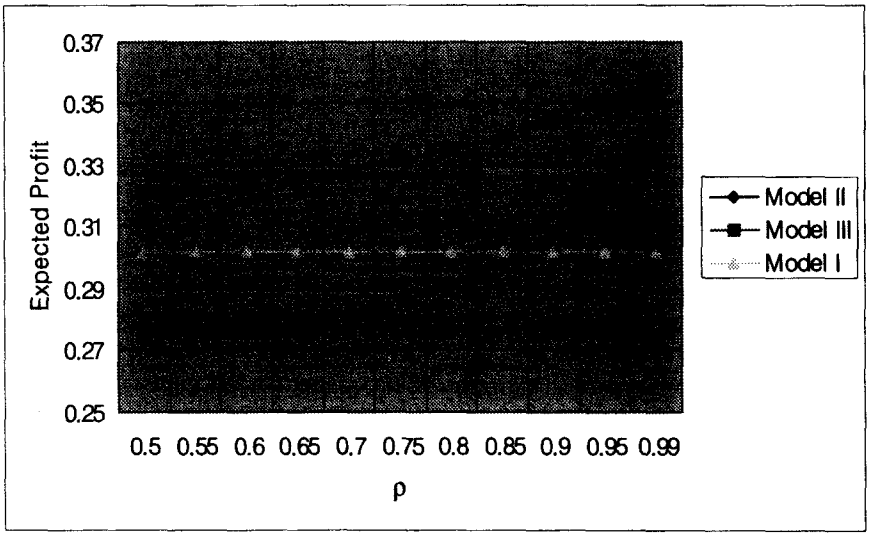


< Figure 1 > Expected Profits as a function of c_y

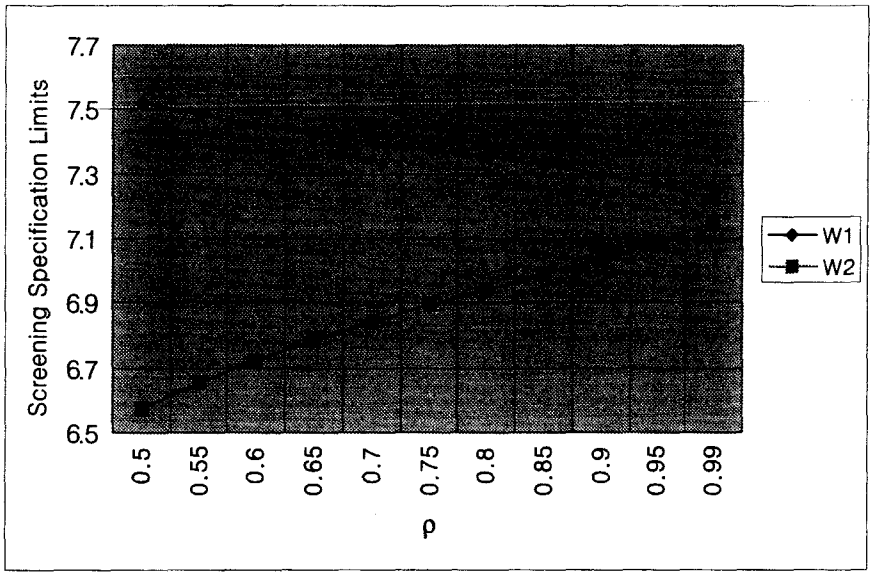


< Figure 2 > Proportion of inspection as a function of c_y

(iii) Effects of ρ: The expected profit per item and the optimal screening specification limits on X are given in <Figures 3> and 4 for selected values of ρ [0.50(0.05) 0.95,0.99] <Figure 3> shows that $E(P_{II})$ and $E(P_{III})$ increase as ρ increase. $E(P_{III})$ is higher than $E(P_{II})$ and the difference between $E(P_{III})$ and $E(P_{II})$ tends to decrease as ρ increases; these computational results agree with our intuition. ω_1^* tends to decrease and ω_2^* tends to increase as ρ increases.



< Figure 3 > Expected profits as a function of ρ



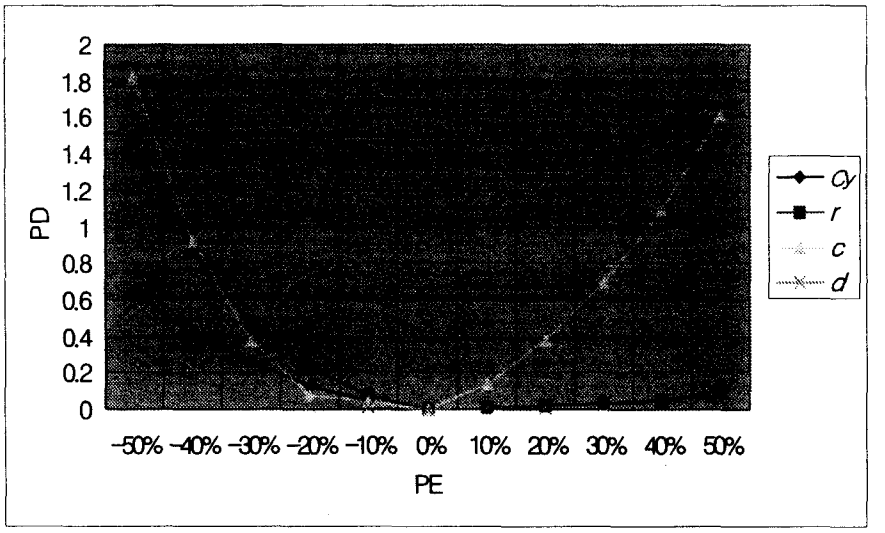
< Figure 4 > Screening specification limits for two-stage screening procedure as a function of ρ

(iv) Effects of using improper cost factors: It is sometimes difficult to obtain accurate estimates of cost parameters (c, c_y, d, r). Use incorrect values for these parameters, $(\mu_y^*, \omega_1^*, \omega_2^*)$ will not guarantee the optimality of the procedure. As a result, the expected profit is expected to be lower than the expected profit with true values. To study the sensitivity of Model III to the changes in cost parameters, the percentage decrease (PD) is given in <Figure 5> for selected values of c, c_y, d and r keeping the remaining parameters constant. PD and PE are expressed as

$$PD = \frac{E(P_{III})^* - E(P_{III})'}{E(P_{III})^*} \times 100(\%)$$

$$PE = \frac{\text{incorrect values of } (c, c_y, d, \gamma) - \text{ true values of } (c, c_y, d, r)}{\text{true values of } (c, c_y, d, \gamma)} \times (100\%)$$

where $E(P_{III})^*$ and $E(P_{III})'$ are the expected profit obtained by using the actual value and incorrect cost parameters, respectively. <Figure 5> indicates that PD is more affected by c than by c_y, d and r, however, Model III is robust to the changes in cost parameters because PD is less than 2%.



< Figure 5 > Graph of PD versus PE for Cost Factors

5. Conclusions

An economic two-stage screening procedure is presented using a performance and surrogate variable for the filling processes, and reviewed two single-stage screening procedures to explain single-stage screening and compare the performance of single- and two-stage screeing procedures. Assuming that the quality characteristic of interest and surrogate variable are jointly normally distributed, the optimal process mean and the screening limits are obtained by maximizing the expected profit which includes selling price, production, reprocessing, inspection and penalty costs. The optimal solution is not given in a closed form expression. Using software such as FORTRAN and IMSL libraries, however, it can be obtained by a numerical search algorithm such as Gauss-Siedels iterative method without difficulty.

Numerical analyses show that the expected profit for the two-stage screening procedure is somewhat higher than that of the single screening procedure based on performance variable and the difference increases as c_y increases. Expected profit for the two-stage screening procedure is higher than that of single screening procedure based on surrogate variable and the difference between two and single-stage screening procedures tends to decrease as ρ increases. Expected profit for single-stage screening procedure based on performance variable is higher

than that of single-stage screening procedure based on surrogate variable if c_y takes small values, but the expected profit for single-stage screening procedure based on performance variable is lower than that of single-stage screening procedure based on surrogate variable if c_y takes large values. The two-stage screening procedure is robust to the changes in the cost parameters. Expected profit for Models I, II and III decreases as σ_y increases, and the process mean tends to increase as σ_y increases.

Appendix : Derivation of Equation (4)

The expected profit per item is given by

$$E(P)=\int_{\omega_1}^{\infty}\int_{L}^{\infty}(a-b-cy-c_x)f(x,y)dy\,dx+\int_{\omega_1}^{\infty}\int_{-\infty}^{L}(a-b-cy-c_x-d)f(x,y)dy\,dx$$

$$+\int_{\omega_2}^{\omega_1}\int_{L}^{\infty}(a-b-cy-c_x-c_y)f(x,y)dy\,dx+\int_{\omega_2}^{\omega_1}\int_{-\infty}^{L}(E(P)-r-c_x-c_y)f(x,y)dy\,dx$$

$$+\int_{-\infty}^{\omega_2}\int_{-\infty}^{\infty}(E(P)-r-c_x)f(x,y)dy\,dx. \qquad \text{(A.1)}$$

Using the following relationships

$$\int_{\omega_2}^{\omega_1}\int_{-\infty}^{L}g(x,y)dy\,dx=\Psi(-\delta_2,\eta;-\rho)-\Psi(-\delta_1,\eta;-\rho),$$

$$\int_{\omega_2}^{\omega_1}\int_{L}^{\infty}g(x,y)dy\,dx=\Psi(-\delta_2,-\eta;\rho)-\Psi(-\delta_1,-\eta;\rho),$$

$$\int_{\omega_2}^{\omega_1}\int_{L}^{\infty}cx\,f(x,y)dy\,dx=\int_{-\eta_2}^{-\eta_1}\phi(z)\beta(z)dz,$$

after some computation, equation (A.1) can be rewritten as equation (4)

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