궤환성을 갖는 단층신경회로망의 Inhibitory Synapses

論 文
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Inhibitotory Synapses of Single-layer Feedback Neural Network

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Abstract - The negative weight can be often seen in Hopfield neural network, which is difficult to implement negative conductance in circuits. Usually, the inverted output of amplifier is used to avoid negative resistors for expressing the negative weights in hardware implementation. However, there is some difference between using negative resistor and the inverted output of amplifier for representing the negative weight. This difference is discussed in this paper.

Key Words : Hopfield neural network, negative weight, inverted outputs.

1. Introduction

The single-layer neural networks discussed in this paper are inherently feedback-type nonlinear networks. Neurons with a continuous activation function can be used in such systems. The networks discussed here are based on the papers of Hopfield[1,2,3]. Many years of development in the area of continuous system have contributed to the existing state of the single-layer feedback networks. As known, the networks can be useful in many ways. They can provide associations or classifications, optimization problem solution, restoration of patterns, and they can be viewed as mapping networks. Despite some unsolved problems and limitations of the fully coupled single-layer networks, their impressive performance has been documented in the technical literatures[2,4]. Both hardware implementations and their numerical simulations indicate that single-layer feedback networks provide a useful alternative to traditional approaches for pattern recognition, association, and optimization problems.

One among the existing problems in hardware implementation is to express an inhibitory synapse. The common method to avoid using a negative resistor for an inhibitory synapse, the amplifier used for neuron has both outputs: one is a normal output(+), the other is an inverted output(-). An excitatory synapse is connected to a normal output of neuron and an inhibitory synapse is connected to an inverted output[2]. In this paper, we discuss the difference between the negative resistor and inverting output for constructing an inhibitory synapse. Also, we discuss the considerations when the inverting output is used for implementing an inhibitory synapse.

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2. Neural Networks For Optimization Problems

Most of the combinatorial optimization problems can be replaced by the effort to search for the minimum of the function F on $\{-1,1\}^n$ defined by

$$F(v)=\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}w_{ij}v_iv_j-\sum_{i=1}^{n}i_iv_i \qquad (1)$$

where w_{ij} and i_i are constants satisfying $w_{ij}=w_{ji}(i\neq j)$ and $w_{ii}=0$ for i=,j =1,...,n. We call this function an "objective function"[7]. We review an electrical model for the Hopfield neural networks.

The Hopfield networks is a simple neural circuit which consists of synaptic connections and neurons. In the actual network, the non-linear input-output relation of neuron is determined by a sigmoid function[4,6]

$$f(u_i)=\frac{1}{1+e^{-\alpha u_i}} \qquad (2)$$

where u_i is neuron input and α is sigmoid gain. The neurons are coupled together by a set of non-linear differential equations,

$$c_i\frac{du_i}{dt} = \sum_{j=1}^{n} w_{ij}v_j + i_i - (\sum_{j=1}^{n} w_{ij} + g_i)u_i, \quad i=1,\dots,n \tag{3}$$

where n is the number of neurons, v is the neuron output, u is neuron input, w is the synaptic connection, c is the input capacitance, and similarly g_i represents the input conductance between the i_th neuron input and ground. Conductance w_{ij}connects the output of the j_th neuron to the input of i_th neuron. Current i_i is the bias incoming current into the i_th neuron input.

Denoting the total conductance connected to the i_th neuron input node as G_i, where

$$G_i = \sum_{j=1}^{n} w_{ij} + g_i \tag{4}$$

Eq. (1) can be simplified to the form of a single state equation as

$$c_i\frac{du_i}{dt} = \sum_{j=1}^{n} w_{ij}v_j + i_i - G_i u_i, \quad i=1,\dots,n \tag{5}$$

Hopfield showed that provided $w_{ij} = w_{ji}$, and $w_{ii}=0$ for all i and j, the state of the networks v_i converges to a local minima of a Lyapunov energy function[1]

$$E(v) = -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} W_{ij}v_iv_j - \sum_{i=1}^{n} i_iv_i + \sum_{i=1}^{n} G_i\int_{0.5}^{v_i} f^{-1}(z)\,dz \tag{6}$$

It is claimed that $dE/dt \le 0$ for all i, therefore, it shows that the Lyapunov energy function will always be minimized.

3. Inhibitory Synapses using Inverting Neuron

The elementary nerve cell, called a neuron, is the fundamental building block of the biological neural network. A typical neuron has three major regions: the cell body, the axon, and the dendrite. The synapse is a contact organ which connects the axon of neuron to the neighboring neurons' dendrites. There are two kinds of synapses, one is a excitatory one which causes a neuron firing, the other is inhibitory one which hinders the firing of neuron[9]. We often see the negative weights in optimization problem using neural networks. These negative weights can be regarded as inhibitory synapses. When designing Hopfield neural networks, the synapse w_{ij} which connects the jth neuron output to the ith neuron input can be expressed by the resistance $R_{ij}=1/w_{ij}$. To provide for both excitatory and inhibitory synaptic connections between neurons while using conventional electrical components, each amplifier used for neuron is given two outputs: a normal output(+) and an inverted output(-) of the same magnitude but opposite in sign. If the synapse is excitatory($w_{ij} > 0$), this resister ($R_{ij}=1/|w_{ij}|$) is connected to the normal output(+) of the jth amplifier. For an inhibitory synapse ($w_{ij} < 0$), it is connected to the inverted output(-) of the jth amplifier. Thus, the normal and inverted outputs for each neuron allow for the construction of both excitatory and inhibitory connections through the use of normal (positive valued) resistors.

3.1 The two Models: Using Inverting Output and Negative Conductnace

In this section, we use Norton's equivalent circuit to analyze the difference between using inverting output and negative resistor to represent inhibitory synapse. Fig. 1 shows the circuit seen from the ith neuron input node,

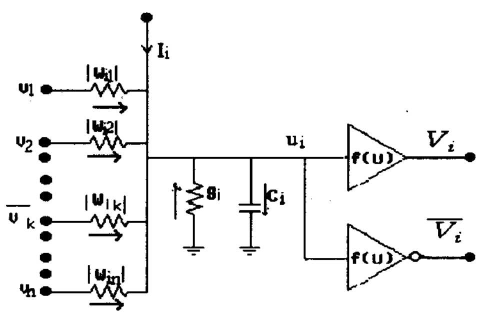


Fig. 1 Input node i connecting inhibitory synapse w_{ij} using inverting output $\overline{V_k}$.

where the synapse w_{ik} is inhibitory, all other synapses are excitatory. Inverting output of the kth amplifier and $|w_{ik}|$ are used for the inhibitory synapse w_{ik}. Fig. 2 shows the equivalent circuits of the negative conductance and the inverting output used for implementing the inhibitory synapse w_{ik}. For two circuits to be same, the total incoming currents and total conductance at the ith input node should be same. Let's check the total incoming currents at the ith input node, then the total incoming currents using the negative conductance is as

$$I_i neg = \sum_{j=1, j\neq k}^{n} |w_{ij}|v_j - |w_{ik}|v_k + i_i \tag{7}$$

and the total incoming currents using the inverting output is as

$$I_i inv = \sum_{j=1, j\neq k}^{n} |w_{ij}|v_j + |w_{ik}|\overline{v_k} + i_i \tag{8}$$

As seen from Eq. (6) and (7), it can be known that the total incoming currents in both circuits are same. However, the total conductances in both circuits are different, the total conductance using negative conductance is

$$G_i neg = \sum_{j=1, j\neq k}^{n} |w_{ij}| - |w_{ik}| + g_i \quad (9)$$

and , the one using the inverting output is as

$$G_i inv = \sum_{j=1}^{n} |w_{ij}| + g_i \quad (10)$$

As we have seen, these two models are not same. However, these two models have been assumed same so far. In the next two sections, we discuss when this assumption can be accepted or not.

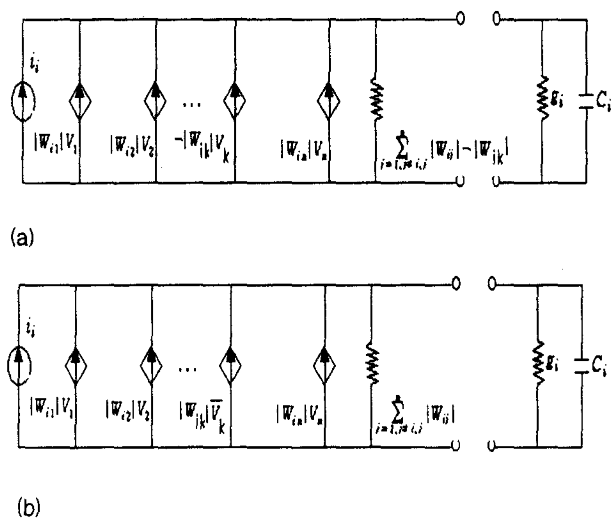


Fig. 2 Equivalent circuits seen from input node i where W_{ij} is inhibitory,
(a) Using non-inverting neuron, (b) Using inverting neuron.

3.2 The High Sigmoid Gain System

As mentioned at the previous section, the popular method of designing neural networks involves an energy function of the form Eq. (6). The first two terms of this equation are chosen in design process to correspond to quadratic cost function which when minimized over the range of vi, i=1,..,n, will yield desired solutions to the problem of interest. The usual justification for ignoring this term is that the integral vanishes in the high sigmoid gain.

The negative gradient of the energy function(6) can be computed as

$$-\nabla E(v) = \sum_{j=1}^{n} w_{ij} v_j + i_i - G_i u_i, \quad i=1, \ldots, n \quad (11)$$

By comparing (7) with the right hand side of (5), it can be written as

$$-\nabla E(v) = C_i \frac{du_i}{dt}, \qquad i=1, \ldots, n \quad (12)$$

Using the inverse function $f^{-1}(z)$, the third term in Eq. (6) can be expressed as

$$E_3 = \frac{1}{\alpha} \sum_{i=1}^{n} G_i \int_{\frac{1}{2}}^{v_i} \ln\left(\frac{z}{1-z}\right) dz \quad (13)$$

As the sigmoid gain(α) is approaching infinity, it can be seen that this third term is disappearing and also is disappearing the third term of Eq. (11). By comparing (11) with (12), the state equation (5) can be simplified as follows,

$$c_i \frac{du_i}{dt} = \sum_{j=1}^{n} w_{ij} v_j + i_i \qquad i=1, \ldots, n \quad (14)$$

Thus, the incoming current into the ith capacitor is not affected by the total conductance G_i. In the Hopfield networks with high sigmoid gain, both two systems which are using inverting neuron and negative resistor for the inhibitory synapses yield the same results in solving the problem of interest.

3.3 The Finite Sigmoid Gain System

As mentioned before, the common method for ignoring the third term is to use high sigmoid gain. However, as a sigmoid gain α goes infinite, the output of neuron becomes discrete rather than continuous[5,7]. In this section, we discuss new method with which both two systems for inhibitory synapses yield same result with a finite sigmoid gain. However, the system efficiency of solving problem decreases because of existence of the third term of energy. This problem and the solution are discussed in section 3.3.2.

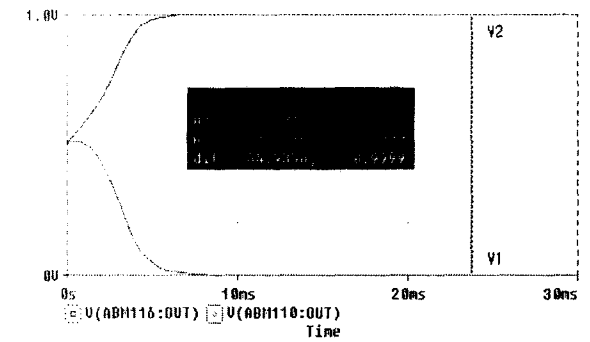
3.3.1 Inverting Neuron with the Finite Sigmoid Gain

With this new method, the system can keep the continuous output of neuron as well as yield same result in solving the optimized problems as using negative resistor. With the finite sigmoid gain, the third term of energy function (6) exists, and therefore the incoming current into the ith capacitor is affected by the total conductance G_i. As known in Eq. (9) and (10), the total conductance in two models for implementing inhibitory synapses are different.

Thus, for the two systems to yield same results, the energy function in both cases should be same. Thus, the total conductance in both models should be same as follows

$$G_i neg = G_i inv \quad (15)$$

However, synapses w_{ij} is the determined value from the beginning when to design system, therefore w_{ij} is unchangeable. But, the parasitic conductance g_i , which is connected in parallel with capacitor c_i, can be adjustable. This parasitic conductance is only known to partially define the time-constant with the parasitic capacitor and to contribute to system's stability. However, it has not been analyzed how to contribute to system's stability. Anyway, the value for this conductance has been selected experimentally, in other words, a trial and err method has been used to select the value of this conductance when to implement hardware so far. The method how to select optimized value for this conductance is discussed in next section.



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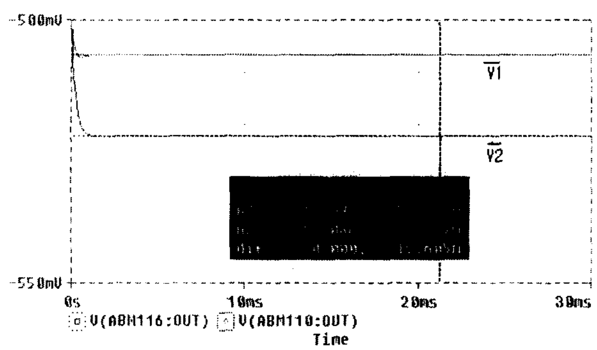

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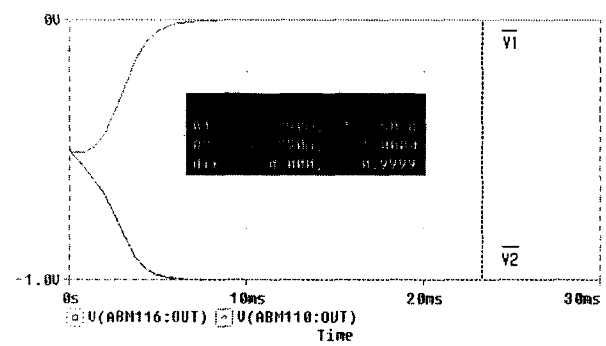


(c)

Fig. 3. 2bit A/D converter transient simulation of output. in case x=1.6, and α=2:

(a) Using noninverting neuron and g1=g2=2.2

(b) Using inverting neuron and g1=g2=2.2

(c) Using inverting neuron and g1=g2=-1.8

To make the total conductance in both models same, this parasitic conductance can be adjustable by comparing Eq.(9) with Eq.(10) as follows

$$g_i inv = \sum_{j=1, j\neq i}^{n} |w_{ij}| - |w_{ij}| + g_i neg - \sum_{j=1}^{n} |w_{ij}| \quad (16)$$

Case Study

The 2-bit A/D converter is selected for a case study[3]. The energy function for 2-bit A/D converter .including weights and bias currents can be expressed as follows [3], where x is analog input

$$E = -\frac{1}{2}[v_1 \ v_2]\begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - [v_1 \ v_2]\begin{bmatrix} x-\frac{1}{2} \\ 2x-2 \end{bmatrix} + \begin{bmatrix} \frac{G_1}{\alpha}\int_{0.5}^{v_1} \ln\left(\frac{z}{1-z}\right)dz \\ \frac{G_2}{\alpha}\int_{0.5}^{v_2} \ln\left(\frac{z}{1-z}\right)dz \end{bmatrix} \quad (17)$$

Fig. 3a, 3b shows the transient results in case of x=1.6, α=2, and $g_1 = g_2 = 2.2$using noninverting neuron and using inverting neuron respectively. For using noninverting output of amplifier, -0.5ohm is selected for $-1/|w_{12}|$ and $-1/|w_{21}|$ and 1/2.2 ohm is selected for $1/g_1$ and $1/g_2$. And for using inverting output of amplifier, 0.5ohm is selected for $1/|w_{12}|$ and $1/|w_{21}|$ and same value is selected as before for $1/g_1$ and $1/g_2$. As expected, these figures show totally different results with finite sigmoid gain α=2. Fig. 3c shows result when the parasitic conductance g_1, g_2 are adjusted to -1.8 respectively using (16) and this system yields same result as in Fig. 3a.

3.3.2 Effects of the total conductance on convergence

The third term energy (13) is zero for vi=1/2 and positive otherwise, getting large as vi approaches 0 or 1 because of slowness with which $f(v)$ approaches its asymptotes (Fig. 4a). As mentioned early in this section, it can be known from Eq. (13) that the common method for neglecting the third term of energy function E_3 is to lect the sigmoid gain α very high.

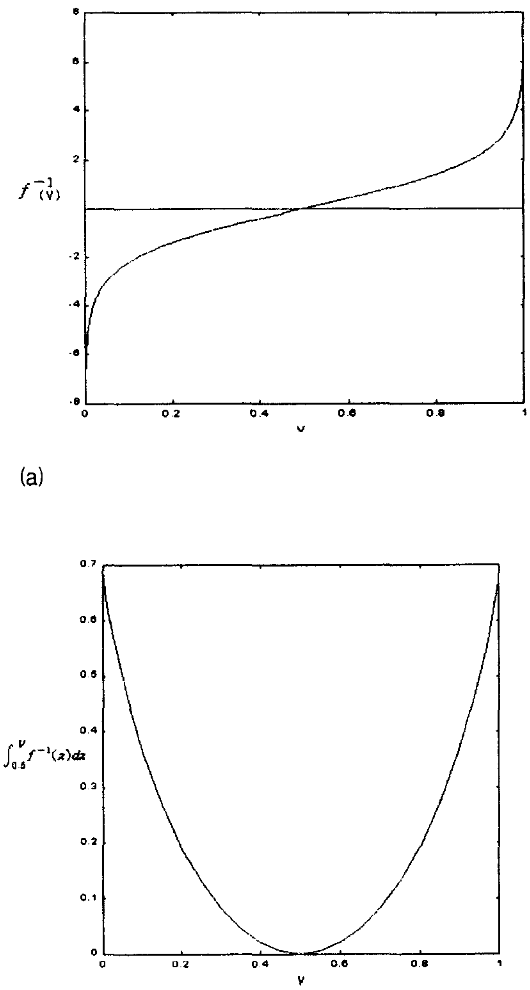
Consider now new method of ignoring the third term of energy function with the finite sigmoid gain using . E_3 is a function of vi, the sigmoid gain α, and the total conductances G_i. Here, we introduce a new method to eliminate this term by adjusting the total conductances G_i. To eliminate the third term energy, the total conductances G_i can be adjusted to zero.

$$G_i = g_i + \sum_{j=1}^{n} w_{ij} = 0 \qquad i = 1, 2, \ldots, n.$$

(18)Thus, the input conduces g_i can be adjusted as follows

$$g_i = -\sum_{j=1}^{n} w_{ij} \qquad i=1,2,\ldots,n. \qquad (19)$$

This choice of g_i in the system would ensure



(a)

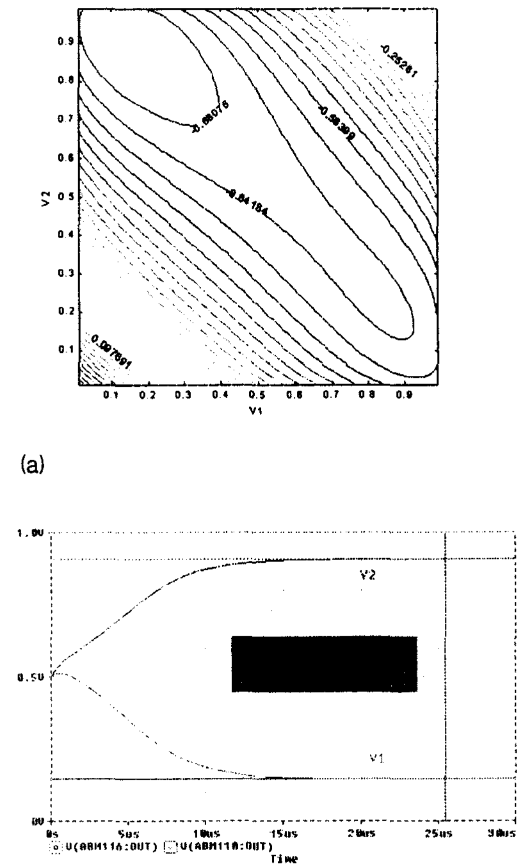
(b)

Fig. 4. (a) The output-input transfer characteristic, $u=f^{-1}(V)$.
(b) The contribution of f^{-1} to the third term of energy function.

convergence exactly to the objective function. However, it has been known that the total conductances G_i equal to zero causes the input space to be unstable, that is, the input space changes linearly to infinite in time. Therefore, the total conductances G_i should be selected as small as possible to neglect the third term energy, but should be greater than zero to make the input space stable[8].

Case Study

The 2-bit A/D converter is also selected for this case study. Fig. 5 shows the energy map and transient result

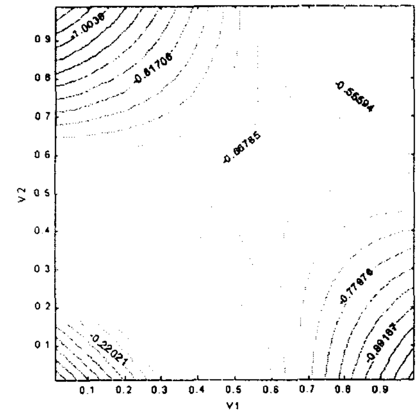


(a)

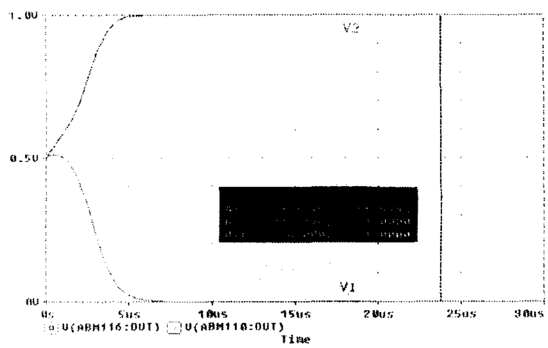
(b)

Fig. 5. 2bit A/D converter in case x=1.6, g1=g2=2.8, and α=2: (a) The energy map, (b) Transient simulation of output.

in case of x=1.6, α=2, and $G_1=G_2=0.8$. -0.5ohm isselected for $-1/|w_{12}|$ and $-1/|w_{21}|$ and 1/2.8ohm is selected for $1/g_1$ and $1/g_2$. We can see that the system converges near to the answer but not correct one(the digital outputs should be v1=0 and v2=1 for analog input x=1.6). However the system would converge farther from the answer as the total conductances G_i grows, because this minima moves toward the center of the energy map as the third term energy grows. Fig. 6 shows the energy map and transient result where $G_1=G_2=0.1$. are selected to reduce the third term energy. As seen in Fig. 6, the minima is located almost at the correct answer compared as in Fig. 5, and so converges the system in transient simulation.



(a)



(b)

Fig. 6. 2bit A/D converter in case x=1.6, g1=g2=2.1, and α=2: (a) The energy map, (b) Transient simulation of output.

4. Implementing the optimized input conductance

4.1 Negative Impedance using Amplifier.

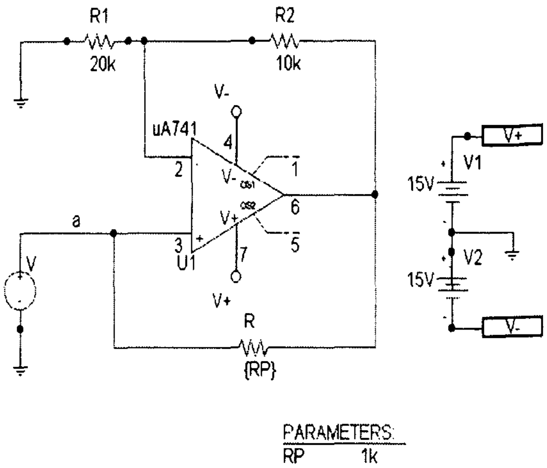
Fig. 7a shows the well known circuit to implement negative impedance using operational amplifier. The equivalent resistance seen from node a to ground can be easily obtained as

$$R_{eq} = \frac{V_s}{i_s} = -\frac{R_1}{R_2}R \qquad (20)$$

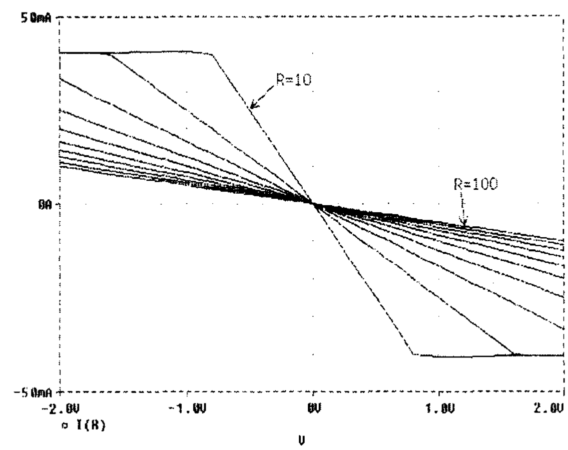
Therefore, provided $R_1 = R_2$, the equivalent resistance can be simplified as

$$R_{eq} = -R \qquad (21)$$

The resistance R has varied in region [10,100] with step size of 10 applied to the circuit shown in Fig. 7b. As shown in simulation, the satisfied results is obtained at the resistance value above 30ohm in the region of input voltage [-2, 2], because the maximum output current of up741 OpAmp is about 40mA. Therefore, we



(a)



(b)

Fig. 7. Negative resistance circuit, (a) Schematic diagram for Pspice (b) Simulation for different values of resistor R.

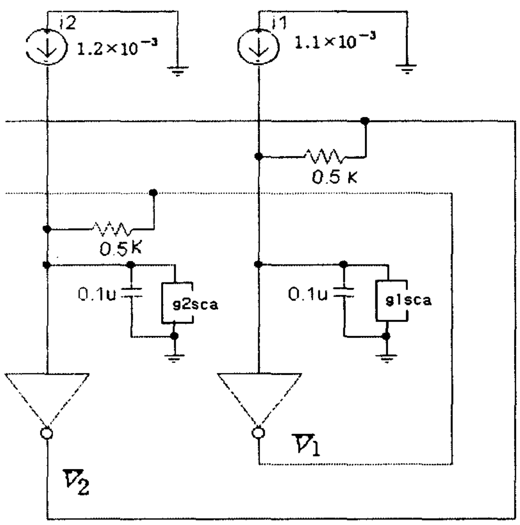
have to remind the fact that this circuit is limited to the high value of resistance. So, it is not possible to use this circuit for Hopfield networks where the weights are in range of high value, that is, the resistors for those weights are very small value. Now, we return to discussing the method to use this circuit by scaling input conductance.

4.2 Scaling input conductance

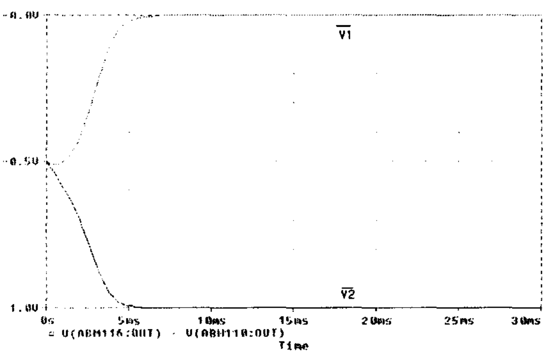
The energy function E(v) of the Hopfield networks can be scaled without changing the shape the energy contour but changing level as follow

$$E(v)_{scaled} = K[-\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}v_iv_j - \sum_{i=1}^{n} i_iv_i + \sum_{i=1}^{n} G_i\int_{0.5}^{v_i} f^{-1}(z)\,dz] \qquad (22)$$

where k is a scale factor. The scale factor k does not change the shape of the energy function, but only scales the entire contour with same ratio. Therefore, both the scaled and unscaled network have a minimum at the same point so that the scaled Hopfield network converges to the same point as does the unscaled network.



(a)



(b)

Fig. 8. The scaled circuit for 2bit A/D converter,

(a) Using negative resistance circuit for g_{isca},

(b) Simulation for $g_{1sca} = g_{2sca} = -1.9 \times 10^{-3}$℧.

The conductance, bias current, and total conductance for the scaled Hopfield network are obtained by rearranging Eq.(22) as follows

$$E(v)_{scaled} = -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}(Kw_{ij})v_i v_j - \sum_{i=1}^{n}(Ki_i)v_i + \sum_{i=1}^{n}(KG_i)\int_{0.5}^{v_i} f^{-1}(z)\,dz] \qquad (23)$$

Therefore, the conductance, bias current, and total conductance for the scaled Hopfield network are

$$w_{scaled} = Kw, \quad i_{scaled} = Ki, \quad G_{scaled} = KG \qquad (24)$$

Case Study

The 2-bit A/D converter is also selected for this case study. The scaled weights, bias currents and input conductance can be expressed as follows [3], where the scaled fator k is 10^{-3}.

$$w_{sca} = 10^{-3}\begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}, \quad i_{sca} = 10^{-3}\begin{bmatrix} x - \frac{1}{2} \\ 2x - 2 \end{bmatrix}, \quad g_{sca} = 10^{-3}\begin{bmatrix} g_1 \\ g_2 \end{bmatrix}$$

(20)Fig.8a shows the circuit diagram in case of x=1.6, α =2, and $G_{1sca} = G_{2sca} = 0.1 \times 10^{-3}$℧. 500ohm is used in the scaled system for $1/|w_{12sca}|$ and $1/|w_{21sca}|$. Thecircuit in Fig. 7a is used to construct the input conductance g_{1sca} and g_{2sca} shown in Fig. 8a. To implement $g_{1sca} = g_{2sca} = -1.9 \times 10^{-3}$℧, $(-1/1.9)\mathrm{k}\Omega$ is used for R in the negative resistance circuit. As seen in Fig. 8b, the system converges to the same point as in the unscaled system.

5. Conclusion

To represent an inhibitory synapses, usually the inverted output of amplifier is used. The total incoming currents of an input node in both circuits, where the inverted output of amplifier and the negative resistor are used for the inhibitotory synapses, are same. However, it has been shown that the total conductances are different in two circuits, and this total conductances affects the convergence of system. As the total conductance grows, the third term of energy function increases, and thus, it cause the energy function of system differ from the objective function which is to be minimized. So we know that the total conductances should be selected as small as possible, but positive to prevent the input node from being unstable. Also, it has been shown that the well known circuit for a negative resistance is limited in using the comparatively high value of resistance. Thus, it is shown in this paper how to scale the energy function to use this circuit for constructing the optimized input conductance.

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