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Optimization of Gradient-index Antireflection Coatings

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A sequence of functions are examined for the gradient-index AR thin film between two dielectric media and are used as the starting profiles in optimization to improve AR performance. Sinusoidal functions were quite efficient to use as components of the index change in the optimization. It is shown that there exist a number of gradient-index profiles which exhibit excellent AR-performance after control of the gradient-index profiles.

Gradient-index layers have been studied for antireflection coatings in the visible and infrared regions [1–3], and recently we have reported step-index multilayer AR-coatings of excellent performance [4]. The gradient-index design is motivated by coevaporation for the deposition of inhomogeneous dielectric layers [5]. Most AR-coatings aim at the reduction of reflectivity on a surface when the incident medium is air, whose index is one. For these cases the index profiles of the AR-coatings can not be continuous from the incident medium index because the available index range has a lower limit which is larger than one.

In any optical system, the index difference of two adjacent optical components whose surfaces border on each other brings on unwanted reflectivity. Here, we investigate the performance of the gradient-index AR-coatings between two dielectric media whose indices are within the available index range of AR-coating materials. We are also concerned about the optimization starting from gradient-index profiles and intend to find the possible optimum profiles which exhibit minimum average reflectivity.

We consider antireflection of the interface between two dielectric media having $n_i = 1.6$ and $n_f = 2.4$ with a graded interface region of thickness a. Other index combinations could be taken. Here, we choose the index combination of $n = 1.6$ and 2.4 for comparison with the optimization results in reference [1]. The index-profile curve is chosen to be a smooth function whose derivative is symmetric in the interface region where the index is such that $1.6 \leq n \leq 2.4$; the index profile along the depth z is taken as

$$n_p(z/a) = \frac{n_f - n_i}{N} \int_0^{z/a} [y(1-y)]^p \, dy + n_i, \qquad (1)$$

where p is a positive integer (or a positive real number), $0 < z < a$ and the normalized constant N is given by

$$N = \int_0^1 [y(1-y)]^p \, dy. \qquad (2)$$

Equation (1) is a generalization of the linear $(n_0(z/a))$, cubic $(n_1(z/a))$ and quintic $(n_2(z/a))$ index profiles of reference [1] and it generates an index distribution which is everywhere continuous and passes the mid value $n = 2.0$ at the midpoint of the interface region $z = a/2$. For integer $p \geq 2$, Eq. (1) exhibits the quintic, seventh, ninth order index distribution, and so on:

$$\begin{aligned}
n_2(t) &= (n_f - n_i)(6t^5 - 15t^4 + 10t^3) + n_i \\
n_3(t) &= (n_f - n_i)(-20t^7 + 70t^6 - 84t^5 + 35t^4) + n_i \\
n_4(t) &= (n_f - n_i)(70t^9 - 315t^8 + 540t^7 - 420t^6 + 126t^5) + n_i \\
n_5(t) &= (n_f - n_i)(-252t^{11} + 1386t^{10} - 3080t^9 + 3465t^8 - 1980t^7 + 462t^6) + n_i \\
&\vdots
\end{aligned} \qquad (3)$$

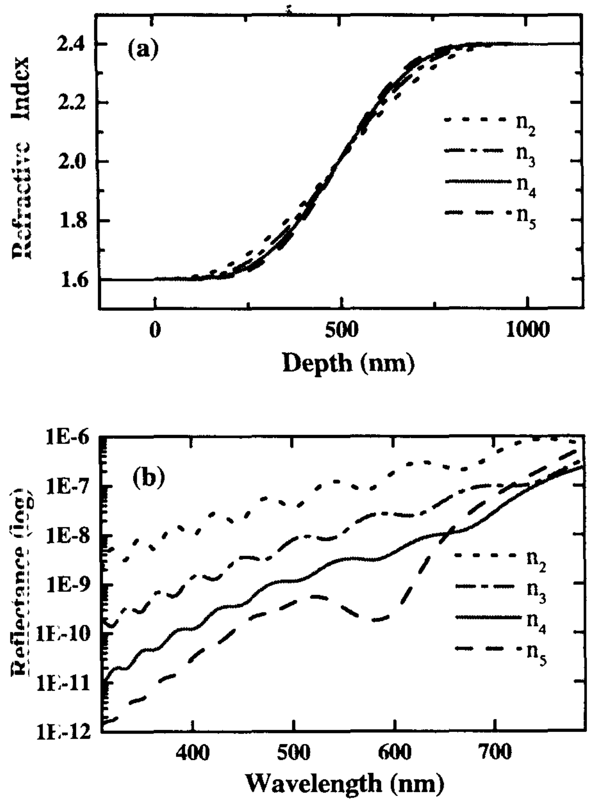

FIG. 1. (a) The index profiles of $n_2(z/a)$, $n_3(z/a)$, $n_4(z/a)$, and $n_5(z/a)$ for $a = 1000$ nm. (b) The reflectivity curves for $n_2(z/a)$, $n_3(z/a)$, $n_4(z/a)$, and $n_5(z/a)$ in the visible range.

The index profiles $n_p(z/a)$ for $p = 2$, 3, 4 and 5 are shown in Fig. 1(a) and the calculated reflectivity spectra for those profiles are plotted in Fig. 1(b). The reflectivity spectra of index profile $n_4(z/a)$ shows the average reflectivity smaller than that of any other $n_p(z/a)$ in the visible region 400 nm$\leq \lambda \leq$ 700 nm. The reflectivity for each index profile rapidly decreases with decreasing wavelength as shown in Fig. 1(b). The reflectivity of a given thin film structure $n_p(z/d_1)$ at a wavelength λ_1 equals that of a thicker thin film structure $n_p(z/d_2)$ at a larger wavelength λ_2 with $d_2/d_1 > 1$, if $\lambda_2/\lambda_1 = d_2/d_1$. Thus, one can obtain significantly lower reflectivity in a particular wavelength region using a thicker antireflection coating of index profile shape $n_p(z/a)$ with thickness a μm.

We adopt the Snedaker's synthesis technique [6] for optimizing the index profiles of the antireflection coatings in the visible region. A gradient-index profile is expressed as $n_p(z)$ plus a linear combination of sinusoidal functions whose values at two end points ($z = 0$ and $z = L$) are zero:

$$n(z) = n_p(z) + \sum_{j=1}^{N} a_j \sin\left(\frac{j\pi}{L}z\right) \tag{4}$$

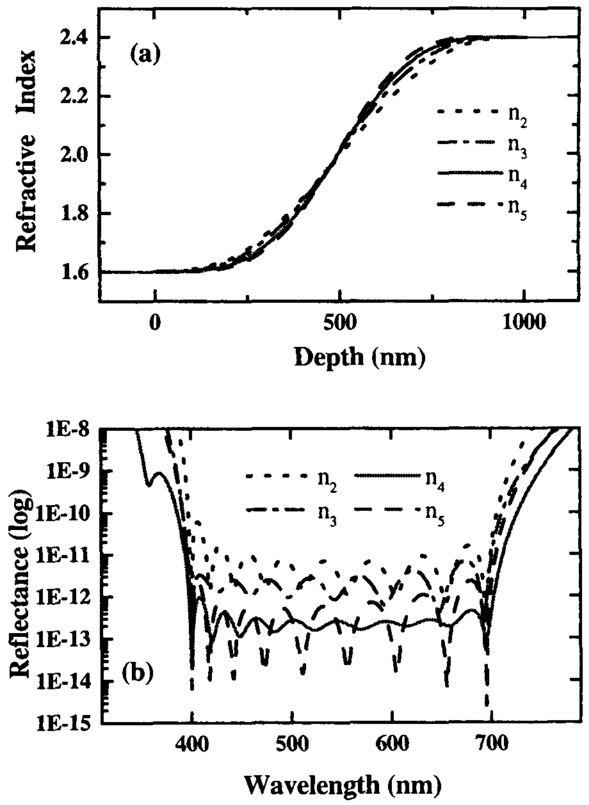

FIG. 2. (a) The resultant index profiles optimized with the starting index profiles $n_2(z/a)$, $n_3(z/a)$, $n_4(z/a)$, and $n_5(z/a)$ with $a = 1000$ nm. (b) The reflectivity curves for the optimized results in the visible range.

For the calculations of optical performance of the antireflection filters, the gradient index profile $n(z)$ is treated as a multilayer of thin slices whose physical thicknesses are identical. The optical performance of the thin film system can be calculated by performing successive multiplications of the characteristic matrices of the slices. The sum of the reflectivities at twenty one evenly spaced wavelengths including two end wavelengths is taken for the merit function.

Starting from the index profile $n^0(z) = n_p(z) + \Sigma_j a_j^0 \sin(j\pi z/L)$ with initial coefficients $a_j^0 = 0$, we adjust one of the coefficients a_j in (4) to reach the smallest value of the merit function with the other coefficients fixed, and repeat the same procedure for the next coefficient with the adjusted coefficient retained. A successive search for all coefficients (N) of the sinusoidal terms is called one pass. With a certain number of values in a range of a_j, the optimization is repeated until the merit function is not changed during a full pass. A similar procedure is continued with the range of the a_j value being reduced.

The optimization through about several hundreds of passes provides the index profiles for antireflection coatings which have excellent AR-performance. The index profiles optimized from the initial profiles $n_p(z/a)$ for $2 \leq p \leq 5$ are shown in Fig. 2(a) and their

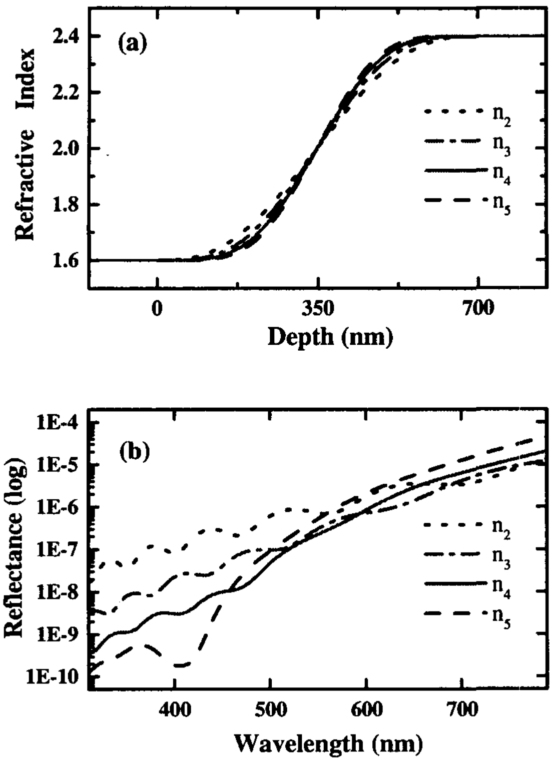

FIG. 3. (a) The index profiles of $n_2(z/a)$, $n_3(z/a)$, $n_4(z/a)$, and $n_5(z/a)$ for a = 700 nm. (b) The reflectivity curves for $n_2(z/a)$, $n_3(z/a)$, $n_4(z/a)$, and $n_5(z/a)$ for a = 700 nm in the visible range.

reflectivity spectra are plotted in Fig. 2(b). The average reflectivity is the order of 10^{-12} in the visible range. This reflectivity calculated with (4) is excellent compared with the result obtained by Southwell [1].

We also calculated the reflectivity spectrum for a gradient antireflection film of smaller thickness. The index $n_p(z/a)$ with p=2, 3, 4 and 5 for $a = 0.7\mu$m are shown in Fig. 3(a) and their reflectivity curves, in Fig. 3(b). The optimization using Eq. (4) led to the resultant curves in Fig. 4(a). As shown in Fig. 4(b), the average reflectivity of the optimized thin film with thickness $a = 0.7\mu$m in the visible region is about less than 10^{-10}. Although the thickness is smaller, the antireflection performance is even better than that of the filter of thickness 1.0 μm obtained in the optimization by Southwell [1].

Thinner gradient antireflection filters can be optimized with Eq. (4). When the filter thickness is less than the maximum wavelength in the spectral range of antireflection, the optimized index profiles are rapidly varying bumpy curves which are not useful for the design of an antireflection coating. In the optimization Eq. (4) is not appropriate for thickness smaller than the maximum wavelength in the spectral range.

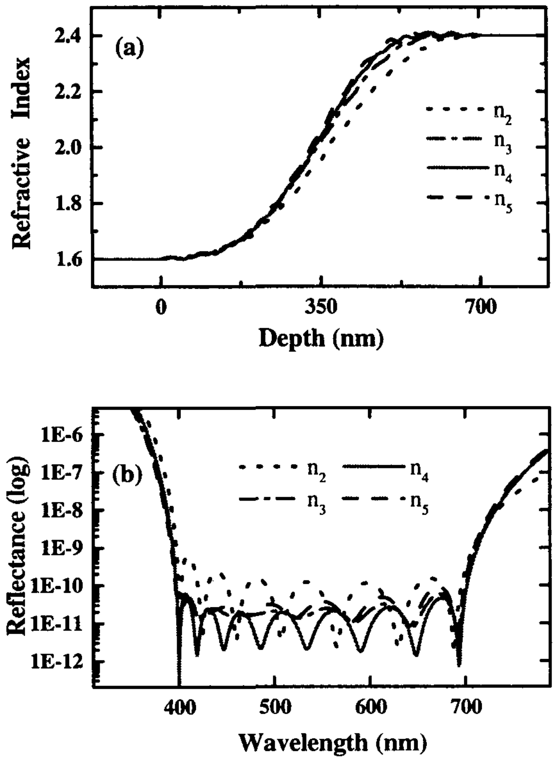

FIG. 4. (a) The resultant index profiles optimized with the starting index profiles $n_2(z/a)$, $n_3(z/a)$, $n_4(z/a)$, and $n_5(z/a)$ with a = 700 nm. (b) The reflectivity curves for the optimized results in the visible range.

In conclusion, for gradient antireflection filters we chose a reasonable series of functions which include those taken by Southwell [1], and found that among them the ninth order function is the best for antireflection performance. The sinusoidal variation (4) wasfound to be very useful for the optimization of the antireflection filters whose spectral wavelength region of antireflection is between $\lambda = 0.6a$ and $1.0a$. The average reflectivity of the optimized structure was less than 10^{-12} for the ninth order function $n_4(z/a)$ with the thickness $a = 1.0\mu$m.

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