다층 유전체위의 다중 결합선로에 대한 유한차분법(FDTD)을 이용한 해석

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ABSTRACT

A general characterization procedure based on the extraction of a 2n-port admittance matrix corresponding to n uniform coupled lines on the multi-layered substrate using the Finite-Difference Time-Domain (FDTD) technique is presented. The frequency-dependent normal mode parameters are obtained from the 2n-port admittance matrix, which in turn provides the frequency-dependent distributed inductance and capacitance matrices. To illustrate the technique, several practical coupled line structures on multi-layered substrate, including a three-line structure, have been simulated. It is shown that the FDTD based time domain characterization procedure is an excellent broadband simulation tool for the design of multiconductor coupled lines on multilayered PCBs as well as thick or thin hybrid structures.

1. Introduction

Accurate characterization of multiconductor coupled lines using electromagnetic modeling and simulation tools plays an important role in the design of high speed transmission line structures in multilayer media. Uniformly multiple coupled line systems, including the symmetric three-line structures are widely used in filters, directional couplers and impedance matching networks at microwave frequencies(1-4). In the past years, a general characterization procedure suggested for these structures consists of obtaining the frequency-dependent distributed inductance and capacitance matrices directly or by extracting the modal parameters by exciting n multiconductor coupled transmission lines [5,6]. In all of these characterization procedures, the assumption that measurement of voltages and currents at the input ends provides all normal mode parameters, and thus the corresponding distribute inductance and capacitance matrices, is valid only for multiconductor coupled lines in homogeneous media or symmetrical coupled line structures(8) in inhomogeneous media. An alternate approach for solving such structures is similar to(7) where two different structures are simulated with and without dielectric layer leading to the capacitance and inductance matrices. This alternate approach, however, increases the simulation time by a factor of two, and, in addition, cannot be used for the characterization of complex structures such as coupled microstrip bends [9].

Many methods for modeling multiconductor interconnection lines have been based on quasi-TEM assumption, and the frequency dependence of the

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distributed circuit parameters has been neglected. A frequency-dependent circuit modeling based upon full-wave analysis is necessary. In this paper, the Finite-Difference Time-Domain (FDTD) method for extracting the frequency-dependent propagation constant and characteristic impedance of multiconductor inter-connection lines on multi-layered substrate is presented.

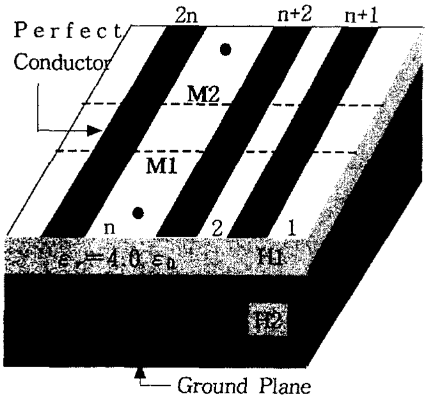
2. EQUIVALENT CIRCUIT MODEL FOR NORMAL MODE APPROACH

The equivalent circuit model can be readily derived from the admittance (impedance) matrix by characterizing the 2n-port network shown in Fig.1.

The procedure for deriving the expression for the admittance or impedance matrix of general 2n-port is well known and is based on the solution of coupled transmission lines equations shown in (1) and (2).

$$\frac{\partial[V]}{\partial z} = -[Z(\omega)][I] \quad (1)$$

$$\frac{\partial[I]}{\partial z} = -[Y(\omega)][V] \quad (2)$$



[Fig. 1] 2n-port multi-layer multiconductor transmission line.

where vectors [V] and [I] represent voltages and currents on the lines and $[Z(\omega)]$ and $[Y(\omega)]$ represent $2n \times 2n$ impedance and admittance matrices, respectively. $[Z(\omega)]$ and $[Y(\omega)]$ are related to [R], [L], [G], [C] matrices from basic equivalent circuit model of the general transmission line equations.

$$[Z(\omega)] = [R] + j\omega[L] \quad (3)$$

$$[Y(\omega)] = [G] + j\omega[C] \quad (4)$$

[R], [L], [G] and [C] are the per-unit-length line constant matrices whose elements are, in general, frequency dependent. The coupled transmission line equations (1) and (2) are decoupled with the help of voltage and corresponding current eigenvector matrices $[M_V]$ and $[M_I]$($[M_I] = [M_V]^{-T}$), respectively,

leading to the characterization of the general n lines 2n-port by its admittance matrix. Following Chin[8],

$$[Y] = \begin{bmatrix} [Y_A] & [Y_B] \\ [Y_B] & [Y_A] \end{bmatrix} \quad (5)$$

with

$$[Y_A] = [Y_{LM}] * [M_V] \cdot [\coth(ri^d)]_{diag} \cdot [M_I]^T \quad (6)$$

and

$$[Y_B] = -[Y_{LM}] * [M_V] \cdot [\csc h(ri^d)]_{diag} \cdot [M_I]^T \quad (7)$$

$$[Y_{LM}]_{n \times n} = \begin{bmatrix} Y_{LM11} & \cdot & Y_{LM1n} \\ Y_{LMn1} & \cdot & Y_{LMnn} \end{bmatrix} \quad (8)$$

where γ_i is the ith normal mode propagation constant and represents the ith eigenvalue, and d is the length of the uniformly coupled multiconductor system. $[Y_{LM}]$ is the line mode admittance matrix whose element Y_{LMkm} represents the characteristic admittance of the kth line for mth mode. The operator '*' was defined in[8] for [C]=[A]*[B], as a product of corresponding terms of matrices [A] and [B].

From equations (5), (6), (7) and (8), a matrix [P] which includes $[Y_{LM}]$, $[M_V]$, and $[(\gamma_i d)]_{diag}$ can be defined as

$$\begin{aligned}[P] &= [Y_A]^T\,[Y_B]^{-T} \\ &= [M_V]^{-7}\,[\coth(ri^d)]^T_{diag} \\ &\quad \cdot [\operatorname{csc}h(ri^d)]^{-T}_{diag}\,[M_V]T \end{aligned} \tag{9}$$

This can be simplified to

$$\begin{aligned}[P] &= [M_V] - T\,[\cosh(ri^d)]^T_{diag} \\ &\quad \cdot [M_V]T \end{aligned} \tag{10}$$

Equation (10) shows that the normal mode propagation constants $\gamma_i\ (i=1,2....n)$ can be directly obtained from the eigenvalues of matrix [P]. Likewise, $[M_V]$ is directly found from the eigenvectors of [P]. In the case of asymmetric multiple coupled line structure, the conventional even and odd mode technique can not be applied due to asymmetry, and, hence, the normal mode parameter approach is used as an alternative. In the case of an asymmetric coupled line, the line mode admittance matrix $[Y_{LM}]$ and the voltage and corresponding current eigenvector matrices $[M_V]$ and $[M_I]$ given by equation (8) reduce to a 2 by 2 matrix with the voltage ratio R_c and R_π for the c and π modes, respectively. The line mode admittance matrix and the two eigenvectors can be written as

$$[Y_{LM}]_{2\times 2} = \begin{bmatrix} Y_{c1} \cdot Y_{\pi 1} \\ Y_{c2} \cdot Y_{\pi 2} \end{bmatrix} \tag{11}$$

$$[M_V] = \begin{bmatrix} 1 & 1 \\ R_c & R_\pi \end{bmatrix} \tag{12}$$

$$[M_I] = ([M_V]^{-1})^T \tag{13}$$

Here line mode impedances such as Z_c and Z_π are readily found from equation (11) and also equation (12) give us the line mode voltage ratio for symmetric and asymmetric coupled line structure.

Furthermore, for the general three coupled line case, the line mode admittance matrix $[Y_{LM}]$ and the voltage eigenvector $[M_V]$ reduce to 3 by 3 matrices for the normal modes a, b and c, respectively[8], which can be expressed as

$$[Y_{LM}]_{3\times 3} = \begin{bmatrix} Y_{a1} & Y_{b1} & Y_{c1} \\ Y_{a2} & Y_{b2} & Y_{c2} \\ Y_{a3} & Y_{b3} & Y_{c3} \end{bmatrix} \tag{14}$$

$$[M_V] = \begin{bmatrix} 1 & 1 & 1 \\ R_{a2} & R_{b2} & R_{c2} \\ R_{a3} & R_{b3} & R_{c3} \end{bmatrix} \tag{15}$$

From equations (14) and (15), we can easily calculate line mode characteristic impedances and the voltage ratios of an asymmetric three coupled line for the three modes as a, b and c, respectively.

For the case of a symmetric three coupled lines,

$Y_{a1} = Y_{a3}$, $Y_{b1} = Y_{b3}$ and $Y_{c1} = Y_{c3}$.

3. FINITE-DIFFERENCE TIME- DOMAIN (FDTD) SIMULATIONS

3.1 BRIEF REVIEW OF FDTD

The FDTD method was first introduced by K.S. Yee[10] to solve electromagnetic scattering problems. Recently, the FDTD method has been widely extended to analyze microstrip based structures. The technique as such is well described by a number of researchers[9,10,11,12], and hence, not repeated in this paper. Application of the FDTD method to simulate the characteristics of the transmission lines provides broadband frequency information in one time simulation. Basically, in this method, the two Maxwell's curl equations are discretized both in time and space and the field values on the nodal points of the space-time mesh are calculated.

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \tag{16}$$

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} + \sigma \vec{E} \tag{17}$$

The entire computational domain is discretized into a number of cells of size Δx, Δy and Δz in x, y and z directions, respectively. As an input excitation, a Gaussian pulse is desirable because its frequency spectrum also being Gaussian, provides frequency domain information from DC to the desired cutoff frequency by adjusting the width of the pulse. A Gaussian pulse is defined by

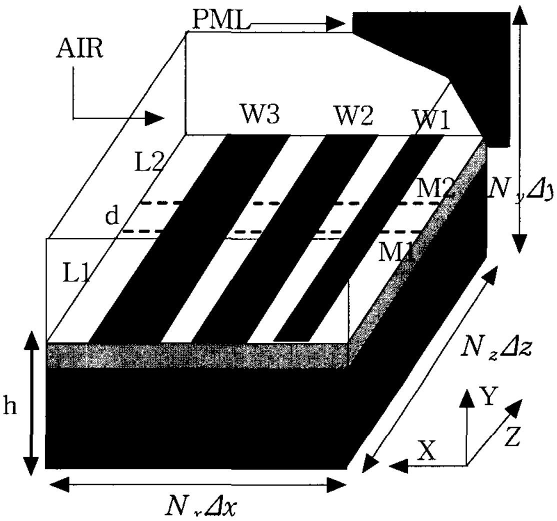
$$g(t) = \exp\left[\frac{-(t-t_0)^2}{T^2}\right] \tag{18}$$

The electromagnetic fields calculated by the FDTD algorithm were used to compare the voltages and currents on multiconductor lines. Voltage and current are defined as

$$I = \oint_c \vec{H} \cdot d\vec{l} \tag{19}$$

$$V = -\int_0^h \vec{E} \cdot d\vec{l} \tag{20}$$

where c is the transverse contour of a conductor and d is the distance between a conductor and the ground plane.



[Fig.2] Computational domain by FDTD simulation for multiple coupled lines on multi-layer substrates.

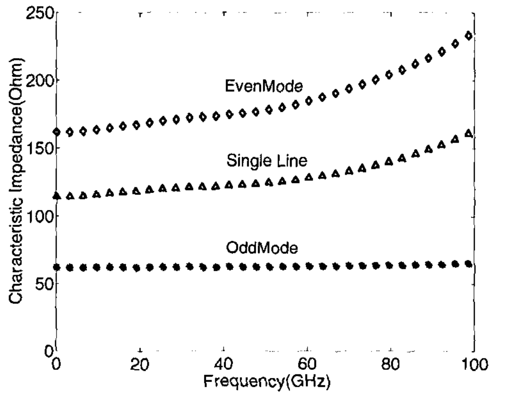
Figure 2 shows the total computational domain of multiconductor transmission lines on multi-layered substrate including the Perfectly Matched Layer(PML) absorbing boundary condition.

3.2 SIMULATION RESULTS

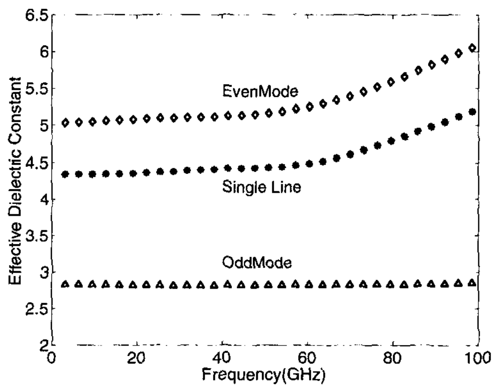
3.2.1 Symmetric Coupled Line

The multiple conductor lines on single layer substrate have been characterized using FDTD technique by Y. Kim[12] since June of 1999. The symmetric coupled line can be analyzed in terms of even- and odd-mode approach because c-(common mode) and π-mode (anti-common) of symmetric coupled line for normal mode approach are identical to even- and odd-mode parameters. For the structure of n multiple conductor interconnection lines shown in Fig.1, n orthogonal excitations are needed. The conductor which is excited is called active line, the other conductors are sense lines. As a simple example, a symmetric coupled microstrip line of widths, W1=W2=50 μm, separation between the two conductors, S=30 μm, substrate thickness, H1=40 μm, H2=200 μm and dielectric constants, $\varepsilon_{r1} = 4.0$, $\varepsilon_{r2} = 12.0$ is considered. The metal strips and the ground plane are assumed to be perfectly conducting and infinitely thin, and are defined by setting the tangential component of the electric field to zero.

The conductor lines are simulated on an $N_x \Delta x$ by $N_y \Delta y$ by $N_z \Delta z$ computational domain with Δx=10 μm, Δy=10 μm and Δz=15 μm. This corresponds to a conductor width of W=5 Δx and substrate heights of H1=4 Δy, H2=20 Δy. The width, N_x, and height, N_y, of the simulation box are chosen to be large enough to not disturb the field distributions near the strips. In all, the entire computational domain including the PML boundary is divided into 65 by 54 by 350 grid cells.



(Fig. 3) Characteristic impedance as a function of frequency for a symmetric coupled line and a single line on the top layer with 40um and bottom layer of 200um substrate thickness. Conductor line widths are W1=W2=50um and the spacing between the two conductors is 30um.

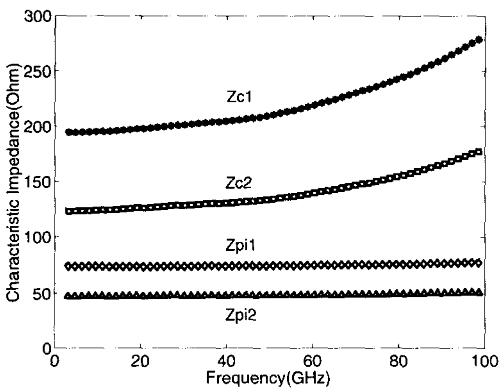


(Fig. 4) Effective dielectric constant as a function of frequency for a symmetric coupled line and a single line as the same structure shown in Fig. 3.

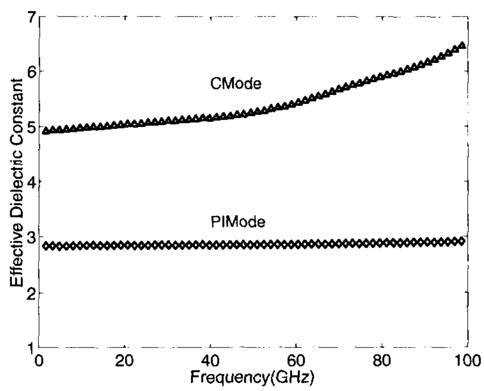
A time step of Δt=0.0192 ps is used and the total number of time steps is 2600. The input is excited with a Gaussian pulse with T=2.05 ps and t_0=6.14 ps. To apply the normal mode parameter approach, each port is excited and then voltages and currents are recorded at the positions M1 and M2 with the actual length of conductor, L1=1500 μm, d=30 μm, and L2=3450 μm, respectively. Quantitative frequency-domain information contained in the time-domain data is extracted via the Fast Fourier Transform (FFT). Figures 3 and 4 show the variation of the characteristic impedance and effective dielectric constant for the symmetric coupled line and a single line on the two-layered different substrates using the proposed normal mode approach and the FDTD technique. Here, we can see that the characteristic impedance and effective dielectric constant rapidly increase following frequency for the single line and even-mode, but odd-mode characteristics are slightly different for the static value.

3.2.2 Asymmetric Coupled Line

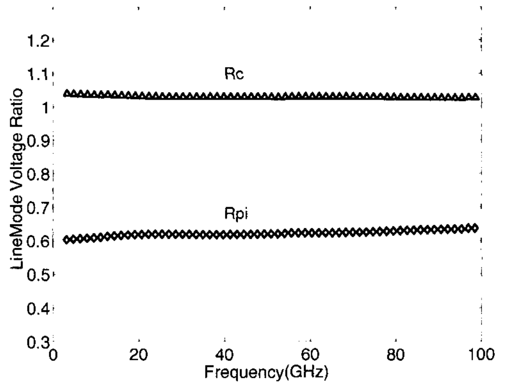
An asymmetric coupled line can be characterized in terms of c and π modes using the normal mode parameter approach. The dimensions of the structure are chosen to be W1=40 μm, S=30 μm and W2=80 μm with the same dielectric substrate specifications as for the case of symmetric coupled line. For the purpose of FDTD simulation, the computational domain is considered to be 77 by 54 by 350 grid cells with time step Δt=0.0192 ps. To apply the normal mode parameter approach, voltages and currents are recorded after the FDTD simulations for the active and sense line at positions M1 and M2, respectively, as indicated Fig.2. From the voltage and current recorded on the active and



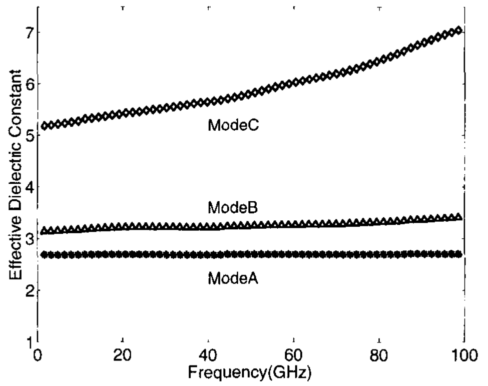
[Fig. 5] Line mode characteristic impedance as a function of frequency for an asymmetric coupled line on the top layer with 40um and bottom layer of 200um substrate thickness. Conductor line widths are W1=40um, W2=80um and the spacing between the two conductors is 30um.



[Fig. 6] Effective dielectric constant as a function of frequency for asymmetric coupled line on the top layer with 40um and bottom layer of 200um substrate thickness. Conductor line widths are W1=40um, W2=80um and the spacing between the two conductors is 30um.



[Fig. 7] Line mode voltage ratio as a function of frequency for an asymmetric coupled line on the top layer with 40um and bottom layer of 200um substrate thickness. Conductor line widths are W1=40um, W2=80um and the spacing between the two conductors is 30um.



[Fig. 8] Effective dielectric constant as a function of frequency for asymmetric three coupled line on the top layer with 40um and bottom layer of 200um substrate thickness. Conductor line widths are W1=40um, W2=60um, W3=80um and the spacings between the three conductors are S1=20um and S2=40um.

sense line, the line mode admittance matrix $[Y_{LM}]$ and the voltage and corresponding current eigenvector matrices, $[M_V]$ and $[M_I]$, are derived following the procedure described in section 2.

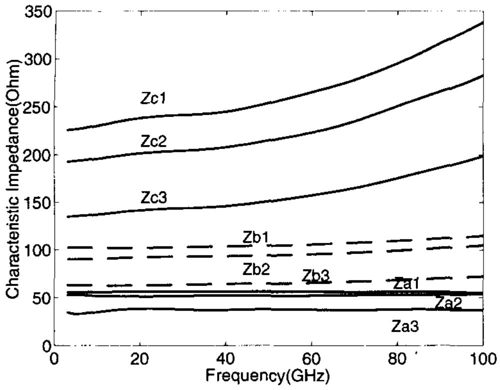
Figures 5, 6 and 7 show the variation of line mode characteristic impedance, effective dielectric constant and line mode voltage ratio for asymmetric coupled line structure as a function of frequency, respectively. As shown in figures 5 to 7, the line characteristics are very similar to symmetric coupled line structure except for mode named as c-(common) and pi-mode(anti-common).

3.2.3 Asymmetric Three Coupled Line

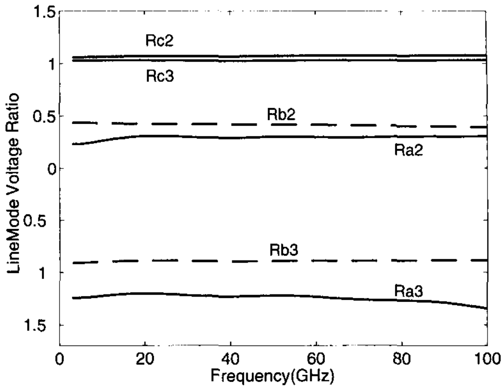
The third example is an asymmetric three coupled interconnection line. The dimensions of the structure are W1=40 μm, S1=20 μm, W2=60 μm, S2=40 μm and W3=80 μm with dielectric substrate specifications the same as for the case of symmetric coupled line. The entire structure is divided into 86 by 54 by 350 grid cells for calculation with Δt=0.0192 ps. Figures 8, 9 and 10 show the variation of effective dielectric constant, line mode characteristic impedance and line mode voltage ratio for each mode of asymmetric three coupled line structure as a function of frequency, respectively. These line parameters such as the effective dielectric constants, characteristic impedance and line mode voltage ratio for the three modes are easily given by equations (10), (14) and (15).

4. CONCLUSION

In this paper, the full-wave Finite-Difference Time-Domain (FDTD) method for the computation of all the frequency-dependent characteristics of general



〔Fig. 9〕 Characteristic impedance as a function of frequency for an asymmetric three coupled line as the same structure shown in Fig. 8



〔Fig. 10〕 Line mode voltage ratio as a function of frequency for an asymmetric three coupled line as the same structure shown in Fig. 8.

asymmetric coupled lines in a multilayer substrate structure has been presented. Especially, asymmetric coupled line and three line structures are analyzed over the FDTD technique and normal mode parameter approach. The effective dielectric constant founded is a convenient way to describe the frequency-dependent phase velocity along the coupled microstrip transmission lines. Application of the FDTD technique for the characterization of the multiple coupled lines provides broadband frequency data in one time simulation. The results for all the frequency-dependent propagation characteristics should be very useful in the analysis and design of multiple coupled line structures on multi-layered substrate such as couplers, matching networks and filters.

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