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A Didactic Comparision between basic concept of the theory of Crisp Set and the theory of Fuzzy Set

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1. Crisp Set and its limitation

"Fuzzy" according to the dictionary means "vague" or "indistinct". For example, if there is a group selection for those who are good at golf, the term "good" according to each individual has a vague meaning and cannot be decided by simply "yes" or "no".

Also, when we are asked to bring "a few things", this connotation is vague too. In our everyday life, we come in contact with many such vague expressions such as "young", "good", "quick", "hot", "many", "close to" etc. Though the expressions contain vague nature but we use them as if they are without errors in our everyday life.

We call these expressions with vague nature, the "fuzzy concept". In contrast, there is a term which has a distinct meaning such as "a is a student of ~~ school". We call this "crisp set", a theory first proposed by Georg Cantor. We will begin this paper from this kind of classification.

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1.1 Crisp Set

What is a set? This is a very difficult question to answer. In this paper, we shall not go into any complicated axiomatic approach to set theory, but shall content ourselves to accept the following.

"Set is any collection into a whole of definite, distinguishable objects, called 'element', of our intuition or thought." This intuitive definition of a set was first given by Georg Cantor(1845-1918), who originated the theory of sets in 1895. For example, "a collection of natural numbers" and "a group of a company employees" are sets.

With this kind of definition, we can define 'finite set', 'infinite set', 'empty set' etc., and can define general set operation. A reference to this example is, if what is being mentioned in this paper is different then, it cannot be a reference any more.

According to Cantor, the crisp set theory concerns with what is belonging or not belonging to a set. When we say that it belongs to a set, then all the elements

of a set are treated to have the same meaning. For example, when we consider apples in a box, the definition according to crisp set rules that there is no consideration on whether the apples are big or small, delicious or not, rotten or good, red or green apples ……. There is no consideration to the nature of the object but it only considers whether it is in the box or not.

1.2 The basic structure of existing beings and the barrier of crisp set.

Every existing beings consist of "Internal Character" (internal, invisible, faculty aspect) and "External Form" (external, visible, shape, element aspect). For example in mankind, internal character is the mind while the external form is the body. In animal, there are the invisible instinctive mind and the external body. In plants, life and the various invisible internal characteristics are the internal whereas the material part made up of the cells is the external. For molecules, atoms and particles the physicochemical character is the internal and the invisible matter and structure are their external. For human intellect, the system of intellect is not independent but exist connected to the elements of the internal character and external form through a harmonious "Give and Take" relationship. As a result, thought and reasoning is possible. Internal Character and External Form together are called the Dual Characteristics. All existing beings consist of dual characters and exist in harmony through the Give and Take action within itself and amongst one another through this relationship. Internal Character is the subject, casual and vertical while the External Form is the object, resultant and horizontal. All objects of the existing world consist of internal element(casual, subjective, invisible) and external elements(resultant, object, visible).

When we observe that all existing beings have this special characteristics. It might be true that all mathematical objects too have subjective elements of an internal aspect and we must attract it to the mathematical realm gradually. Then, this will add a higher dimension to the mathematical realm, of course, in science we cannot but to feel that natural phenomena and closeness are necessity factors. Even though the definition of the theory of sets by Cantor has great contribution in the world of science but, from the argument of the Dual Characteristics, it only considers the objective and subordinated parts of all the mathematical things. According to the theory of crisp sets, it is only the external aspect and therefore, cannot be sufficient. This is the limitation of the theory of crisp sets.

1.3 The effect of the logic of crisp set

Logic is the study of laws and forms of things. Formal logic, started by Aristotle, dealt with universal laws and forms of thinking, which is variously different in content. The basic laws of valid thinking, in traditional formal logic, are four principles, as follows ;

- The principle of identity :

Which is expressed in the form "A is A".

- The principle of contradiction :

Which is expressed in the form "A is not non-A".

- The principle of excluded middle :

Which is expressed in the form "A is either B or not B". This means that in the assertion "B and not B" there is no third assertion in the middle.

- The principle of sufficient reason :

"All thinking has a reason -Leibniz-." And there is no thinking without a reason. When applied to phenomena, we get the proposition, "every phenomenon has a cause". We get the law of cause and effect.

The pattern of the crisp set theory in the "world of crisp" composed basic logic of the crisp set theory. Mathematical object is requested to answer in only two ways, either "yes" or "no". According to "the principle of excluded middle", the logic which only has the "true or false" term has controlled the overall area of science. So, the unnatural tendency which ignore the middle elements, has developed in the culture and history of humankind.

But realistically, in the world of human thought and logic, if sufficient middle elements were not considered then the non-remuneration would be beyond the possibility of numbering. Let's see two examples:

① (In the case of election)

The choice of selection is approved (All) or opposed(Nothing).

But in reality : there is a possibility of approval (52%) and opposed (48%).

② (The administration of manpower)

According to the theory of crisp set " 1 person + 1 person = 2 persons", but this is only based upon form calculation and do not consider the actual situation. The theory of crisp set only considers the form calculation and does not take into consideration such aspect as the capability, unity, positive factors and human relationship. In reality, when we consider the unity of two persons and the many invisible elements:

1 person + 1 person = efficiency above that of two person's or 0 efficiency or minus efficiency is also possible

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In this kind of thought, when the middle consideration is removed, what is the effect on our society? It is not easy to summarized but if we attempt to, then they are as follow: The effect of the logic which based on two-value system is at first we make the "culture of objective" and make the "culture of immediate", exclusive, so aggressive.

2. Fuzzy Set

Prof. L.A. Zadeh of California University first introduced the Theory of Fuzzy Set in 1965. He had attempted that set's elements were determined by the contained individual characteristics (internal character elements).

Usually, we can express crisp set A, subset of X as follow using characteristic function χ_A. i.e

$\chi_A : X \to \{0,1\}$, defined by

$$x \mapsto \chi_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$$

If x belongs to A, then assigned

the value 1. When it is not, then it has a value of 0. That is, 1 that matchs x is the element of A. The element of A is not the match of 0. The existence and non existence of nature is 1 or 0 (true or false). This is the argument according to crisp set.

So making reference to the objects of the world of existence, it cannot be distinguished by "yes" or "no", "1" or "0". For example when saying "2.5 is close to 3" we must show to a compatible degree (to the satisfaction of the nature) according to the need of the situation and circumstances.

Basic concept of fuzzy theory is extent the value {0,1} to interval [0,1]. We can express it using membership function $m_A : X \rightarrow [0,1]$.

A fuzzy set is a class of objects with a continuum of grades of membership. More often than not, the class of objects encountered in the real world do not have precisely defined criteria of membership. Yet, the fact remains that such imprecisely defined "classes" play an important role in human thinking, particularly in the domains of pattern recognition, communication of information and abstraction.

Concerning subjectivity among the characteristics of men which are superior to those of machines, L.A. Zadeh presented in 1965, the concept of fuzzy set methods of mathematics as well as science.

A fuzzy set $\tilde{A}$ which defined on the entire set X be characterized by a "membership function" $m_A : X \rightarrow [0,1]$, with the value of $m_A(x)$ at x representing the grade of membership of x in A. We can denote sometimes a fuzzy set $\tilde{A} = \{(x, m_A(x)) \mid x \in X\}$ (Here, the first coordinate, the element x and the second coordinate mean the degree(grade) of x belongs to A).

These attempts attract the subjective elements of mathematical objects to the scientific field. So, partially overcoming the limitation of two-value logic and adding higher dimension on the view of the mathematics.

How can Fuzzy Sets treats the inner character of the elements as mathematical objects? Let us see more examples: Let's look at the set (fuzzy set) composed of "tall students". In the Theory of Crisp set, fundamentally the set cannot be constructed. But, according to the following method, fuzzy set can be acquired.

Tall students ; $\tilde{A} = \{(a, \alpha)\}$.

(Here, a; student, α; degree of tall hight $\alpha \in [0,1]$)

The disposition that the elements of the given set can have, let's think in close proximity. Even though we use it in everyday life, but it cannot be managed by mathematical calculation. <The number close to 2> + <The number close to 3>=<The number close to 5>.

The above method is common but the problem is how can this be handled mathematically. After the introduction of fuzzy set, this problem can be solved.

It is possible by the same law of calculation like the following method ;

$$m_{\tilde{A}+\tilde{B}}(z) = \bigvee_{z=x+y} \{m_{\tilde{A}}(x) \wedge m_{\tilde{B}}(y)\}.$$

Fuzzy has the meaning of vagueness, and it has many aspects of uncertainty in the real circumstances of science. The definition of operations among those is very interesting. On one hand, similar calculation as well as by one meaning is not defined and varieties can be seen. Until the present, in the development of calculation, the most commonly used concept of calculation is like the above max-min law of calculation.

Fuzzy set began from this basic concept, and developed into various calculation definition of fuzzy measurement, fuzzy integration, fuzzy derivatives, fuzzy function, fuzzy numbers etc.. Fuzzy is practically applied freely in the field of control and information system and in the fields of automatic translation, intelligent robots, intelligent terminal, home automation, nursing robots, social health system, decision support system, environmental evaluation, farm and fishery products evaluation and etc., in many fields achieving quality surpassing that of human.

3. Conclusion

Human thought has no limit. Now science is approaching the part that is passing the utmost smallest portion. But through the limited scientific access mankind can be substantially helped to a frightening degree through the achievement during the past time. Nevertheless, the huge area which can not be explained by scientific theory still remains. Looking in a board sense this world of existence consists of the physical world (natural world) and the invisible world (spiritual world ; a world that can be experienced through the feeling). Invisible world clearly coexist with physical world and its area is unlimited. The Theory of fuzzy from the theory of fuzzy set gives mathematics a wider realm of access and if we are to summarize this applicable theory, they are:

By the introduction of fuzzy set, we can treat with causal and subjective part of every existing beings in the mathematical and science field partially. Ultimate goal of science be giving best interpreted by natural phenomena and has provided humankind the direction to a comfortable and happy environment. Man's thought and reasoning faculty, more than representing it mathematically, can be expanded through human natural science. The "principle of excluded middle", "principle of contradiction" are invalid under the fuzzy logic. The culture of conflict from this divalent(black-white logic) theory entered upon a stage with the theory of "Give and Take", thus Man's thought can be expanded and we hope this can develop into the result of peace and understanding.

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길 병 문[1)]

요 약

본 논문의 목적은 G. Cantor 에 의하여 출발된 집합론을 보통집합 이론이라고 구별하여 부를 때, 보통 집합 이론이 그 바탕에 깔고 있는 논리적 제한 점들 곧, 배중률이라든지 모순의 법칙 등을 어떻게 보완할 수 있을 것인가? 하는 점과 그러한 점을 보완하여야 할 필요성에 대하여도 생각하고자 한다. 그런 관점에서 보통집합 이론과 퍼지집합 이론의 기본개념을 상호 비교함으로써 앞서 제기한 문제의 보완 요소를 찾아보려고 한다. 실제에 있어 인간의 사고 가운데에서는 중간을 배제하는 일이 없음에도 불구하고 이를 수학적으로 접근하고 표현하는 수단이 부족함으로 인하여 부자연 스러운 논리의 법칙을 받아들일 수 밖에 없었던 것도 사실이다. 특히, 논리적 응용력이 부족한 중등과정의 학생들에게 있어서 수학이 전적으로 2가 논리에 의하여 지배되고 있다는 방식으로만 지도하는 것은 여러 가지 측면에서 그 내용의 보완이 요구된다. 보다 다양한 수학적 표현의 여지를 열어주는 지도법은 쉼없이 연구되어야 할 것이다. 무엇보다도 배우는 학생들이 보다 폭 넓은 사고의 영역을 소유하고, 그를 바탕으로 창의적이고 자유로운 발상이 이어질 수 있도록 하기 위하여는 교사의 수학적 시야가 보다 넓고 유연해져야 한다함은 재론할 필요가 없을 것이다. 그런 의미에서 본 논문이 작은 역할을 할 수 있기를 바란다.

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