International Journal of Reliability and Application
Vol.1, No.2, pp.155–165, 2000

# On Estimating of Kullback–Leibler Information Function using Three Step Stress Accelerated Life Test

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**Summary.** In this paper, we propose some estimators of Kullback–Leibler Information functions using the data from three step stress accelerated life tests. This acceleration model is assumed to be a tampered random variable model. Some asymptotic properties of proposed estimators are proved. Simulations are performed for comparing the small sample properties of the proposed estimators under use condition of accelerated life test.

**Key Words** : *accelerated life tests, tampered random variable, step stress, Kullback–Leibler Information function, use condition.*

## 1. INTRODUCTION

In general, most of the life testings are conducted at the usual conditions. But with modern high reliability devices, life testing may tend to require long time and excessive expenses. As a common approach to avoid this problem, such circumstances call for testing the sample under conditions which are more severe than the operating conditions, with the result that failure data can be obtained in a short period of test time. Such life tests are called the accelerated life tests (ALTs).

To begin with, we review the ALT and this method has several types according to the method which we give stresses for models. And method commonly used in engineering practice is called the step stress test. In a step stress ALTs, stress on each unit is not constant but is increased by planned steps at planned times. A test unit starts at a specified low stress. If the unit does not fail in a specified time, stress on it is raised and held a specified time. specimen stress is repeatedly increased and held, until the specimen fails. The step stress pattern is chosen to assure failures quickly.

Again, there are three types of models used commonly on analysis of step stress ALTs. Those are the tampered random variable (TRV) model, the cumulative exposure (CE) model and the tampered failure rate (TFR) model.

DeGroot and Goal (1979) considered a two partially ALTs in which a test item is first run at use condition and, if it does not fail for a specified time $\tau$, then it is run at accelerated condition until failure. Thus, $Y = T$, if $T \leq \tau$; and $Y = \tau + \alpha(T - \tau)$, if $T > \tau$; where $T$ is the lifetime of an item at use condition and $Y$ is called the tampering random variables, $\tau$ is called the tampering point and $\alpha$ is called the tampering coefficient. In general, $\alpha$ is smaller than 1 since the effect of changing to the higher stress is shorten the lifetime of the test unit. Assuming that $T$ follows an exponential distribution with mean $\theta$ and using a Bayes approach, they obtained estimators of $\alpha$ and $\theta$, and optimal change time $\tau^*$. This model is called TRV (See; Nelon (1990), etc).

In the past, many authors suggested the estimators of Kullback–Leibler Information (KLI) and used statistics based on the KLI to test a normality, exponentiality etc., for the classical models. But in ALT models, nobody uses statistics based on the KLI and Besseler, Chernoff and Marshall (1962) studied an optimal sequential design for ALTs using the KLI. Therefore, we need to develop the KLI in ALTs.

In this paper, we are going to use the TRV model among the above three models.

Letting $T_{(1)} \leq T_{(2)} \leq \cdots \leq T_{(n)}$ be the order statistics based on a random sample from the distribution function (df), $F$ with the probability density function (pdf) $f$ and assuming the observed distribution to approximate model with the df $F_0(t)$ and the pdf $f_0(t)$, then the K–L information is defined by Kullback and Leibler (1951) as

$$\begin{aligned} I(f : f_0) &= \int_{-\infty}^{\infty} f(t) \ln \frac{f(t)}{f_0(t)} dt \\ &= -H(f) - \int_{-\infty}^{\infty} f(t) \ln f_0(t) dt, \end{aligned} \tag{1.1}$$

where, $H(f) = -\int_{-\infty}^{\infty} f(t) \ln f(t) = \int \left(\frac{d}{dt} F^{-1}(t)\right) dt$ is the distance between the observed distribution $F(t)$ with the pdf $f(t)$ and the true model distribution $F_0(t)$ with the pdf $f_0(t)$ and remaining term will be a constant under the null hypothesis $H_0$. Therefore, $H(f)$, the parametric term, is called the entropy and is introduced by Shannon (1948) as a measure of information and uncertainty. So, the K–L Information is an extension of the entropy and we can estimate the K–L Information by estimating the entropy. But there are not many papers devoted to the entropy estimation problem. Now the most prominent estimator of entropy based on spacings, was proposed by Vasicek (1976), van Es (1992) and Correa (1995). And we also use these three entropies in this paper.

Therefore, in this paper, we propose three estimator of KLI functions using the data from three step stress ALT, this acceleration model is assumed to be a tampered random variable model (See ; Degroot and Goel (1979)). Also, Vasicek's estimator

(1976), van Es' estimator (1992), Correa's estimator (1995) are considered. Through a simulations are performed for comparing the sample properties of the proposed three estimators in regard to the bias and mean square error (MSE) based on use condition and we show which one is best in them.

In Section 2, we review a three step stress ALT model and estimators of parameters using the maximum likelihood method. In Section 3, we discuss the entropy estimation. At first, we review three entropy estimators (Vasicek's estimator, van Es' estimator, Correa's estimator) and estimate them by transforming the sample of the ALT model into the sample under the use condtion. Then we estimate three KLI and these are proper statistics, by mentioned their asymptotic properties. In Section 4, through a simulation study for the comparison of three proposed estimators in small sample, we investigate which proposed estimator is best in bais and mean squared error(MSE).

## 2. ESTIMATION OF PARAMETERS FOR THE THREE STEP STRESS ALTS

In this section, we consider KLI under three step stress ALTs model at tampered random variable and it calculate the maximum likelihood estimators (MLEs) for parameters. For the three step stress of ALTs, all units are initially subjected to the use condition stress $s_1 = s_u$ until a preassigned time $\tau_1$ , but if all units do not fail before time $\tau_1$, the stress is increased to $s_2$ at time $\tau_2$ , For the surviving units at time $\tau_2$, the stress is also increased to a larger stress $s_3$ and held constant until the remaining units fail.

It is assumed that the effect of changing the stress is to multiply the remaining lifetime by unknown factors $\alpha$, $\gamma(0 < \alpha, \gamma < 1)$ subsequent to changing points $\tau_1$, $\tau_2$ . Let $T_{ij}$ with the df, $F_1(t_{ij}|\theta)$, denote life time under the use condition and $Y_{ij}$ denote the $j$th life time with the df, $F(y_{ij}|\theta, \alpha, \gamma)$ under the $i$th step stress. Then, the TRV model under the three step stress is given by

$$Y_{ij} = \begin{cases} T_{1j}, & T_{1j} \le \tau_1 \\ \tau_1 + \alpha(T_{2j} - \tau_1), & \tau_1 < T_{2j} < \frac{1}{\alpha}(\tau_2 - \tau_1) + \tau_1 \\ \tau_2 - \gamma(\tau_2 - \tau_1) + \alpha\gamma(T_{3j} - \tau_1), & T_{3j} \ge \frac{1}{\alpha}(\tau_2 - \tau_1) + \tau_1. \end{cases} \tag{2.1}$$

Acceleration factors $\alpha$, $\gamma$ depend on the stresses $s_1$, $s_2$ and $s_3$ and possibly also on the preassigned time $\tau_1$, $\tau_2$. Also $\alpha$, $\gamma$ called a tampered coefficients, will be less than 1 because the effect of changing the stress to the higher level is to subject the unit to a greater failure.

Since we assume the life time $T_{ij}$, is distributed exponential before, the pdf is as follows

$$f(y_{ij}|\theta,\alpha,\gamma)=\begin{cases}\theta\exp\left[-\theta y_{1j}\right], & y_{1j}<\tau_1,\\ \frac{\theta}{\alpha}\exp\left[-\theta\left(\tau_1+\frac{y_{2j}-\tau_1}{\alpha}\right)\right], & \tau_1<y_{2j}\le\tau_2\\ \frac{\theta}{\alpha\gamma}\exp\left[-\theta\left(\tau_1+\frac{\tau_2-\tau_1}{\alpha}+\frac{y_{3j}-\tau_2}{\alpha\gamma}\right)\right], & y_{3j}>\tau_2\end{cases}\tag{2.2}$$

for $i=1,2,3,\;\; j=1,2,\cdots,n_i$.

From the Equation (2.2), we can obtain the likelihood function is,

$$\begin{aligned}L(\theta,\alpha,\gamma)=&\prod_{j=1}^{n_1}\theta\exp\left[-\theta y_{1j}\right]\prod_{j=1}^{n_2}\frac{\theta}{\alpha}\exp\left[-\theta\left(\tau_1+\frac{y_{2j}-\tau_1}{\alpha}\right)\right]\\ &\times\prod_{j=1}^{n_3}\frac{\theta}{\alpha\gamma}\exp\left[-\theta\left(\tau_1+\frac{\tau_2-\tau_1}{\alpha}+\frac{y_{3j}-\tau_2}{\alpha\gamma}\right)\right]\\ =&\theta^n\left(\frac{1}{\alpha}\right)^{n_2}\left(\frac{1}{\alpha\gamma}\right)^{n_3}\\ &\times\exp\left[-\theta\left(U+n_2\tau_1+\frac{V}{\alpha}+\frac{W}{\alpha\gamma}+n_3\tau_1+n_3\frac{\tau_2-\tau_1}{\alpha}\right)\right],\end{aligned}\tag{2.3}$$

where $n=n_1+n_2+n_3$, $U=\sum_{j=1}^{n_1}y_{1j}$, $V=\sum_{j=1}^{n_2}(y_{2j}-\tau_1)$, $W=\sum_{j=1}^{n_3}(y_{3j}-\tau_2)$, $n_1$ be the number of observations under the stress $s_1$, $n_2$ be the number of the observations under the stress $s_2$, and $n_3=n-n_1-n_2$ be the number of observations under the stress $s_3$.

From the Equation (2.3), we can obtain the MLEs of $\theta$, $\alpha$ and $\gamma$ as follows.

$$\begin{aligned}\hat{\theta}=&\frac{n_1}{U+(n_2+n_3)\tau_1}\\ \hat{\alpha}=&\frac{n_1\left[V+n_3(\tau_2-\tau_1)\right]}{n_2\left[U+(n_2+n_3)\tau_1\right]}=\frac{\hat{\theta}\left[V+n_3(\tau_2-\tau_1)\right]}{n_2}\\ \hat{\gamma}=&\frac{n_1W}{\hat{\alpha}n_3\left[U+(n_2+n_3)\tau_1\right]}=\frac{W\hat{\theta}}{n_3\hat{\alpha}}.\end{aligned}\tag{2.4}$$

Now we are to find the statistics for this model. But to do so, we need the life time under the use condition. So, after replacing the parameters with the MLEs, we transform the life time under the stresses into the life time under the use condition. Then, the observed life time, $\hat{T}_{ij}$, under the use condition are given by

$$\hat{T}_{ij}=\begin{cases}\hat{T}_{1j}=Y_{1j}, & j=1,2,\cdots,n_1\\ \hat{T}_{2j}=\frac{Y_{2j}-\tau_1}{\hat{\alpha}}+\tau_1, & j=1,2,\cdots,n_2\\ \hat{T}_{3j}=\frac{Y_{3j}-\tau_2+\hat{\gamma}(\tau_2-\tau_1)}{\hat{\alpha}\hat{\gamma}}+\tau_1, & j=1,2,\cdots,n_3.\end{cases}\tag{2.5}$$

Moreover, $\hat{T}_{ij}$ , $i = 1, 2, 3,\ j = 1, 2, \cdots, n_i$ has an exponential distribution with the failure rate $\theta$ because $\hat{T}_{ij}$ is an observation of $T_{ij}$ .

Then we will show strong consistencies for $\hat{T}_{ij}$, $i = 1, 2, 3,\ j = 1, 2, \cdots, n_i$ using the MLEs of parameters.

**Theorem 2.1** Let $T_{11}, \cdots, T_{1n_1},\ T_{21}, \cdots, T_{2n_2},\ T_{31}, \cdots, T_{3n_3}$ be a sample from three step stress ALT under exponential distribution with $\theta$ . Then for $\hat{T}_{11}, \cdots, \hat{T}_{1n_1}$, $\hat{T}_{21}, \cdots, \hat{T}_{2n_2}$, $\hat{T}_{31}, \cdots, \hat{T}_{3n_3}$ ,

$$T_{ij} - \hat{T}_{ij} \ \xrightarrow{a.s} \ 0 \tag{2.6}$$

for $i = 1, 2, 3,\ j = 1, 2, \cdots, n_i$ .

**proof** We already know the MLE $\hat{\theta}$, $\hat{\alpha}$, and $\hat{\gamma}$ are consistent estimators of $\theta$, $\alpha$ and $\gamma$ , respectively, and $\hat{T}_{ij}$ is a function of $\hat{\theta}$, $\hat{\alpha}$, and $\hat{\gamma}$. Using the invariance property of the MLEs, we can easily prove it.

## 3. ESTIMATORS OF KULLBACK-LEIBLER INFORMATION FUNCTION USING THREE STEP STRESS ALTS

In this section, we proposed three estimators ( Vasicek (1976), van Es (1992), and Correa (1995)) of KLI functions using the data from ALTs. This acceleration model is assumed to be a TRV model.

To begin with, we think of the KLI. As mentioned before, we deal with KLI under the given hypothesis testings. Here, if we let the pdf of $\hat{T}_{ij}$ with Equation (2.5) be $f_{\hat{T}_{ij}}$, we can give the hypothesis as follows, whether the pdf is equivalent with the specific pdf $f_0$ or not. Then, the K–L Information is given by,

$$\begin{aligned} I(f_{\hat{T}_{ij}} : f_0) &= \int_{-\infty}^{\infty} f_{\hat{T}_{ij}}(t) \ln \frac{f_{\hat{T}_{ij}}(t)}{f_0(t)} dt \\ &= -H(f_{\hat{T}_{ij}}) - \int_{-\infty}^{\infty} f_{\hat{T}_{ij}}(t) \ln f_0(t) dt, \end{aligned} \tag{3.1}$$

where the entropy, $H(f_{\hat{T}_{ij}}) = -\int_{-\infty}^{\infty} f_{\hat{T}_{ij}}(t) \ln f_{\hat{T}_{ij}}(t) = \int_{-\infty}^{\infty} \left( \frac{d}{dt} F_{\hat{T}_{ij}}^{-1}(t) \right) dt$.

Here, because the remaining term except the entropy term is a constant, we get the estimated KLI by estimating the entropy only. So, in classification, pattern recognition, statistical physics, stochastic dynamics, statistics, etc.. Just in statistics, Shannon's entropy is used as a descriptive parameter (measure of dispersion), for testing normality (Vasicek (1976), Arizono and Ohta (1989)), exponentiality (Ebrahhimi, Habibullah, and Soofi (1992), Wieczorkowski and Grzegorzewski (1999)) and uniformity (Dudewicz, van der Meulen et al.(1995)).

Here, by assumption that $\hat{T}_{ij}$ has an exponential distribution, the KLI $I(f_{\hat{T}_{ij}}, f_0)$ is given by

$$I(f_{\hat{T}_{ij}} : f_0) = -\hat{H}(f_{\hat{T}_{ij}}) - \ln\hat{\theta} + 1. \tag{3.2}$$

And we use the prominent Vasicek's estimator, van Es' estimator and Correa's estimator as the entropy estimator, $\hat{H}(f_{\hat{T}_{ij}})$.

First, an estimator, $H_{mn}$, of an entropy $H(f)$, proposed by Vasicek (1976) is the Equation (3.3). The estimator was constructed by using a difference operator instead of the differential operator. The derivative of $F^{-1}(p)$ is then estimated by a function of the order statistics. So, Assuming that $T_{(1)}, T_{(2)}, \cdots, T_{(n)}$ is the odered sample of $\hat{T}_{ij}$, the estimator is given by

$$H_{mn} = \frac{1}{n}\sum_{i=1}^{n} \ln\left\{\frac{n}{2m}(\hat{T}_{(i+m)} - \hat{T}_{(i-m)})\right\}, \tag{3.3}$$

where $\hat{T}_{(1)} \le \hat{T}_{(2)} \le \cdots \le \hat{T}_{(n)}$ are the order statistics, $m$ is a positive integer, $(m < \frac{n}{2})$,and $T_{(i)} = T_{(1)}$ for $i < 1$, $T_{(i)} = T_{(n)}$ for $i > n$.

Therefore, from the Equations (3.2) and (3.3), an estimator of KLI, or $I(f_{\hat{T}_{ij}}, f_0)$ is proposed by

$$I_1 = I_1(f_{\hat{T}_{ij}}, f_0) = -H_{mn} - \ln\hat{\theta} + 1. \tag{3.4}$$

But, to put $I_1$ as the statistic of this model, we must show its consistency. Vasicek already proved the consistency of $H_{mn}$ for classical model and we know that $H_{mn}$ of the observation $\hat{T}_{ij}$ is also consistent because we know that the transformed $\hat{T}_{ij}$ is exponential distributed under use condition. So, we know $I_1$ is consistent through the following theorem.

**Theorem 3.1** Let $\hat{T}_{11}, \cdots, \hat{T}_{1n_1}, \hat{T}_{21}, \cdots, \hat{T}_{2n_2}, \hat{T}_{31}, \cdots, \hat{T}_{3n_3}$ with transformation be a random sample from the exponential distribution. Then

$$I_1(f_{\hat{T}_{ij}} : f_0) \xrightarrow{p} I(f_{T_{ij}}, f_0) \tag{3.5}$$

as $n \longrightarrow \infty$, $m \longrightarrow \infty$, $\frac{m}{n} \longrightarrow \infty$ .

**proof** With consistency of $H_{mn}$, the MLE $\hat{\theta}$ of $\theta$ converges to $\theta$ in probability. So, according to the Slutsky Theorem, we proved it.

Secondly, van Es suggested an entropy estimator of the Equation (1.1) based on the differences of random samples as follows

$$\begin{aligned} HE_{mn} =& \frac{1}{n-m}\sum_{i=1}^{n-m} \ln\left\{\frac{n+1}{m}(\hat{T}_{(i+m)} - \hat{T}_{(i-m)})\right\} \\ &+ \sum_{k=m}^{n}\frac{1}{k} + \ln(m) - \ln(n+1), \end{aligned} \tag{3.6}$$

where $\hat{T}_{(1)} \leq \hat{T}_{(2)} \leq \cdots \leq \hat{T}_{(n)}$ are the order statistics, $m$ is a positive integer($m < \frac{n}{2}$), and $\hat{T}_{(i)} = \hat{T}_{(1)}$ for $i < 1$, $\hat{T}_{(i)} = \hat{T}_{(n)}$ for $i > n$.

So, the estimated KLI is given by

$$I_2 = I_2(f_{\hat{T}_{ij}} : f_0) = -HE_{mn} - \ln\hat{\theta} + 1. \tag{3.7}$$

van Es also proved his estimator is consistent and we can know $I_2$ is also consistent by replacing $I_1$ into $I_2$ from the above Theorem.

As the last estimator, in 1995, Correa suggested a modification of Vasicek's estimator and prove his estimator is consistent. In estimation the density $f$ of $F$ in the interval $(T_{(i-m)}, T_{(i+m)})$. This yields a following estimator

$$HC_{mn} = -\frac{1}{n}\sum_{i=1}^{n} \ln(b_i), \tag{3.8}$$

where $b_i = \frac{\sum_{j=i-m}^{i+m}(\hat{T}_{(j)} - \bar{\hat{T}}_{(i)})(j-i)}{n\sum_{j=i-m}^{i+m}(\hat{T}_{(j)} - \bar{\hat{T}}_{(i)})^2}$, $\bar{\hat{T}}_{(i)} = \frac{1}{2m+1}\sum_{j=i-m}^{i+m}\hat{T}_{(j)}$, $\hat{T}_{(i)} = \hat{T}_{(1)}$ for $i < 1$, and $\hat{T}_{(i)} = \hat{T}_{(n)}$ for $i > n$. $m$ is a positive integer smaller than $\frac{n}{2}$.

Correa showed that for a few distribution (standard normal, exponential with mean equal to 1 and uniform U(0,1)) his estimator produces smaller mean squared error than Vasicek's estimator and van Es' estimator in simulation. And the KLI is estimated by

$$I_3 = I_3(f_{\hat{T}_{ij}}) = -HC_{mn} - \ln\hat{\theta} + 1. \tag{3.9}$$

Finally, we can suggest the above three estimators of KLI as the statistics in our three step stress ALT model.

## 4. MONTE CARLO SIMULATION

In this section, we compare three estimators $I_1$, $I_2$, $I_3$ in bias and MSE and know which one is best in them.

For the Equations (3.4), (3.7), (3.9), i.e., $I_1$, $I_2$, $I_3$, we suppose the failure rate $\theta = 0.5$, the significant level $\alpha = 0.5$ and each tampering point $\tau_1 = \frac{-\ln(0.3)}{\theta}$, $\tau_2 = \frac{4}{3}\tau_1$ as simulation conditions.

For each distribution 500 samples of sizes 10, 15, 20, 25, 30, 35, 40 and 50 were generated and the estimators of $I_1$, $I_2$, $I_3$ and their bias and MSE(mean squared error) were computed : each experiment was repeated 10 times and average was taken. And from Figure 4.1 and Figure 4.2, we observed the following facts.

First, in a three step stress ALT model, both the bias and the MSE decrease as $m$ and $n$ increase. In the case of Bias, for fixed $n$, the biases of the estimators of $I_1$, $I_3$, decrease and have also same results as n increases. For $I_2$, though the bias increases a little for the fixed $n$, the change becomes very small. Moreover the change becomes smaller as n increases. MSEs of three estimators, $I_1$, $I_2$, $I_3$ also decrease both fixed $n$ and increased $n$.

Second, as regard to bias and MSE comparisons, van Es' estimator is the best in the above three estimators. And especially, we can think that $I_2$ goes to zero faster and stabler than other two estimators from the Bias figure and the MSE figure.

Through comparison for entropy estimators, Correa (1995) showed that his estimator is better than Vasicek's estimator in the sense that the MSE and the bias are smaller. In 1999, Wieczorkowski and Grzegorzewski suggested the corrected Correa's estimator and had the same results with Correa (1995). And recently Park, Yoon and Cho (2000) studied the KLI in ALTs.

Our simulation results also indicate that regarding the MSE and the bias, the van Es' estimator is better than Vasicek's estimator and Correa's estimator in three step stress ALTs.

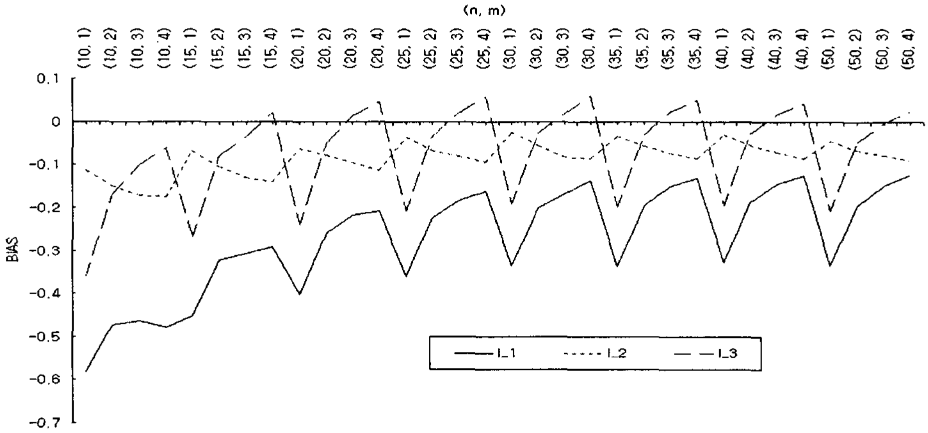


**Figure 4.1** Bias for $I_1$, $I_2$, $I_3$

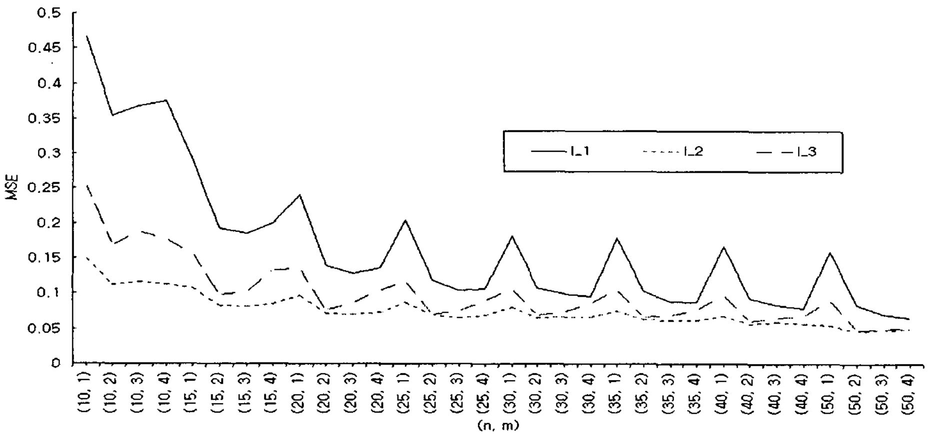


**Figure 4.2** MSE for $I_1$, $I_2$, $I_3$

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