Journal of the Korean Society of Groundwater Environment, Vol. 7, No. 1, pp. 20~23, March, 2000

A General Solution of Determining Storage Coefficient From Multi-Step Pumping Test Recovery Data

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Abstract: A general solution for determining the storage coefficient from multi-step pumping test recovery data is suggested. This solution is essentially based on the method of Banton and Bangoy (1996), which used single-step pumping test recovery data. The suggested solution can be applied to any-step pumping test recovery data. We have demonstrated the applicability of the general solution to single-, double-, and triple-step pumping and/or step-drawdown test data partially described in Lee and Lee (1999). The estimates of storage coefficient as well as transmissivity are well consistent with the values from other methods for pumping phase data.

요 약: 양수시험시 획득되는 회복자료만을 이용하여 저유계수를 추정하는 일반해를 제시하였다. Banton and Bangoy(1996)가 제시한 단 단계(single-step) 양수시험에서 얻는 회복자료를 이용한 저유계수 추정해를 일반적인 다단계(multi-step) 시험 회복자료에도 적용 가능하게 확장한 것이다. 본 연구에서 구한 일반 해를 이용하여 저유계수와 투수계수를 구한 결과 다른 방법으로 구한 결과 값과 잘 일치하였다.

Introduction

Conventional methods such as Theis (1935) recovery method and Cooper-Jacob (1946) method can not be used to compute storage coefficient from recovery data. As indicated in Banton and Bangoy (1996), a few methods have been suggested to estimate storage coefficient from recovery data (Bardsley *et al.*, 1985; Ballukraya and Sharma, 1991, Shapiro *et al.*, 1998). Unfortunately most of these methods are associated with large errors and/or complexity in the computation of storage coefficient (Shapiro *et al.*, 1998). Banton and Bangoy (1996) suggested a simple method to estimate storage coefficient from pumping test recovery data. The method was applied to solely single-step pumping test recovery data. For the case of variable rate pumping test recovery data, they used an average pumping rate to compute storage coefficient. In this study, we suggested a general solution of estimating storage coefficient from any-step pumping test recovery data (variable rate pumping test recovery data) without averaging pumping rates as in Banton and Bangoy (1996). The suggested solution was applied to single-, double-, and triple-step pumping test recovery data in Lee (1998) and Lee and Lee (1999).

Mathmathetical Implementation

The mathematical developments described below are essentially based on Banton and Bangoy (1996). Therefore, all assumptions and limitations related to the suggested solution are the same as Banton and Bangoy (1996).

If n pumping-step (including Q=0 at the time of pumping shutdown) exists in a pumping test or step-drawdown test, the residual drawdown by the superposition rule can be described as follows:

$$s(r,t) = \frac{Q_1}{4\pi T}W(u_1) + \frac{Q_2 - Q_1}{4\pi T}W(u_2) + \frac{Q_3 - Q_2}{4\pi T}W(u_3) + \cdots + \frac{0 - Q_{n-1}}{4\pi T}W(u_n) \quad (1)$$

Q_i: pumping rate of *i*th step

$u_i = \frac{r^2 S}{4Tt_i}$, t_i : time elapsed since start of the ith pumping step

Substituting the first three terms of Theis (1935) infinite series for the well function yields

$$\begin{aligned} s(r,t) &= \frac{Q_1}{4\pi T}\left(\ln\frac{u_2}{u_1} + u_1 - u_2\right) + \frac{Q_2}{4\pi T}\left(\ln\frac{u_3}{u_2} + u_2 - u_3\right) + \\ &\quad \frac{Q_2}{4\pi T}\left(\ln\frac{u_4}{u_3} + u_3 - u_4\right) + \cdots + \frac{Q_{n-1}}{4\pi T}\left(\ln\frac{u_n}{u_{n-1}} + u_{n-1} - u_n\right) \\ &= \frac{Q_1}{4\pi T}\left(\ln\frac{t_1}{t_2} + \frac{r^2 S}{4Tt_1} - \frac{r^2 S}{4Tt_2}\right) + \frac{Q_2}{4\pi T}\left(ln\frac{t_2}{t_3} + \frac{r^2 S}{4Tt_2} - \frac{r^2 S}{4Tt_3}\right) + \cdots \\ &= \frac{Q_{n-1}}{4\pi T}\left(\ln\frac{t_{n-1}}{t_n} + \frac{r^2 S}{4Tt_{n-1}} - \frac{r^2 S}{4Tt_n}\right) \end{aligned} \quad (2)$$

If the natural logarithm is changed to the common logarithm, then Equation 2 becomes:

$$s(r,t) = \frac{2.3Q_1}{4\pi T}log\frac{t_1}{t_2} + \frac{Q_1 r^2 S}{16\pi T^2}\left(\frac{1}{t_1} - \frac{1}{t_2}\right) + \frac{2.3Q_2}{4\pi T}\log\left(\frac{t_2}{t_3}\right) + \cdots$$

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$$+\frac{2.3Q_{n-1}}{4\pi T}\log\left(\frac{t_{n-1}}{t_n}\right)+\frac{Q_{n-1}r^2S}{16\pi T^2}\left(\frac{1}{t_{n-1}}-\frac{1}{t_n}\right) \quad (3)$$

Arranging Equation 3 with respect to r gives;
Therefore,

$$s(r, t_1, t_2, \cdots, t_n) =$$

$$A_1+B_1r^2+A_2+B_1r^2+\cdots+A_{n-1}+B_{n-1}r^2$$

$$=(A_1+A_2+\cdots+A_{n-1})+(B_1+B_2+\cdots+B_{n-1})r^2 \quad (4)$$

where $A_i(t_i, t_{i+1}) = \left(\frac{2.3Q_i}{4\pi T}\right)\log\left(\frac{t_i}{t_{i+1}}\right)$

and $B_i(t_i, t_{i+1}) = \left(\frac{Q_iS}{16\pi T^2}\right)\left(\frac{1}{t_t}-\frac{1}{t_{i+1}}\right)$

By plotting the residual drawdown, $s(r, t_1, t_2,\cdots t_n)$ versus r^2, straight lines are obtained. From the straight line, the intercept $(A_1+A_2+...+A_{n-1})$ and the slope $(B_1+B_2+...B_{n-1})$ can be computed using an appropriate optimization method. The intercept is expressed as follows:

$$A=\frac{2.3Q_i}{4\pi T}\log\left(\frac{t_1}{t_2}\right)+\frac{2.3Q_2}{4\pi T}\log\left(\frac{t_2}{t_3}\right)+\cdots+\frac{2.3Q_{n-1}}{4\pi T}\log\left(\frac{t_{n-1}}{t_n}\right)$$

$$=\frac{2.3}{4\pi T}\log\left[\left(\frac{t_1}{t_2}\right)^{Q_1}\left(\frac{t_2}{t_3}\right)^{Q_2}\cdots\left(\frac{t_{n-1}}{t_n}\right)^{Q_{n-1}}\right] \quad (5)$$

Plot of the intercept A versus $\log\left[\left(\frac{t_1}{t_2}\right)^{Q_1}\left(\frac{t_2}{t_3}\right)^{Q_2}\cdots\left(\frac{t_{n-1}}{t_n}\right)^{Q_{n-1}}\right]$ gives a slope, $\frac{2.3}{4\pi T}$, from which the transmissivity can be computed. The slope B in Equation 4 is:

$$B=\frac{Q_1S}{16\pi T^2}\left(\frac{1}{t_1}-\frac{1}{t_2}\right)+\frac{Q_1S}{16\pi T^2}\left(\frac{1}{t_2}-\frac{1}{t_3}\right)+\cdots+\frac{Q_{n-1}S}{16\pi T^2}\left(\frac{1}{t_{n-1}}-\frac{1}{t_n}\right)$$

$$=\frac{S}{16\pi T^2}\left[\left(\frac{1}{t_1}-\frac{1}{t_2}\right)Q_1+\left(\frac{1}{t_2}-\frac{1}{t_3}\right)Q_2+\cdots+\left(\frac{1}{t_{n-1}}-\frac{1}{t_n}\right)Q_{n-1}\right]$$

Therefore, plot of the slope B versus $\left[\left(\frac{1}{t_1}-\frac{1}{t_2}\right)Q_1+\left(\frac{1}{t_2}-\frac{1}{t_3}\right)Q_2\right]$ $Q_2+\cdots+\left(\frac{1}{t_{n-1}}-\frac{1}{t_n}\right)Q_{n-1}\Big]$ enables the computation of a slope, $\frac{S}{16\pi T^2}$, from which the storage coefficient can be obtained.

Applications

To validate the suggested solution, three sets of field data were used. The details of these data and characteristics of the aquifer can be referred to Lee (1998) and Lee and Lee (1999). The test site is located at Buron-myun, approximately 23 km southwest of the city of Wonju, Korea. The test site is in an area of low topographic relief. Stratigraphic units underlying the test site include Precambrian gneiss intruded by Jurassic granite, and covered by alluvium and a local reclamation layer. The test site was equipped with a pumping well, three deep observation wells (Lee and Lee, 1999; see Figure 1). In the validation, single-, double-, and triple-step pumping test recovery data were used.

Figure 2(a) shows single-step pumping test result. The radial distances from the pumping well to the observation wells are 5.02 and 22.15 meters. Pumping rate was 17.55 L/min. Drawdown and recovery of ground water level were monitored for about 1200 min using an automatic data logger with pressure transducers. Using these data, the intercept A and the slope B were obtained from plots of residual drawdown versus square of distance (r^2) at each time. Plot of

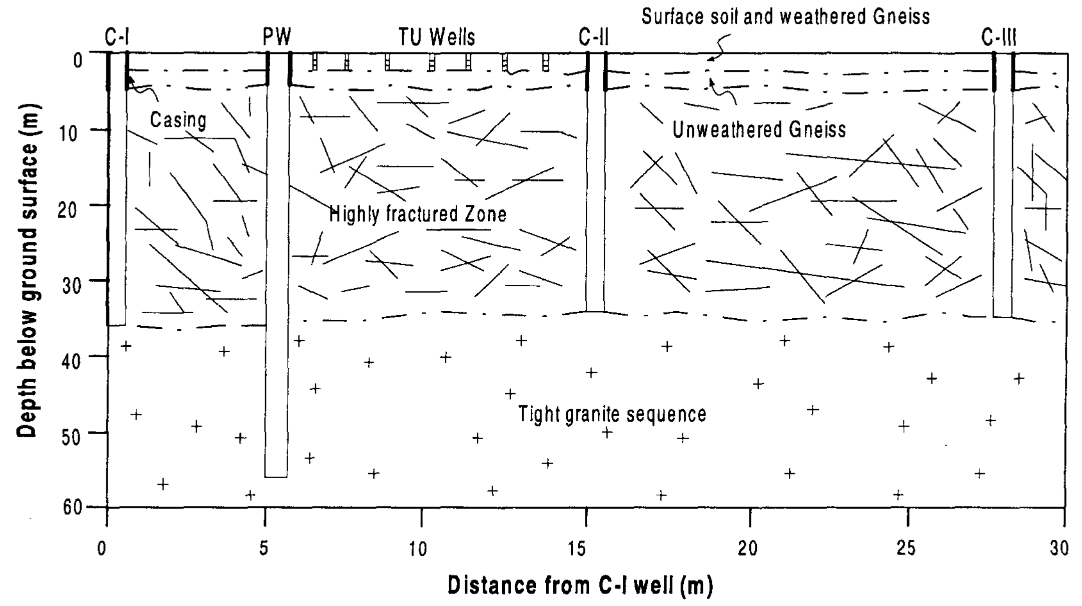

Figure 1. Location of the test wells.

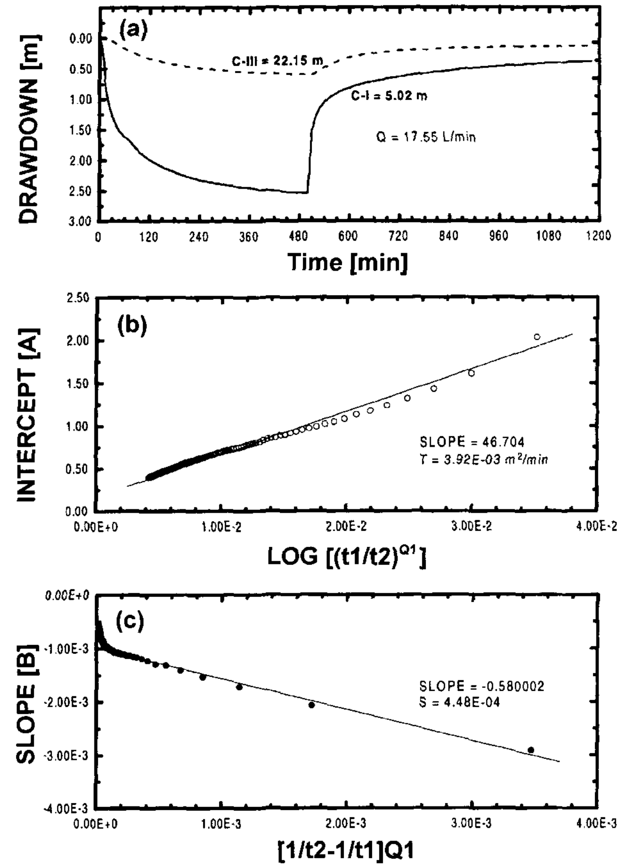

Figure 2. Determination of transmissivity and storage coefficient from the plots of drawdown vs time, intercept vs log[t_1/ t_2]Q_1, and slope vs $1/t_2-1/t_1$. The observation wells are C-I and C-III, and the pumped well is PW.

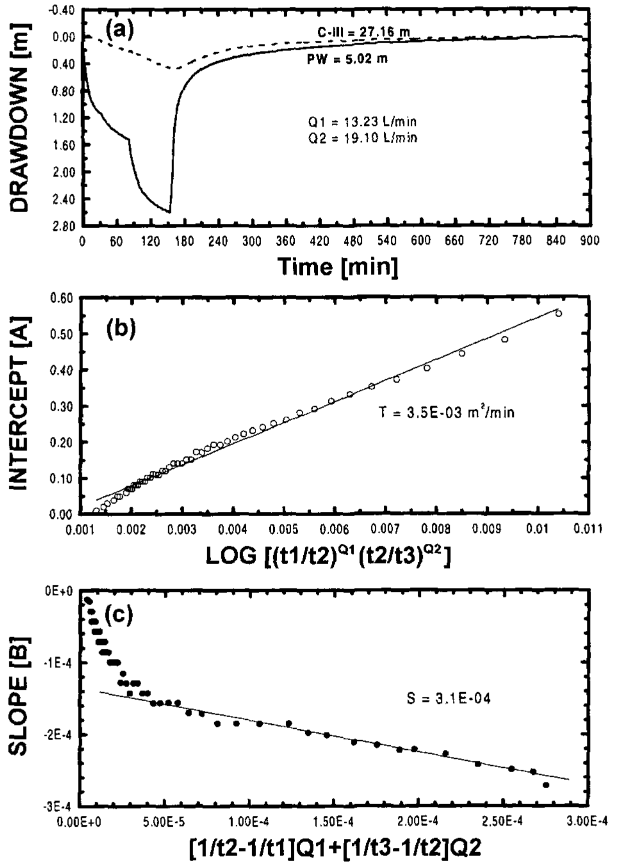

Figure 3. Determination of transmissivity and storage coefficient from the plots of drawdown vs time, intercept vs log [$(t_1/t_2)Q_1(t_2/t_3)Q_2$], and slope vs [$1/t_2-1/t_1$]Q_1+[$1/t_3-1/t_2$]Q_2. The observation wells are PW and C-III, and the pumped well is C-I.

intercept A versus Q_1 log(t_1/t_2) in Figure 2(b) yields a slope of 46.7, which gives a transmissivity of 3.92E-02 m^2/min. Also the storage coefficient was computed from the plot of slope B versus $Q_1(1/t_2-1/t_1)$ yielding a slope of 0.580 and a storage coefficient of 4.48E-04.

Figure 3 also shows an application of the suggested solution to double-step pumping test recovery data. The pumping rate was firstly 13.23 L/min and later was increased up to 19.10 L/min. As in the case of analysis for the single-step recovery data, the plots of the optimized intercept and the slope versus $log[(t_1/t_2)^{Q_1}(t_2/t_3)^{Q_2}]$ and $[1/t_2-1/t_1]Q_1+[1/t_3-1/t_2]Q_2$ gave a transmissivity estimate of 3.5E-03 m^2/min and a storage coefficient of 3.1E-04, respectively. Figure 4 is associated with the analysis of the triple-step pumping test recovery data using the suggested solution. As in the same manner, the transmissivity and storativity were computed.

Table 1 shows a comparison of the estimated values of

Table 1. Estimates of transmissivity and storage coefficients obtained by the suggested solution and the two classical methods. Transmissivities are in m^2/min.

Pumping Well	Obs Wells	Cooper-Jacob (1946) Pumping Phase	Theis Recovery (1935) Recovery Phase	The Suggested Soultion Recovery Phase
PW	C-I, C-III	*T=4.14E-03 **S=5.31E-04	T=4.55E-03	T=3.92E-03 S=4.48E-04
C-I	PW, C-III	T=4.02E-03 S=3.99E-04	T=3.98E-03	T=3.50E-03 S=3.10E-04
C-II	C-III, PW, C-I	T=5.08E-03 S=8.42E-04	T=8.34E-03	T=5.97E-03 S=6.20E-04

*, **: geometirc mean

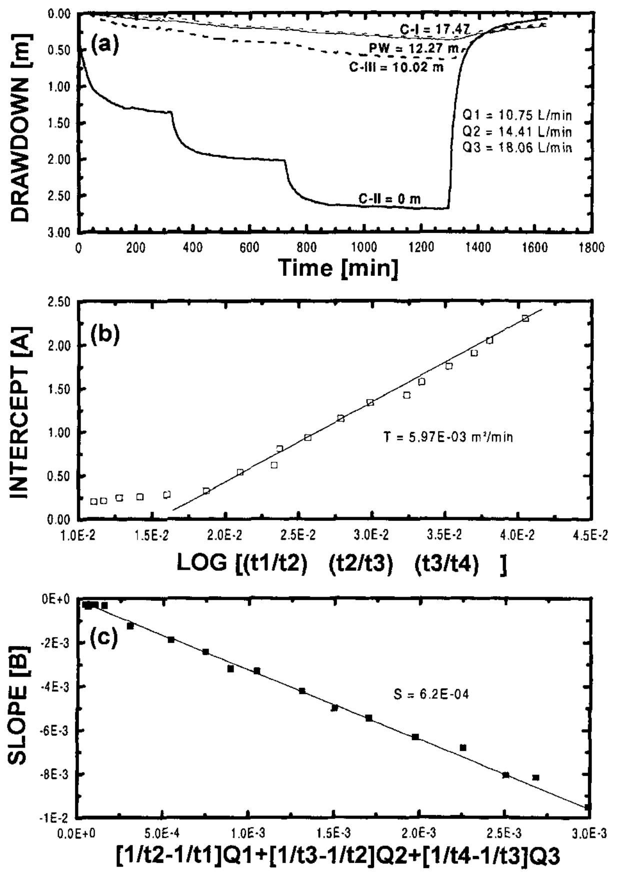

Figure 4. Determination of transmissivity and storage coefficient from the plots of drawdown vs time, intercept vs $\log[(t_1/t_2)Q_1(t_2/t_3)Q_2(t_3/t_4)Q_3]$, and slope vs $[1/t_2-1/t_1]Q_1+[1/t_3-1/t_2]Q_2+[1/t_4-1/t_3]Q_3$. The observation wells are C-III, PW and C-I, and the pumped well is C-II.

transmissivity and storage coefficient by the suggested solution with the classical Cooper-Jacob (1946) approximation and Theis (1935) recovery methods. The values of transmissivity and storage coefficient from the three methods are well consistent. In the considerations of these results, it is concluded that the suggested solution, in which only recovery phase data used, will produce a good estimate of transmissivity and especially storage coefficient.

Conclusion and Discussion

In this study, we have suggested a general solution to estimate storage coefficient from pumping test recovery data. This solution was derived from an general extension of a method for single-step pumping test recovery data of Banton and Bangoy (1996). Therefore, the suggested solution can be applied to any-step pumping test and/or step drawdown test recovery data. The three-case validations indicate that the suggested solution can be comparable to the other classical methods, such as Cooper-Jacob (1946) and Theis (1935) recovery methods in estimating transmissivity and storage coefficient. Especially this solution has an advantage of estimating storage coefficient with a good quality from recovery data. The potential effects of nonlinear well losses in the vicinity of the pumped well developed during pumping phase, however, can not be explained by this solution as in Banton and Bangoy (1996). A more complicated method considering turbulent head losses in the pumped well was suggested by Shapiro *et al.* (1998).

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