Computer-Aided Decision Analysis for Improvement of System Reliability

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Abstract

Nowadays, every kind of system is changed so complex and enormous, it is necessary to assure system reliability, product liability and safety. Fault tree analysis(FTA) is a reliability/safety design analysis technique which starts from consideration of system failure effect, referred to as "top event", and proceeds by determining how these can be caused by single or combined lower level failures or events. So in fault tree analysis, it is important to find the combination of events which affect system failure. Minimal cut sets(MCS) and minimal path sets(MPS) are used in this process. FTA-I computer program is developed which calculates MCS and MPS in terms of Gw-Basic computer language considering Fussell's algorithm. FTA-II computer program which analyzes importance and function cost of VE consists of five programs as follows : (1) Structural importance of basic event, (2) Structural probability importance of basic event, (3) Structural criticality importance of basic event, (4) Cost-Failure importance of basic event, (5) VE function cost analysis for importance of basic event. In this study, a method of initiation such as failure, function and cost in FTA is suggested, and especially the priority rank which is calculated by computer-aided decision analysis program developed in this study can be used in decision making determining the most important basic event under various conditions. Also the priority rank can be available for the case which selects system component in FMEA analysis.

1. Introduction

FTA is a reliability/safety design analysis technique which starts from consideration of system failure effect, referred to as "top event", and proceeds by determining how these can be caused by single or combined lower level failures or events. It is a top-down analysis, and takes multiple failure modes into account[8].

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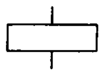
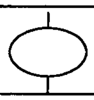
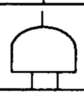
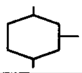
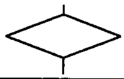
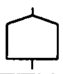
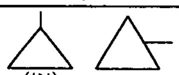
No.	Symbol	Name of symbol	Meaning of symbol
1		Event	An event that results from the combination of basic events
2		Basic Event	A basic fault event that requires no further development
3		AND Gate	Failure will occur if all inputs fail
4		OR Gate	Failure will occur if any input fails
5		Inhibit Gate	Casual relationship between one fault and another. The input event directly produces the output event if the indicated condition is satisfied.
6		Undeveloped Event	Event which could be subdivided into basic events but while this is not done through lack of information or usefulness.
7		House Event	Basic event which is a normal occurrence while the system is operating.
8	 (IN) (OUT)	Triangles	A line from the apex of the triangle indicates a transfer in a line from the side denotes a transfer out.

<Fig. 1> Standard logical symbol used in Fault Tree Analysis

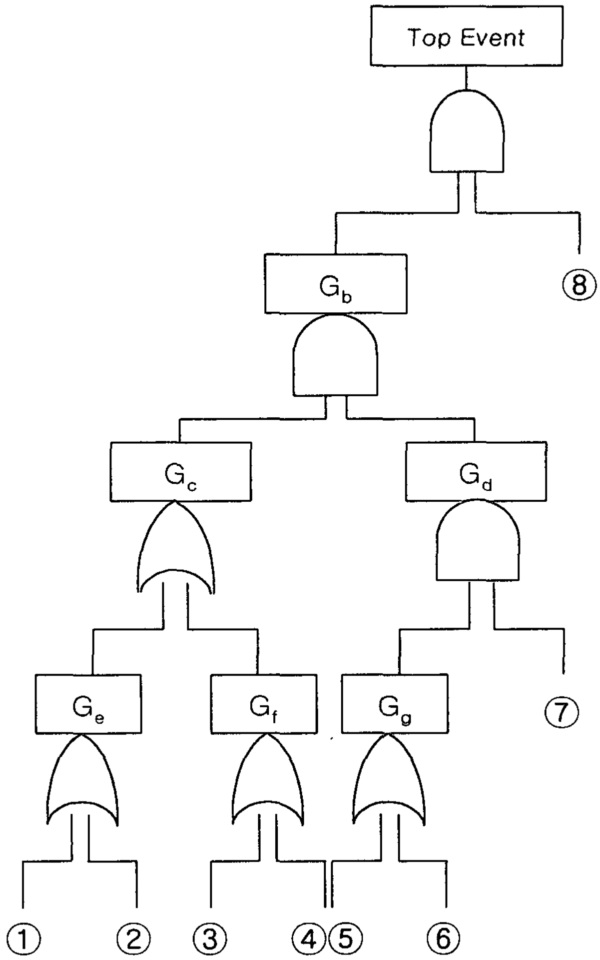
This deductive method first appeared in 1962 in the Bell laboratories and was used to eliminate several weak points in the Minuteman project. Later Mearns in Bell laboratories improved this technique, and significant computerized improvement was done with Haasl, Schroder, Jackson in Boeing company[3,4].

In this study, standard type of logical symbol[2,8] is used as indicated in <Fig. 1> and <Fig. 2> is used in example data for efficiency test and result analysis.

Fault tree analysis is aimed to find undesirable events that are concerned with system failure and improve weak point in the system to improve system safety and reliability.

So in fault tree analysis, it is important to find the combination of events which affect system failure. Minimal cut sets and minimal path sets are used in this process.

A cut set is a set of component whose failure causes the system to fail. Minimal cut set is a cut set not containing any other cut sets. A path set is a set of component whose functioning insures the functioning of the system. Minimal path set is a minimal set of path set[3].



<Fig. 2> Simple Fault Tree

The result of minimal cut sets and minimal path sets in <Fig. 2> which are calculated by computer-aided decision analysis program developed in this study is as below.

Minimal Cut Sets :	Minimal Path Sets :	Probability of Top Event : 0.417410E-05
{ 1 , 8 , 5 , 7 }	{ 1 , 3 , 2 , 4 }	Probability of Gate A : 0.417410E-05
{ 3 , 8 , 5 , 7 }	{ 8 }	Probability of Gate B : 0.417410E-03
{ 2 , 8 , 5 , 7 }	{ 5 , 6 }	Probability of Gate C : 0.195051E+00
{ 1 , 8 , 6 , 7 }	{ 7 }	Probability of Gate D : 0.214000E-02
{ 4 , 8 , 5 , 7 }		Probability of Gate E : 0.144400E+00
{ 3 , 8 , 6 , 7 }		Probability of Gate F : 0.592000E-01
{ 2 , 8 , 6 , 7 }		Probability of Gate G : 0.107000E+00
{ 4 , 8 , 6 , 7 }		

2. Theoretical analysis

2.1 FTA structure function

Generally, for the purpose of analyzing complex system, it is convenient to initiate Boolean Algebra or mathematical structure function which shows relationship between system and component. Also in this study, in order to change complex statement in FT into simple expression, the system structure function which consists of n component is defined[5] as below.

Statement of i_{th} component :

$$x_i = \begin{cases} 1 : & \text{if component } i \text{ is functioning,} \\ 0 : & \text{if component } i \text{ is failed,} \end{cases}$$

$$\text{for } i = 1, 2, \ldots, n.$$

Statement of system :

$$\phi = \begin{cases} 1 : & \text{if the system is functioning,} \\ 0 : & \text{if the system is failed.} \end{cases}$$

We assume that the state of the system is determined completely by the state of the components, so that we may write structure function of system[5] :

$$\Phi = \Phi(x),$$

where

$$x = \{x_1, \cdots, x_n\}.$$

If a system structure is a type of series structure, i.e., the structure function of OR gate is

$$\phi(x) = \coprod_{i=1}^{n} x_i = \min\{x_1, \ldots, x_n\}.$$

where $\coprod_{i=1}^{n} x_i = 1 - \prod_{i=1}^{n}(1 - x_i).$

If a system structure is a type of parallel structure, i.e., the structure function of AND gate is given by

$$\phi(x) = \prod_{i=1}^{n} x_i = \max\{x_1, \ldots, x_n\}.$$

2.1.1 Structural importance of basic event

In FT, top event occurrence depends on whether the basic event occurs or not. The structural importance of basic event is the analysis of structural importance without probabilistic analysis of basic event occurrence.

This method gives a priority ranks according to structural importance, i.e., it shows how basic event occurrence affects top event occurrence.

The combination of top event changes as to each basic event changes in cut sets is written as :

$$n_{\phi}(i) = \sum_{(x|x_i=1)} [\phi(1_i, x) - \phi(0_i, x)].$$

So, we can define the structural importance of i_{th} basic event :

$$I_{\phi}(i) = \frac{1}{2^{n-1}} n_{\phi}(i).$$

The results of structural importance in <Fig. 2> which is calculated by computer-aided decision analysis program developed in this study are

$I_{\Phi}(1)$ = 0.234375E-01,
$I_{\Phi}(2)$ = 0.234375E-01,
$I_{\Phi}(3)$ = 0.234375E-01,
$I_{\Phi}(4)$ = 0.234375E-01,
$I_{\Phi}(5)$ = 0.117188E+00,
$I_{\Phi}(6)$ = 0.117188E+00,
$I_{\Phi}(7)$ = 0.351563E+00,
$I_{\Phi}(8)$ = 0.351563E+00.

Thus, the priority of the structural importance of basic event is

8 = 7 > 6 = 5 > 4 = 3 = 2 = 1.

As a result, we can find the sequence of priority rank in decision makings is 8 and 7.

2.1.2 Structural probability importance of basic event

The structural probability importance of basic event is the method to analyze how much basic event occurrence probability has influence on top event occurrence probability on the basis of basic event occurrence probability. If we can control the basic event which has the first priority of probability importance, it means we can reduce top event occurrence effectively.

In FT, we generally assume that basic event is independent of each other. When we define Pi, g as below,

p_i = $Pr\{x_i=1\}$, for $i=1, 2, \cdots, n$.

g = $Pr\{\phi(x)=1\}$.

Hence, by definition,

$g=g(p)$,

$g(p)=p_i g(1_i,p)+(1-p_i)g(0_i,p)$.

From the property of coherent structure function and g function, the probability importance of basic event j is given[3] by

$$I_g(j)=\frac{\partial g(p)}{\partial p_j},$$

$$I_g(j)=g(1_j,p)-g(0_j,p).$$

The results of structural probability importance for <Fig. 2> which is calculated by computer-aided decision analysis program developed in this study are

$I_g(1)$ = 0.187238E-04,
$I_g(2)$ = 0.185225E-04,
$I_g(3)$ = 0.175774E-04,
$I_g(4)$ = 0.179436E-04,
$I_g(5)$ = 0.366697E-04,
$I_g(6)$ = 0.370598E-04,
$I_g(7)$ = 0.208705E-03,
$I_g(8)$ = 0.417410E-05.

Thus, the priority of the structural probability importance of basic event is

7 > 6 > 5 > 1 > 2 > 4 > 3 > 8.

Summarily, we can find the sequence of priority rank in decision makings is 7, 6 and 5.

2.1.3 Structural criticality importance of basic event

The structural criticality importance of basic event is the method which compares transformation in occurrence probability percentage ratio of basic event to top event.

Hence, Criticality importance is written as :

$$CI_g(j)=\frac{\partial \ln g(p)}{\partial \ln p_j}=\frac{p_i}{g}I_g(j).$$

The results of structural criticality importance for <Fig. 2> which is calculated by computer-aided decision analysis program developed in this study are

$CI_g(1)$ = 0.358857E+00,
$CI_g(2)$ = 0.310623E+00,
$CI_g(3)$ = 0.842214E-01,
$CI_g(4)$ = 0.171952E+00,
$CI_g(5)$ = 0.439252E+00,
$CI_g(6)$ = 0.532710E+00,
$CI_g(7)$ = 0.100000E+01,
$CI_g(8)$ = 0.100000E-01.

Thus, the priority of the structural criticality importance of basic event is 7 > 6 > 5 > 1 > 2 > 4 > 3 > 8.

2.1.4 Cost-Failure importance of basic event

The cost failure importance of basic event is the method which compares ratio of relative failure percentage to relative cost percentage basic event occurrence prevention.

Hence, the cost failure importance of basic event is written as :

$$V_I(i) = \frac{Relative\ Failure\ \%}{Relative\ Cost\ \%}.$$

The results of cost failure importance of basic event for <Table 1> which is calculated by computer-aided decision analysis program developed in this study are

$V_I(1)$ = 0.461714E+01,
$V_I(2)$ = 0.269333E+01,
$V_I(3)$ = 0.524675E+00,
$V_I(4)$ = 0.128254E+01,
$V_I(5)$ = 0.961905E+00,
$V_I(6)$ = 0.203697E+01,

<Table 1> Failure percentage and cost for basic event

Basic Event	Failure Rate	Relative %	Cost(₩)	Relative %
1	0.08	22.86	10,000	4.98
2	0.07	20.00	14,000	6.97
3	0.02	5.71	22,000	10.95
4	0.04	11.43	18,000	8.96
5	0.05	14.29	30,000	14.93
6	0.06	17.14	17,000	8.46
7	0.02	5.71	40,000	19.90
8	0.01	2.86	50,000	24.88

$V_I(7)$ = 0.288571E+00,

$V_I(8)$ = 0.115429E+00.

Thus, the priority of the cost failure importance of basic event is
1 > 2 > 6 > 4 > 5 > 3 > 7 > 8.

2.1.5 VE function cost analysis for importance of basic event

Value Engineering is defined as a systematic, creative approach to ensure that the essential function of a product, process, or administrative procedure is provided at a minimum overall cost. It involves five basic phases : information, speculation, analysis, planning and decision, and summary and conclusion. Although none of these phases are new to the engineering profession, the systematic approach employed in value engineering has led to many startling cost improvements in industry[6].

A quantitative formula for the value of product or service is written as :

$$V=\frac{F}{C} \quad \textit{where} \quad V= V\text{alue}, \quad F=F\text{unction}, \quad C= C\text{ost}.$$

In this study, the function cost analysis of VE, in which structure and probability analysis are considered along with cost and function, is suggested to FTA, so that priority rank of basic event can be determined. <Table 2> is obtained from the function analysis for <Fig 2> which takes advantage of cost and function value of basic event.

The results of function cost analysis which has the lowest value for <Table 2> calculated by computer-aided decision analysis program developed in this study are
1 > 2 > 3 > 7 > 8 > 4 > 5 > 6.

<Table 2> Functional cost and value for basic event (unit : ₩)

Basic event	Cost	Function	Value(F/C)	C - F
1	10,000	8,000	0.80	2,000
2	15,000	14,000	0.93	1,000
3	22,000	25,000	1.14	-3,000
4	18,000	30,000	1.67	-12,000
5	30,000	45,000	1.50	-15,000
6	17,000	40,000	2.35	-23,000
7	40,000	45,000	1.13	-5,000
8	50,000	60,000	1.20	-10,000

2.2 Computer Program

In this study, FTA-I computer program which calculate MCS and MPS in terms of Gw-Basic computer language considering Fussell's algorithm[3] is developed.

The procedures of computer program are as below :

① Calculate number of low and column in $\Delta x,y$ matrix.

② Eliminate gate name in $\Delta x,y$ matrix.

③ Repeat ② until $\Delta x,y$ matrix consist of all basic event.

④ Calculate BICS(Boolean Indicated Cut Sets).

⑤ Calculate Minimal Cut Sets.

⑥ Transform original FT into (Dual)FT.

⑦ Calculate Minimal Path Sets.

With : $\rho_{w,i}$: i_{th} input in gate w,
λw : number of input in gate w,
x : x_{th} BICS,
y : y_{th} element in certain BICS,
$\Delta x,y$: variable in x_{th} BICS and y_{th} element in certain BICS,
$x_{\max}$: maximum used value of x,
$y_{\max}$: maximum used value of y in x_{th} BICS.

The routines for more detailed procedure of computer program are :

(1) Calculate the number of column in $\Delta x,y$ matrix :

$$X_i = \begin{cases} x_{i.1} \cdot x_{i.2} \cdots x_{i.j_{\max}} & \text{if } i \text{ is AND gate,} \\ x_{i.1} + x_{i.2} + \cdots + x_{i.j_{\max}} & \text{if } i \text{ is OR gate.} \end{cases}$$

Calculate the number of row in $\Delta x,y$:

$$Y_i = \begin{cases} y_{i.1} + y_{i.2} + \cdots + y_{i.j_{\max}} & \text{if } i \text{ is AND gate,} \\ \max(y_{i.1}, y_{i.2}, \cdots, y_{i.j_{\max}}) & \text{if } i \text{ is OR gate.} \end{cases}$$

If gate i is a next gate of top event, the final value of *Xtop* or *Ytop* is obtained. (2), (3) Eliminate of gate name :

Eliminate gate w from (x,y) in $\Delta x,y$ matrix,

$\Delta x,y = \rho_{w,1}$.

- If i is AND gate :

  $\Delta x,y_{\max}+1 = \rho_{w,\pi}$, for $\pi=2, \ldots, \lambda w$.

  where $y_{\max}$ is increased every π.

- If i is OR gate :

  $\Delta x_{\max}+1,n = \Delta x,n$ for $n=1, \ldots, y_{\max}$. if $n \neq y$,

  $= \rho_{w,\pi}$ if $n=y$.

  for $\pi=2, \ldots, \lambda w$.

where x_{max} is increased every π.

(4) Calculate BICS.

(5) Calculate Minimal Cut Set.
Eliminate duplicate event or cut from BICS.

(6) Transform original FT into dual FT.
Transform AND gate into OR gate, OR gate into AND gate and each event into complementary event.

(7) Calculate Minimal Path Set.
Repeat step (1) - (5).

FTA-II computer program which analyzes importance and function cost of VE consists of five programs as below and is automatically linked with FTA-I computer program output file, so only you input data-file name in FTA-I :

(1) Structural importance of basic event,
(2) Structural probability importance of basic event,
(3) Structural criticality importance of basic event,
(4) Cost-Failure importance of basic event,
(5) VE function cost analysis for importance of basic event.

3. Conclusion

In this study, a method of initiation such as failure, function and cost in FTA is suggested, and especially the priority rank which is calculated by computer-aided decision analysis program developed in this study can be used in decision making determining the most important basic event under various conditions. Also the concept of priority rank can be available for the case which select system component in FMEA analysis.

In conclusion, FTA-II computer program consists of five programs which analyze importance and the function cost of VE can be used for system reliability improvement as below :

(1) Structural importance of basic event :

a) Structural Analysis
FMEA which consists of component order of criticality failure event, major effect event, medium effect event and minor effect event can be used for the case which prevents system failure caused by single component failure. And it is the primary analysis method concerned with structural point of view.

b) Probability Analysis
It is possible to find which component contribute to system failure. And it

can be used more effectively for outside order component.

(2) Structural probability importance of basic event,

(3) Structural criticality importance of basic event :
By this analysis, we can find which component most influences system failure sensitively.

(4) Cost-Failure importance of basic event :
This method stresses cost point of view, for instance, it can be used for determination of inspection order which has a lot of inspection item.

(5) VE function cost analysis for importance of basic event :
This method can be used for determination of component priority rank by using functional appraisement of VE.

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