# Synthetic risk management over risk of financial assets

# -금융자산의 위험에 관한 종합적 위험관리-

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# 요 약

최근의 추세를 볼 때 위험관리에 관한 중요성이 점점 증대하고 있다. 그럼에도 불구하고 우리나라 은행들의 위험관리 실태는 아직 미흡한 실청이다. 그리고 대부분의 은행들이 현재 위험관리에 대응하기 위하여 ALM의 갭관리, 듀레이션관리 등을 행하고 있지만 BIS에서 중요시하고 있는 VaR의 개발과 운용은 아직 초보단계에 있다. 한국 주식포트폴리오에서 몬테카를로 시뮬레이션과 Full Variance Covariance Model의 VaR값은 비슷한 수준으로 Diagonal Model의 VaR값 보다 작음을 알 수 있다. 이는 좀 더 정교한 계산이 요구되는 Full Variance Covariance Model의 VaR값이 보다 단순한 Diagonal Model의 VaR값 보다 정확성면에서 우수하다는 것을 보여주고 있다. 한편 이자율포트폴리오의 경우에는 델타-감마 분석법과 몬테카를로 시뮬레이션의 경우 95% 신뢰구간의 VaR는 델타-감마 분석법이 작지만 99% 신뢰구간에서의 VaR는 몬테카를로 시뮬레이션방법이 작다는 것을 알 수 있다. 그래서 어느 한 가지 방법에 의한 VaR추정치가 가장 좋은 것이라고 단정하기 어려움을 알 수 있었다.

#### 1. Introduction

Orange County, Barings, Metallgesellschaft. . . Some of the world's largest financial entities have lost billions of dollars in financial markets. In most cases, -

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senior management poorly monitored the exposure to market risks. To address this problem, the world's leading banks and financial firms are turning to value at risk (VaR), an easy-to-understand method for calculating and controlling market risks. The recent debate on *derivatives* has also brought to the forefront the issue of financial instruments that "caused" huge losses and should be curtailed. An opposite view is that, provided they are judiciously used, derlivatives are inherently stabilizing because they allow better allocation of risk. Regulators have also stated that the devotion of "substantial resources to the development of more sophisticated risk management tools

have had favorable spill-over effects on institutions' abilities to manage their total portfolios, not just their derivative activities." In other words, derivatives have started the revolution in financial risk management that is now leading to the widespread use of VaR.

What is VaR? VaR is a method of assessing risk that uses standard statistical techniques routinely used in other technical fields. Formally, VaR measures the worst expected loss over a given time interval under normal market conditions at a given confidence level. Based on firm scientific foundations. VaR provides users with a summary measure of market risk. For instance, a bank might say that the daily VaR of it trading portfolio is \$35 million at the 99 percent confidence level. In other words, there is only 1 chance in a 100, under normal market conditions, for a loss greater than \$35 million to occur. This single number summarizes the bank's exposure to market risk as well as the probability of an adverse move. Equally important, it measures risk using the same units as the bank's bottom line dollars. Shareholders and managers can then decide whether they feel comfortable with this level of risk. If the answer is no, the process that led to the computation of VaR can be used to decide where to trim the risk.

No doubt this is why regulators and industry groups are now advocating the used of VaR system. In 1995, the International Swap and Derivatives Association (ISDA) stated that

ISDA believes that the measurement of market risk is meaningful to readers of financial statements. The measure thought to be appropriate by most of the leading practitioners is some form of Value-at-Risk.

# 2. Background

# 2.1 Additive Capital Requirements

The CAD and the Basle standardized approach are very similar. Heavily influenced by the systems of capital requirements operated by U.K. and U.S. securities regulators, both systems require a firm to hold capital equivalent to a percentages of its hodings in different asset categories, where the percentages are chosen to reflect the price volatilities of generic assets in the relevant categories.

An important drawback of both CAD and the Basle standardized approach is the *additive* nature of the capital required for broad asset categories. The requirement is calculated market by market for equity, foreign exchange(FX), and interest rate risk, and then the separate requirements are summed. Thus, for example, the capital requirement for a long position in U.K. equities takes into account hedging in the same market but not, say, any offset from holding a short position in U.S. equities. Nor does it take into account the benefits in diversification from holding long positions in both markets.

The effect is to favor specialized market makers at the expense of globally diversified banks. Banks that run global portfolios have therefore pressed the Basle Committee to consider approaches to capital requirements that do recognize the benefits of diversification.

Clearly, achieving this in a regime in which the supervisors set the percentage capital requirements and hedging allowances for different types of position would be extremely complex. But firms themselves have been developing methods of measuring the risk of given losses on a total portfolio, and these internal whole book or value at risk(VaR), models have provided a way of making the problem tractable. Hence, it is possible to develop an alternative to the Basle standardized approach.

# 2.2 The Basle Alternative Approach

In the Basle "alternative approach," rather than laying down percentage capital requirements for different exposures, regulators would establish standards for bank's in-house risk models. These models would then form the basis for the calculation of capital requirements. This has the key additional advantage of aligning the capital calculation with the risk measurement approach of the

particular firm.

Using internal models to generate capital requirements is a radical change in approch, but supervisors have for some time been moving steadily in this direction. In the CAD and the Basle standardized method, it is recognized that only by employing the firm's internal models can some positions be correctly processed for inclusion in the capital calculation. This is particularly the case for options, but sensitivity models designed to convert large books of swaps into equivalent bond exposures and assess the risk on foreign exchange books are also allowed.

This does, however, raise a number of issues for supervisors concerning the safeguards that should be put in place to ensure that the capital requirements generated are adequate. Basle has addressed this in several ways. One is to lay down standards for the construction of the models. For example, models must calculate the distribution of losses over a ten-day holding period using at least twelve months of data, and must yield capital requirements sufficient to cover losses on 99% of occasions.

Adopting general standards is necessary both to increase consistency between banks and to ensure that capital requirements really are adequate to the task. In theory, however, they might create inconsistencies between the regulatory model and the one that a firm uses for its own purposes. Typically, firm's VaR models use a 95% confidence interval and a twenty-four-hour holding period. Basle will not, however, prescribe the type of model to be used.

#### 2.3 Regulatory Safeguards

As a post hoc check on the accuracy of the models under the proposed alternative Basle approach, the supervisors will carry out backtesting, the comparison of actual trading results with model-generated risk measures. This may pose problems, first, because trading results are often affected by changes in portfolios in the period following the calculation of the VaR. Because of this, Basle has urged banks to develop the capability to perform backtests using the losses that would have occurred if the book had been held constant over a one-day period. Second, Kupiec(1995) argues that backtesting requires a large number of observations in order to make a judgment about the accuracy of the model's estimate of the tail of the probability distribution.

Nevertheless, backtesting and some kind of penalty are essential to provide incentives for firms to increase the accuracy of the models. The Basle proposals envisage that firms that do not meet the backtesting criterion for accuracy should suffer additional capital charges.

As well as backtesting, the system would include the safeguard of an over-riding multiplier. More precisely, Basle is proposing that the capital requirement should be equivalent to the higher of 1) the current value at risk estimate, and 2) the average VaR estimate over the previous sixty days multiplied by three.

The incorporation of a multiplier has the advantage of making the system more conservative without distorting the treatment of trading books with different risk profiles. Of course, if the multiplier is too high, it could discourage firms from developing in-house models and lead them to select the standardized rather than the alternative approach, because, as mentioned above, banks themselves are free to choose which they adopt.

# 2.4 Value at Risk Analysis

What then is the nature of the "whole book" or VaR models that will be used in capital requirement calculations by banks that take the Basle Committee's alternative approach? The typical VaR models developed by firms for their internal risk management purposes attempt to measure the loss on a portfolio over a specified period (often the next twenty-four hours) that will be exceeded only on a given fraction of occasions (typically 1% or 5%). Two broad types of VaR analysis are employed.

First, under parametric VaR analysis, the distribution of asset returns is estimated from historical data, assuming that this distribution is a member of a given parametric class. The most common procedure is to suppose that returns are stationary, joint normal, and independent over time. Using estimates of the means and covariances of returns, one may calculate the daily loss that will be exceeded with a given probability.

Second, the simulation approach to VaR analysis consists of finding, from a long run of historical data, the loss that is exceeded on a long a given percentage of the days in the sample. As a non-parametric procedure, this approach imposes no distributional assumptions.

In this article, we examine various aspects of VaR analysis and its as an instrument of banking regulation from an empirical point of view. Using data on the equity, interest, and FX rate exposure of a bank with significant trading cativity, we compare the empirical performance of parametric and simulation-based VaR analysis. Even though the proposed Basle Accord Amendment does not specify which approach banks should use, the penalties envisaged for banks whose models fail to forecast loss probabilities accurately make this an important question.

We also look at impact of window length (i.e., the length of returns data series used) and weighting factors for the returns. The alternative Basle system requires the use of at least one year of data, and we assess whether this appears sensible. A finding of considerable practical significance is that adopting different approaches to estimating return volatility for reasonably well-diversified fixed income portfolios makes little difference to the degree to which one can forecast the average size of price changes. The techniques one employs in calculating volatility can affect forecasting accuracy in a statistically significant way, but the improvements are not substantial enough to be economically significant. On the other hand, the various aproaches to VaR modeling differ widely in the accuracy with which they predict the fraction of times a given loss will be exceeded. If this latter criterion is applied, simulation-based rather than parametric VaR techniques appear preferable. Finally, we investigate the precise formula for required capital proposed in the Basle alternative approach. The current proposal is that capital must exceed the maximum of 1) the previous day's VaR, or 2) three times the average VaR of the previous sixty days. It is interesting to ask, with our real-life books, how the scaling factor and the fact that one must take the maximum of two quantities affect the outcome.

### 2.5 The advantages and drawbacks of VaR Management

The advantages of VaR Management are that it

- · Incorporates the mark-to-market approach uniformly.
- · Relies on a much shorter horizon forecast of market variables. This improves the risk estimate as short horizon forecasts tend to be more accurate than long horizon forecasts.

Of course, drawbacks exist. One of them is that it may not be trivial to mark certain transactions to market or even understand their behavior under certain rate environments. This is particularly true for instruments such as demand deposits in a retail banking environment for example. Whatever the difficulties, the aim of getting an integrated picture of a firm's exposure to market risks is worth a number of assumptions, some of which may be reasonable representations of reality.

# 3. Empirical Analysis related methods

Empirical Analysis related methods are involved with models such as delta-gamma

method(variance-covariance method), garch modeling, full variance covariance model, diagonal model, monte Carlo Simulation.

## 3.1 Delta-Gamma Method

Assume that return innovations are jointly normally distributed with mean zero and non-singular M×M covariance matrix  $\Sigma\Delta t$ , e.g.  $\Delta S \sim N(0, \Sigma\Delta t)$ ; define  $\delta$  as the portfolio's M×1 delta vector with respect to each of the market rates (e.g.  $\partial P/\partial S$ ) and  $\gamma$  as its (non-singular), symmetric gamma matrix (e.g.  $(\partial^2 P/\partial S_i S_i)$ .

Define P as the non-singular Cholesky decomposition of  $\Sigma$  defined by  $\Sigma = PP'$ , satisfying the equation  $P^{-1}\Sigma P^{-1} = I$ , and, T as the orthogonal matrix which satisfies the equation  $T'(P'\gamma P)T = \gamma^*$ , where  $\gamma^*$  is the diagonal matrix of the eigenvalues of the matrix  $P'\gamma P$  and T is the matrix of eigenvalues of  $P'\gamma P$ .

Define  $\phi = \text{Min}[\phi^-, \phi^+, 0]$ , where  $\phi^- = (\delta'PTe - \alpha e'\gamma)$  and  $\phi^+ = (\delta'PTe + \alpha e'\gamma)$ , an M×1 vector, where e is an M×1 vector of 1s,  $\alpha$  is the number of standard deviations required to give the desired confidence interval for a standardized univariate normal variate, and the Min[] operator is defined element by element for the vector. Finally, define  $\sigma_{\delta\gamma} = \sqrt{\dot{\phi}}\dot{\phi}$ . Then the capital at risk of the portfolio is approximated by: VaR  $\delta_{\gamma} = \alpha \sigma_{\delta\gamma} \sqrt{\Delta t}$ 

#### 3.2 Monte Carlo Simulation

The Monte Carlo simulation method calculates capital at risk by using the three-step procedure. Just like the empirical simulation method, except that the simulation are based on specific models for market rate innovations over the holding period rather than on the historical innovations.

#### 3.3 Full Variance Covariance Model

A portfolio can be characterized by positions on a certain number of risk factors. Once the decomposition is established, the portfolio return is a linear combination of the returns on underlying assets, where the weights are given by the relative dollar amounts invested at the beginning of the period. Therefore, the VaR of a portfolio can be reconstructed from a combination of the risks of underlying

securities.

Define the portfolio return from t to t+1 as

$$R_{p,t+1} = \sum_{i=1}^{N} w_{i,t} R_{i,t+1}, \qquad (2)$$

where the weight  $w_{i,t}$  were established at the beginning of the period and sum to unity. To shorten notation, the portfolio return can be written using matrix notation, replacing strings of numbers by a single vector:

$$R_{p} = [w_{1}w_{2}\cdots w_{N}] \begin{bmatrix} R_{1} \\ R_{2} \\ \vdots \\ R_{N} \end{bmatrix} = w'R$$
(3)

where w' represents the transposed vector (i.e., horizontal) of weights and R is the vertical vector containing individual asset returns.

By extension of the formulas, the portfolio expected return is

$$E(R_{p}) = \mu_{p} = \sum_{i=1}^{N} w_{i} \mu_{i}$$
 (4)

and the variance is

$$V(R_{p}) = \sigma_{p}^{2} = \sum_{i=1}^{N} w_{i}^{2} \sigma_{i}^{2} + \sum_{i=1}^{N} \sum_{j=1,j\neq 1}^{N} w_{i} w_{j} \sigma_{ij}$$

$$= \sum_{i=1}^{N} w_{i}^{2} \sigma_{i}^{2} + 2 \sum_{j=1}^{N} \sum_{j=1}^{N} w_{i} w_{j} \sigma_{ij}$$
(5)

This sum accounts not only for the risk of the individual securities  $\sigma_i^2$ , but also for all different cross-products, which add up to a total of N(N-1)/2 different covariances.

As the number of assets increases, it becomes difficult to keep track of all covariance terms, which is why it is easier to use matrix notation. The variance is

$$\sigma_{p}^{2} = \left[ \mathbf{w}_{1} \mathbf{w}_{2} \cdots \mathbf{w}_{N} \right] \begin{bmatrix} \sigma_{1}^{2} & \sigma_{12} & \sigma_{13} \cdots \sigma_{1N} \\ \vdots & \vdots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \sigma_{N3} \cdots \sigma_{N}^{2} \end{bmatrix} \begin{bmatrix} \mathbf{w}_{1} \\ \mathbf{w}_{2} \\ \vdots \\ \mathbf{w}_{N} \end{bmatrix}$$

$$(6)$$

Define  $\Sigma$  as the covarance matrix, the portfolio variance can be written more compactly as  $\sigma_p^2 = w' \Sigma w$ .

Using a normal distribution, the VaR measure is then  $a\sigma_p$  times the initial investment.

Lower portfolio risk can be achived through low correlations or a large number of assets. To see the effect of N, assume that all assets have the same risk and taat all correlations are the same, that equal weight is put on each asset.

Start with the risk of one security, which is assumed to be 12 percent. When  $\rho$  is equal 0, thw risk of a 10-asset portfolio drops to 3.8 percent; increasing N to 100 drops the risk even farther to 1.2 percent. Risk tends asymptotically to zero. More generally, portfolio risk is

$$\sigma_{\rm p} = \sqrt{\sigma \frac{1}{N} + (1 - \frac{1}{N})\rho} \tag{7}$$

which tends to  $\sqrt[n]{\rho}$  as N increases. So, when  $\rho = 0.5$ , risk decreases rapidly from 12 percent to 8.9 percent as N goes to 10, but then converges much more slowly toward its minimum value of 8.5 percent. Correlations are essential in lowering portfolio risk.

Covariances can be estimated from sample data as

$$\widehat{\sigma}_{ij} = \frac{1}{(T-1)} \sum_{i=1}^{T} (\mathbf{x}_{t,i} - \widehat{\mu}_i) (\mathbf{x}_{t,j} - \widehat{\mu}_j)$$
(8)

Covariance is a measure of the extent to which two variables move linearly together. If two variables are independent, their covariance is equal to 0. A positive covariance means that the two variables tend to move in the same direction; a negative covariance means that they tend to move in opposite directions.

The magnitude of covariance, however, depends on the variances of the individual components and is not easily interpreted. The correlation coefficient is a more convenient, scale-free, measure of linear dependence;  $\rho_{12} = \sigma_{12}/(\sigma_1 \sigma_2)$ .

The correlation coefficient  $\rho$  always lies between -1 and +1. When equal to unity, the two variables are said to be perfectly correlated. When 0, the variables are uncorrelated.

Correlations help to diversify portfolio risk. With two assets, the "diversified" portfolio variance is  $\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \rho_{12} \sigma_1 \sigma_2$ .

For simplicity, assume now that both assets have the same volatility. When the

correlation is 0,

$$\sigma_{p}^{2} = V(R_{1} + R_{2}) = w_{1}^{2}\sigma_{1}^{2} + w_{2}^{2}\sigma^{2} = (w_{1}^{2} + w_{2}^{2})V(R)$$
(9)

The portfoio risk must be lower than individual risk. When the correlation is exactly unity,

$$V[w_1R_1 + w_2R_2] = w_1^2V[R] + w_2^2V[R] + 2w_1w_2V[R]$$

$$= (w_1 + w_2)^2V[R]$$

$$= V[R],$$
(10)

since the portfolio weights sum to unity. Generally, the "undiversified" VaR is the sum of individual VaR measures - diversification into perfectly correlated assets does not pay.

So far, nothing was said about the distribution of the portfolio return. Ultimately, we would like to translate the portfolio variance into a VaR measure. To do so, we need to know the distribution of the portfolio return. In the "delta-normal" model, all individual security returns are assumed normally distributed. This is particularly convenient since the portfolio, a linear combination of normal random variables, is then also normally distributed. At a given confidence level, the portfolio VaR is value at risk =  $a\sigma_{b}$ .

#### 3.3.1 Diagonal Model

A related problem is that, as the number of assets increases, it is more likely that some correlations will be measured with error. Some models can help simplifying this process by providing a simpler structure for the covariance matrix. One such model is the diagonal model, originally proposed by Shape in the context of stock portfolios.<sup>1)</sup>

<sup>1)</sup> Note that this model is often referred to as the CAPM, which is not correct. The diagonal model is simply a simplification of the covariance matrix and says nothing about expected returns, whose description is the essence of the CAPM.

#### 4. Empirical tests

#### 4.1 Literature Survey: Fallon(1996)

He used weekly data sampled on successive Wednesdays to help mitigate problems others have encountered with daily data. These include spurious negative serial correlation due to "bid-ask" bounce, as well as problems due to non-trading periods such as holidays, which normally occur on Mondays, Tuesdays, and Fridays. He found usage of weekly data (with weeks beginning and ending on successive Wednesdays) largely avoids these problems. The equity prices are contemporaneous daily closing prices for Dow Chemical, Exxon, Union Carbide, Coca-Cola, and Standard & Poors 500 Index over a twenty-six year period starting on January 1, 1969 and ending on December 31, 1994. These five securities are widely held, actively traded, and highly liquid. To compute a weekly return series using the twenty-six years of daily data, he compounded daily returns between successive Wednesdays. This approach netted 1,356 observations for each series.

<Table 1> compares the performance of the six VaR calculation methods in correctly predicting the VaR. Three a values are tested 10% 5% and 1% For

<Table 1> compares the performance of the six VaR calculation methods in correctly predicting the VaR. Three  $\alpha$  values are tested, 10%, 5% and 1%. For each expected  $\alpha$  level for each model, we show the actual number of times over the 706 week testing period that the actual realized loss on the portfolio exceeded the estimated VaR. This count is denoted as C in Table 1. Next to C is the actual frequency implied by C, given the 706 observations, which we denote as A. A is computed as A=C/706. If the VaR calculation method is accurate, then A, the "actual"  $\alpha$ , should equal the expected  $\alpha$ .

The results indicate that in two of three cases, the A for the gamma-GARCH model is closest to the expected  $\alpha$  level. The gamma-WTN model performs slightly better than the gamma-GARCH model at the 1% level, but this may be due to the low number of observations in this category. All of the HOM models perform poorly. In all cases, either or both of the WTN and GARCH models more accurately predict the VaR. This echoes the efficiency regression results. All of the gamma models perform better than the corresponding delta models, usually by wide margins. For example, at  $\alpha$  =5%, the best of the delta models (delta-GARCH) achieves A=1.8%. But this performance is very poor compared to the corresponding gamma-GARCH method, which achieves A=4.7%.

Portfolio Fen.	State Variable	Expected $\alpha$ Level					
Approximation		10%		5%		1%	
Model	Model	С	Α	С	Α	C	A
P-1 - delta	X-1 - HOM	27	3.8%	12	1.7%	0	0.0%
P-1 - delta	X-2 - WTN	29	4.1	10	1.4	1	0.1
P-1 - delta	X-3 - GARCH	32	4.5	13	1.8	3	0.4
P-2 - gamma	X-1 - HOM	51	7.2	27	3.8	4	0.6
P-2 - gamma	X-2 - WTN	53	7.5	29	4.1	7	1.0
P-2 - gamma	X-3 - GARCH	60	8.5	33	4.7	10	1.4

<Table 1> Out-of-Sample VaR Comparison

Note: 1) P-1 - delta is assumed that  $P(t,\chi)$  has one derivative with respect to each argument, denotes  $P_t$  and g, where  $P(t,\chi)$  is portfolio pricing function, with t representing time and  $\chi$  representing an  $n\times 1$  vector of random state variables.  $P_t = \frac{\partial P(t,\chi)}{\partial t} \quad \text{is a scalar and } g = \left[ \begin{array}{c} \frac{\partial P(t,\chi)}{\partial x_1} \,, \, \frac{\partial P(t,\chi)}{\partial x_2} \,, \cdots, \, \frac{\partial P(t,\chi)}{\partial x_n} \, \right] \quad \text{is } n\times 1.$ 

Higher derivatives are assumed equal to 0.

- 2) P-2 gamma is assumed that P(t,  $\chi$ ) has two derivatives wrt t and  $\chi$ , denoted P<sub>t</sub>, P<sub>tt</sub>, g,  $P_{\chi}$ , and H. P<sub>t</sub> =  $\frac{\partial P}{\partial t}$  and P<sub>tt</sub> =  $\frac{\partial^2 P}{\partial t^2}$  are scalars.  $g = \frac{\partial P}{\partial \chi}$  and P<sub>tx</sub> =  $\frac{\partial^2 P}{\partial t \partial \chi}$  are n×1 vectors and H is an n×n matrix. Each element of H, H<sub>ij</sub>, is computed as  $H_{ij} = \frac{\partial^2 P}{\partial x_i \partial x_j}$ . Higher oder derivatives are equal to 0.
- 3) X-1 HOM is assumed that the sequences of simple returns to holdings  $\chi$  over a forecast horizon  $\Delta t$ , R, is jointly normally distributed with mean vector 0 and constant covariance matrix  $\Sigma^{1r}$ , i.e.,  $r \sim N_n(0, \Sigma^{1r})$ , for  $r_{it} = (x_{i,t+\triangle t} x_{it})/x_{it}$ .
- 4) X-2 WTN is assumed that Each elements  $\sigma_{ij(t)}$  of  $\Sigma_t^{2r}$ , the time-varying covariance matrix of  $\gamma$ , is computed as  $\sigma_{ij(t)} = \Sigma_{k=0}^{\infty} \omega_k (r_{i(t-k)} \mu_{ri}) (r_{j(t-k)} \mu_{rj})$  where  $\omega_k = \lambda^k (1-\lambda)$ ,  $0 < \lambda < 1$ , and  $\lambda$  is given exogenously.
- 5) X-3 GARCH is assumed that State variable returns,  $\gamma_t$  are distributed as:

$$\gamma_t = \mu + u_t$$

$$u_t \mid \psi_{t-1} \sim N(0, \Sigma_t^{3r})$$

where  $\gamma, \mu, u_t$  are  $n \times 1$  and  $\Sigma_t^{3r}$  is  $n \times n$ .  $\psi_{t-1}$  is the information set at t-1.

Individual elements of  $\Sigma_t^{3r}$  are given as follows:

$$v_{ii(t)} = \alpha_{0i} + \alpha_{1i} u_{i,t-1}^{2} + \beta_{i} v_{ii(t-1)}$$
$$v_{ij(t)} = \rho_{ij} \sqrt{v_{ii(t)}} \sqrt{v_{ij(t)}}$$

#### 4.2 Empirical tests

#### 4.2.1 Stock portfolio

Analysis materials are daily stock returns of Sila trade company, Sam yang company, Sam sung electronics. Analysis horizon is from October 17, 1996 to December 29, 1997.<sup>2)</sup> The result of <Table 2> shows that VaR of Monte Carlo Simulation and Full Variance Covariance Model is less than that of Diagonal Model. By the way, each of VaR is less than daily maximum loss weight 8% at price of stock on 'financial solid regulation of securities' in securities.

<Table 2> Results of VaR related estimate methods

(basis: %)

Estimate methods	VaR(Value at Risk) confidence level		
Estimate methods	95%	99%	
Monte Carlo Simulation	-3.5804	-5.1100	
Full Variance Covariance Model	-3.5666	-5.2165	
Diagonal Model	-3.8265	-5.5968	

Note: 1) Price of stock is ₩24,000 at Cheil Jedang Company, ₩7,700 at Sam Yang Company, ₩49,000 at Sam Sung Electronics on December 29, 1997.

- 2) Minus(-) of VaR is daily maximum loss weight on price of stock.
- 3) Random variables of Monte Carlo Simulation are 5,000.

<sup>2)</sup> where Cheil Jedang Company is related with food and drink, Sam Yang Company is related with texture, Sam Sung Electronics is related with electronics. The reason structured as different fields is that structure of stock portfolio needs that.

#### 4.2.2 Interest rate

<Table 3> Results of VaR related estimate methods

(basis : ₩)

Estimate methods	Delta-gamma analysis		Monte Carlo Simulation		
Confidence interval	95%	99%	95%	99%	
VaR	-127.4803	-180.0182	-139.4522	-151.9554	

Note: 1) Price of Bond is ₩10,087 at end of period on December 1997.

- 2) Minus(-) of VaR is amount of loss.
- 3) Random variables of Monte Carlo Simulation is 5,000.

where material of interest rate is spot rate computed through 'term structure' as from Jan 1992 to December 1997 due 5 year of sovereign bond. By the way, Method computed on price of bond and interest consists of yield of corporate bond (due 3 years, guarantee by banks) and remaining time horizon of one year. Results of <Table 3> consist of that of delta-gamma analysis and monte carlo simulation. It shows that result of monte carlo simulation is less than that of delta-gamma analysis on 95% confidence level. But, result of 99% is reversed. Therefore, result of which method is not dominated.

By the way, in 'financial solid regulation of securities', bonds in remaining time horizon of one year consist of more than tolerance value of risk.

#### It is why:

First, really adopted regulation needs that details of bond need more exactly than that of stock. Second, problem of sample will be in this tests. spot rate is computed through 'term structure' of sovereign bond, but price of bond and interest rate are computed through yield of corporate bond. And, sovereign bond has a little of issue and trade volume.

Third, because 'financial solid regulation of securities' is established on December 1995, it does not include interest rate volatility caused to economic situation after December 1995.

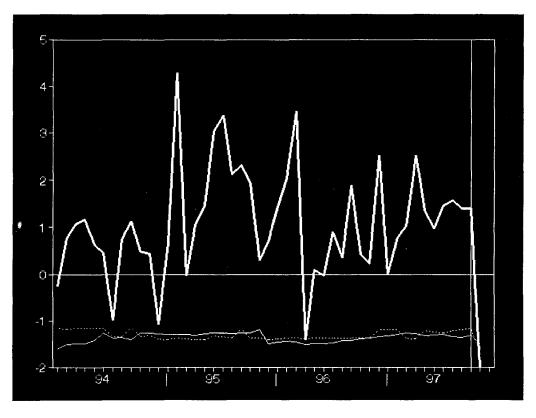
It means two fact at forecast on volatility of stock and interest rate portfolio. First, in Delta-gamma method and Monte Carlo Simulation, assumption of distribution affects Value at Risk. Second, Value at Risk depends on test method. And, if option price is included, test results will have difference between the two.

#### 4.3 Empirical tests on estimate methods

#### 4.3.1 Interest rate

Comparative results of each method use probability of error and confidence interval is 95%.<sup>3)</sup> The meaning for probability of error is followed. If VaR results are estimated through confidence interval of 95%, loss above VaR within 5 days among 100 business days is permitted.

If VaR estimated on probability of error is to keep in real situation, it will do precisive estimation of risk.



<Figure 1> comparative graphs on VaR and return of Interest rate
Note: Bold style is return of Interest rate and light line is VaR result of

<sup>3)</sup> VaR results are monthly pricing and yield of corporate bond from January 1992 to December 1997. VaR results on January 1997 are related with from January 1992 to December 1996(One-step ahead forecast).

delta-gamma method, dotted line is VaR result of monte-carlo simulation

<Figure 1> shows loss and profit of real portfolio on 'estimated VaR' from January 1996 to December 1997. In this horizon, yield of corporate bond is volatile according to fund demand and supply. In this results, each method is not different and it shows low level on probability of error, excepting November 1997 related with financial crisis.

Aim of risk management is precisive measurement and control about risk. Therefore, this results are useful on financial state of korea. If option is inculded to portfolio, monte carlo simulation is likely to exceeded more than delta-gamma method. But, because option market is not opened in korea, this result is not ascertained.

# 5. Summary and Conclusions

The recent trend is that risk management has more and more its importance. Neverthless, Korea's risk management is not developed. Even most banks does gap, duration in ALM for risk management, development and operation of VaR stressed at BIS have elementary level.

In the case of Fallon and Pritsker, Marshall, gamma model is superior to delta model and Monte Carlo Simulation is improved at its result, as sample number is increased. And, nonparametric model is superior to parametric model.

In the case of Korea's stock portfolio, VaR of Monte Carlo Simulation and Full Variance Covariance Model is less than that of Diagonal Model. The reason is that VaR of Full Variance Covariance Model is more precise than that of Diagonal Model.

By the way, in the case of interest rate, result of monte carlo simulation is less than that of delta-gamma analysis on 95% confidence level. But, result of 99% is reversed. Therefore, result of which method is not dominated.

It means two fact at forecast on volatility of stock and interest rate portfolio. First, in Delta-gamma method and Monte Carlo Simulation, assumption of distribution affects Value at Risk. Second, Value at Risk depends on test method. And, if option price is included, test results will have difference between the two.

Therefore, If interest rate futures and option market is open, Korea's findings is supposed to like results of other advanced countries. And, every banks try to develop its internal model.

#### References

- [1] Carol Alexander, The Handbook of Risk Management and Analysis, John Wiley & Dickinsons, ISBN.
- [2] Basle Committee on Banking Supervision(1994), Risk Management Guidelines for Derivatives.
- [3] \_\_\_\_\_\_(1996a), Supplement to The Capital Accord to Incorporate Market Risks.
- [4] \_\_\_\_\_\_\_(1996b), Supervisory Framework for The Use of "Backtesting" in Conjunction with The Internal Models Approach to Market Risk Capital Requirements.
- [5] Cedomir Crnkovic and Jordan Drachman(1996), Quality Control, Risk, September 1996, pp139-143
- [6] Christopher Marshall(1996), "Value at Risk: Implementing a Risk Measurement Standard," Financial Institutions Center, The Wharton School, University of Pennsylvania.
- [7] Darryll Hendricks(1996), Evaluation of Value-at-Risk Models Using Historical Data, FRB NY Economics Policy Review, April 1996, pp39-69
- [8] Hendricks, Darryll(1996), "Evaluation of value-at-risk models using historical data," *Economic Policy Review*, Federal Reserve Bank of New York, Vol. 2, April 1996, pp.39-69
- [9] Jackson, Patricia, David J. Maude & William Perrudin(1997), "Bank Capital and Value-at-Risk," *Journal of Derivatives*.
- [10] Kenneth Leong(1996), The Right approach, Value at Risk: A Risk Special Supplement, June 1996, pp9-14.
- [11] Matthew Pritsker(1996), "Evaluating Value at Risk Methodologies: Accuracy versus Computational Time," Financial Institutions Center, The Wharton School, University of Pennsylvania.
- [12] Paul H. Kupiec(1995), Techniques for Verifying the Accuracy of Risk Measurement Models, The Journal of Derivatives, Winter 1995, pp73-84.
- [13] Phillippe Jorion(1995), Predicting Volatility in the Foreign Exchange Market, Journal of Finance vol.50 no.2, 1995, pp507-528.
- [14] \_\_\_\_\_(1997), "Value at Risk: The New Benchmark for Controlling Market Risk," Irwin, Chicago.
- [15] William Fallon(1996), "Calculating Value-at-Risk," Financial Institutions Center, The Wharton School, University of Pennsylvania.
- [16] JP Morgan(1996), "Value-at-Risk software for use with RiskMetrics<sup>TM</sup>," FourFifteen<sup>TM</sup>.