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The nonlinear interaction between two resonant waves in a waveguide free-electron laser

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Abstract – We extend numerical analysis investigating the waveguide parameter dependency of the two resonant frequencies at the small-signal gain regime in a waveguide free-electron laser to the case that there exists a nonlinear coupling. The properties of the nonlinear interaction between the two resonant waves, one with higher frequency and positive slippage and the another one with lower frequency and negative slippage, are numerically investigated in the high gain regime. The results of numerical work with a set of partial differential equations describing the space and time interaction of the two resonant waves are analyzed.

I. Introduction

In the last few years, a lot of experimental [1-4] and theoretical [5, 6] literatures have been reported in which they showed a waveguide with narrowly spaced parrallel plates could controll the slippage between the radiation and the electron beam [7-10]. As in a free space free-electron laser (FEL) [11], it is known that there exist two resonant frequencies, depending on positive and negative slippage, in the waveguide in a FEL [5, 10]. When a waveguide is used, however, they are quite different and associated with counter-propagating waves: The dispersion causes the two resonant frequencies to move toward each other when the distance between the parallel plates of the waveguide is reduced.

The waveguide operation in the low gain regime has been studied by many authors [5, 6]. Either using a perturbation theory [5] or from the spontaneous emission spectrum applying the Madey's theorem [6], the gain in the low gain regime has already been studied extensively.

A numerical work done by our group [12] showed that, in the case that there does not exist a coupling between two resonant frequencies, the magnitude and the frequency strongly depend on the waveguide parameter.

In this article, we extend our numerical work to the case that there exists a nonlinear coupling between the two resonant frequencies in the high gain regime. We study the emission at the upper frequency induced by the amplification of a signal at the lower frequency.

A set of partial differential equations describing the space and time interaction of the two resonant freqeuicies is numerically analyzed, taking into account their positive and negative slippage.

The numerical results show that the emitted field amplitude at the higher frequency is produced by giving a signal at the lower frequency. On the other hand, the amplitude at the lower frequency is not affected by injecting a signal at the higher frequency.

The remainder of this article is arranged as follows. In section II we intoduce nonlinear two wave model, and the numerical results are analyzed in section III. Section IV contains our summary.

II. The Nonlinear Coupled Wave Equations

It is interesting to investigate the nonlinear coupling, if existed, between the two resonant waves. When the interference effects between the two waves can be ignored, that is, $\alpha - 1 >> \sqrt{\rho_0}$, the set of partial differential equation discribing the space and time can be written as [13] in the Compton limit $|\gamma_j - \gamma_0| << \gamma_0$,

$$\frac{\partial^2 \theta_j}{\partial \eta^2} = -(fA_1 e^{i\alpha\theta_j} + A_2 e^{i\theta_j} + c.c.) \tag{1}$$

$$\left[\frac{\partial}{\partial \eta} + s_1 \frac{\partial}{\partial \tau}\right] A_1 = f\langle e^{-i\alpha\theta_j}\rangle \tag{2}$$

$$\left[\frac{\partial}{\partial \eta} + s_2 \frac{\partial}{\partial \tau}\right] A_2 = \langle e^{-i\theta_j}\rangle \tag{3}$$

where the index $j = 1, \cdots, N$ represents the electrons and $\langle(\cdots)\rangle = N^{-1}\sum_{j=1}^{N}(\cdots)$. The frequency ratio parameter is defined as $\alpha = \omega_1/\omega_2$, with ω_1 upper wave frequency and ω_2 lower wave frequency. The ratio of Bessel factor f is defined as $f = F(\xi_1)/F(\xi_2)$, with $\xi_1 = \alpha\xi_2 = \alpha\xi_0 (1+\alpha)$ and $\xi_0 = a_w^2/2(1+a_w^2)$. The wiggler parameter a_w is given as; $a_w = eB_w/\sqrt{2}\,mc^2 k_w$, with B_w, average wiggler field, k_w, wiggler wave vector. The Bessel factor is defined as $F(\xi) = [J_0(\xi) - J_1(\xi)]$.

The phase of amplitude is for the low frequency wave. The electric field of the TE_{01} mode in the waveguide is to be the sum of two resonant frequency components;

$$E = \hat{x}\sin(k_\perp y)\left[\sum_{m=1}^{2} E_{01}^{(m)}\exp[i(k_m z - \omega_m t)] + c.c.\right],$$

assuming $E_{01}^{(m)}(z,t)$ $(m = 1, 2)$ a slowly varying function of its arguments and $\theta = k_w z + k_2 z - \omega_2 t$, the combination of the lower resonant frequency mode and the wiggler phases. Two dimensionless variables discribing the space and time are introdueced: $\eta = 2\bar{k}_{w2}\rho_2 z = z/l_{g2}$, the scaled position along the wiggler with $\bar{k}_{w2} = k_w + k_2 - \omega_2/c = k_w/(1+\alpha)$, and the scaled time $\tau = 2k_2\rho_2(z - v_\parallel t)$ with the average electron velocity $v_\parallel$ [14]. Also the dimensionless wave amplitude is scaled as $A_m = E_{01}^{(m)}/(4\pi mc^2\gamma_r n_e \rho_2)^{1/2}$, and the slippage coefficients are $s_1 = (\alpha-1)/\alpha = -s_2/\alpha$.

III. Numerical Analysis of the Two Wave Model

Eqs.(1)-(3) are discretized via the finite difference method, and yield the following set of formulae:

$$\frac{\theta_{jn+1} - \theta_{jn}}{\Delta\eta} = p_n \qquad (4)$$

$$\frac{p_{jn+1} - p_{jn}}{\Delta\eta} = -(fA_{1n,m}e^{i\alpha\theta_{jn,m}} + A_{2n,m}e^{i\theta_{jn,m}} + c.c.) \qquad (5)$$

$$\frac{A_{1n,m} - A_{1n-1,m}}{\Delta\eta} + s_1\frac{A_{1n,m} - A_{1n,m-1}}{\Delta\tau} = fb_{1n,m} \qquad (6)$$

$$\frac{A_{2n,m} - A_{2n-1,m}}{\Delta\eta} + s_2\frac{A_{2n,m} - A_{2n,m-1}}{\Delta\tau} = b_{2n,m} \qquad (7)$$

where bunching parameters are defined by $b_1 = \langle\exp(-i\alpha\theta_j)\rangle$ rangle and $b_2 = \langle\exp(-i\theta_j)\rangle$ rangle. Using

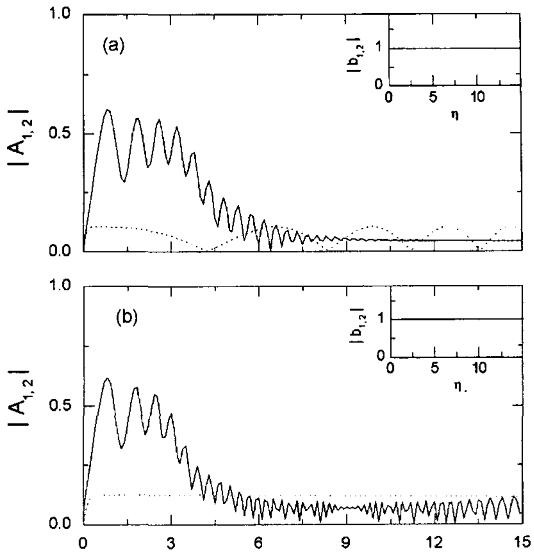

Fig. 1. Emitted field amplitude and bunching at the lower (dot line) and upper (continuous line) resonant frequencies versus η, for a single particle, (a) when the signal at the lower frequency is injected with $A_{20} = 0.05$ $(A_{10} = 0)$, (b) the signal at the higher frequency is injected with $A_{10} = 0.05$ $(A_{20} = 0)$; other parameter are: $\alpha = 10$, $\tau = 2.0$, $\bar{l}_b = 4$, $a_w = 1$ and $f = 0.884$ (Both bunchings versus η is shown in the inset.).

Eqs. (4)-(7), we integrated the set of partial differential equations with boundary conditions on a retangular profile with length $\bar{l}_b$ for the fields on the tailing edge $\tau = 0$, $A_1(\eta, 0) = A_{10}$, and on the leading edge $\tau = \bar{l}_b = 2k_2\rho_2 l_b$, $A_2(\eta, l_b) = A_{20}$. Futhermore, we assumed that $A_1(0,\tau) = A_{10}$, $A_2(0,\tau) = A_{20}$, $p(0,\tau) = (\partial\theta_j/\partial\eta)(0,\tau) = 0$. We choose the number N of electrons within an optical wavelength (realistically sim ~10^5) as $N = 100$. We have checked for more electron numbers N up to 2000, but the output had negligible effects. The initial conditions of the electron phases were $\theta_j(0,\tau) = 2\pi j/N$ for $j = 1, 2, \cdots, N$.

The emitted field amplitude and bunching at the lower and upper resonant frequencies versus η for a single particle are shown in Fig. 1. We can see, as soon as an electron enter the wiggler, the upper frequency fields increases rapidly and then makes several spikes, while the lower field remains almost unchanged in both cases. Both bunching parameters are, as we expected, not changed along the wiggler length in this single electron case. As shown in Fig.

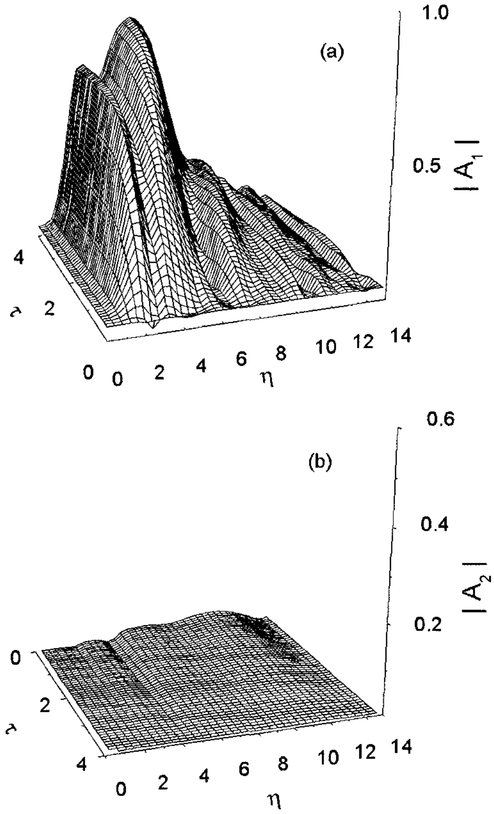

Fig. 2. Emitted field amplitude at the upper (a) and lower (b) resonant frequencies when a signal at the upper frequency is injected with $A_{10} = 0.05$ ($A_{20} = 0$) for $\alpha = 10$, $N = 100$; other parameter are: $\bar{l}_b = 4$, $a_w = 1$ and $f = 0.884$.

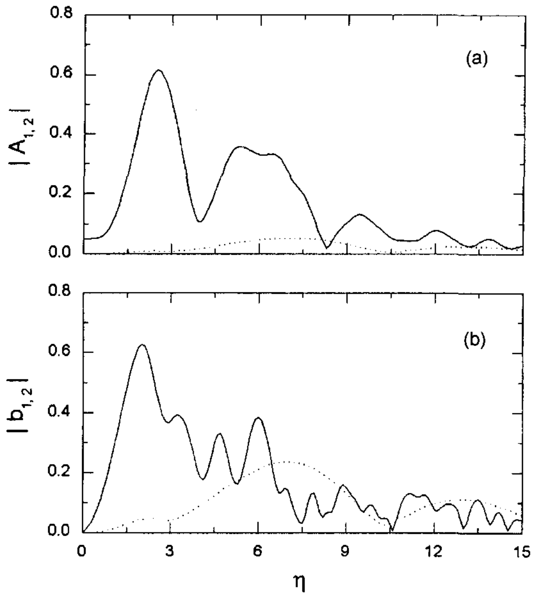

Fig. 3. Cross section of emitted field amplitude (a) and bunching (b) at the lower (dot line) and upper (continuous line) resonant frequencies versus η at $\tau = 2.0$ in Fig. 2.

2 and 3, if a wave at the upper frequency is injected into the wiggler, no appreciable deviations from the customary results from the one-wave model [8]. A_1 exhibits the usual behavior, while A_2 and the bunching parameter b_2 of the lower frequency remain almost zero. The opposite situation is much more interesting as shown in Fig. 4 and 5. When a low frequency wave is injected into the wiggler, an intense signal and strong bunching grow on the upper frequency ω_1. Comparing with the one electron case (Fig. 1) the upper frequency wave gains signal from about η~1. The physical properties of this emitted field and bunching depend on the parameter α as well as f as shown in Figs. 6 and 7. In Figs. 6 and 7, dot line in (a) is numerical fit, which decrease rather slowly with a scaling law approximately given by $|A_1|_{max} = 1.45\ \alpha^{-0.52}$, and $|b_1|_{max} = 1.0\ \alpha^{-0.37}$. As shown in the

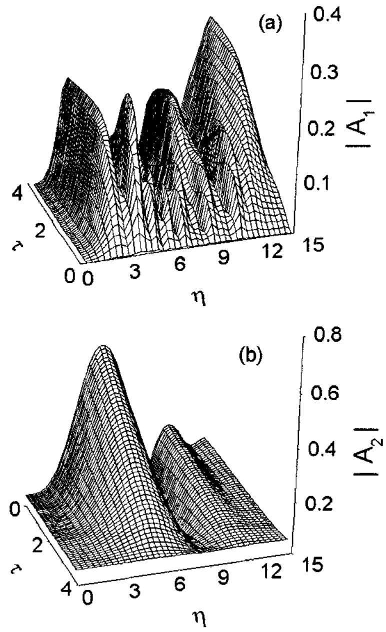

Fig. 4. Emitted field amplitude at the upper (a) and lower (b) resonant frequencies when a signal at the lower frequency is injected with $A_{20} = 0.05$ ($A_{10} = 0$) for $\alpha = 10$, $N = 100$; other parameter are: $\bar{l}_b = 4$, $a_w = 1$ and $f = 0.884$.

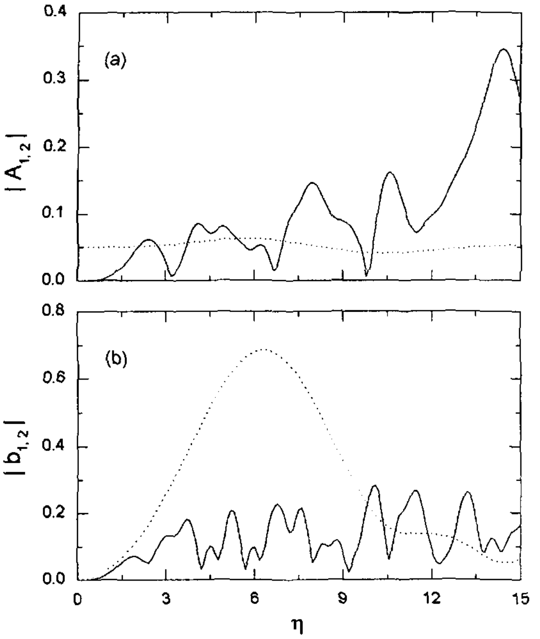

Fig. 5. Cross section of emitted field amplitude (a) and bunching (b) at the lower (dot line) and upper (continuous line) resonant frequencies versus η at $\tau = 2.0$ in Fig. 4.

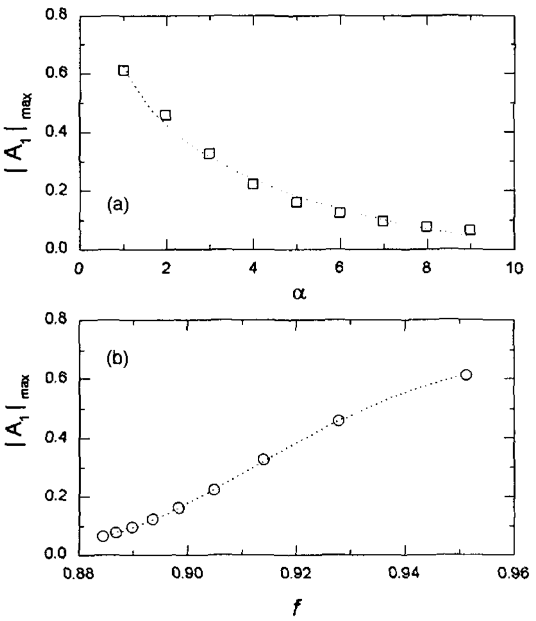

Fig. 6. Dependency on the frequency ratio α (a) and the Bessel factor ratio f (b) of the maximum of $|A_1|$ at $\tau = 2$, $A_{10} = 0$, $A_{20} = 0.05$, $\bar{l}_b = 4$, $N = 100$, $a_w = 1$ and $f = 0.884$. Dot line in (a) is numerical fit, $|A_1|_{max} = 1.45\ \alpha^{-0.52}$. (Dot line in (b) is for the eyeguide.).

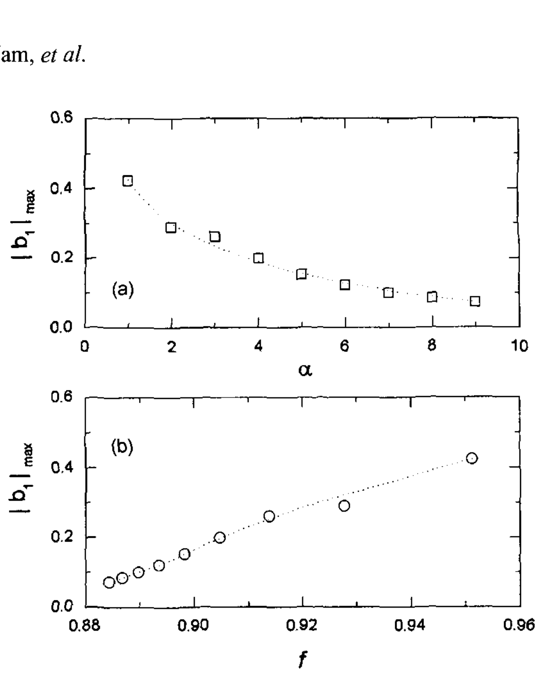

Fig. 7. Dependency on the frequency ratio α (a) and the Bessel factor ratio f (b) of the maximum of $|b_1|$ at $\tau = 2$, $A_{10} = 0$, $A_{20} = 0.05$, $\bar{l}_b = 4$, $N = 100$, $a_w = 1$ and $f = 0.884$. Dot line in (a) is numerical fit, $|b_1|_{max} = 1.0\alpha^{-0.37}$. (The dot line in (b) is for the eyeguide.)

Fig. 4., the first maximum of $|A_1|$ reaches a value of 0.3 through out the time τ. We believe that it is possible to generate a strong bunching and a signal more intense than the injected signal.

The fact that the frequency ratio α (f) increases (decreases) means that the distance between two resonant frequencies increases, and then the nonlinear interaction is getting weaker. The dependency of the first maximum of the emitted field and bunching on α (or f) are shown in Fig. 7.

IV. Conclusion

We studied the nonlinear interaction between the two resonant waves, one with higher frequency and positive slippage and the other one with lower frequency and negative slippage. A set of partial differential equations for the two wave model discribing the space and time was numerically studied. Using the numerical work, we were able to generate a strong bunching and a signal approximately six times of magnitude more intense than the injected signal at the end of wiggler. In the single electron case, the upper frequency field increased rapidly when an electron enter the wiggler and both bunching parameters were remain unchanged. On the other hand, when the number of electron N is 100, injecting low frequency into the wiggler made an intense signal and strong bunching on the upper frequency. The frequency ratio for the amplitude and bunching decreased rather slowly with a scaling law approximately given by $|A_1|_{max} = 1.45\alpha^{-0.52}$, and $|b_1|_{max} = 1.0\alpha^{-0.37}$. We also showed that the nonlinear interaction was getting weaker as the frequency ratio increased.

Acknowledgments

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