Structural Damage Assessment Using Transient Dynamic Response

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요  지

강재진동을 가진 구조물의 재진단 위치에서 측정한 가속도를 사용하여 손상을 확인하고 평가하는 알고리즘을 개발하였다. 개발된 알고리즘에서는 선형적 구속-비선택 특성화에 의해 최적의 구조변수를 구하여 구조물의 안정성 시간영역-시스템 토익기법을 사용하였다. 동적응동방정식의 모음을 최소화하도록 최적의 변수를 추정하였으며, 재진단 위치에서 측정한 가속도 자료를 이용하여 손상된 부재를 찾기 위하여 적합한 변수모음법을 적용하였다. 손상은 측정된 가속도의 시간이익에 시간강의 개별을 적용하여 통계적으로 평가하였다. 가속도가 측정된 자료로부터의 변위와 속도는 측정된 가속도를 직분하여 계산하였으며, 미지속 자료에서는 변위를 추가의 미지변수로 추정하고, 속도와 가속도는 추정된 변위의 차분에 의해 수치적으로 계산하였다. 개발된 알고리즘의 효율성을 검증하기 위하여 트러스에 대한 속도모의 실험을 실시하였다. 손상자치의 한계를 정하고 각 부분의 손상가능도를 계산하기 위하여 자료교란법을 적용하였다.

핵심요인 : 손상평가, 시간영역 SL, 강재진동, 동적응동방정식, 최적화

Abstract

A damage detection and assessment algorithm is developed by measuring accelerations at limited locations of a structure under forced vibrations. The developed algorithm applies a time domain system identification (SI) method that identifies a structure by solving a linearly constrained nonlinear optimization problem for optimal structural parameters. An equation error of the dynamic equilibrium of motion is minimized to estimate optimal parameters. An adaptive parameter grouping scheme is applied to localize damaged members with sparse measured accelerations. Damage is assessed in a statistical manner by applying a time-windowing technique to the measured time history of acceleration. Displacements and velocities at the measured degrees of freedom (DOF) are computed by integrating the measured accelerations. The displacements at the unmeasured DOF are estimated as additional unknowns to the known structural parameters, and the corresponding velocities and accelerations are computed by a numerical differentiation. A numerical simulation study with a truss structure is carried out to examine the efficiency of the algorithm. A data perturbation scheme is applied to determine the thresholds for damage indices and to compute the damage possibility of each member.

Keywords : damage assessment, time-domain SI, forced vibration, transient dynamic response, optimization

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1. Introduction

System identification (SI) methods have been widely applied in various structural engineering fields to verify structural models or to detect damage in structural systems. A SI method requires sets of measured response and a parametric model for the structure. Most of the available SI methods using dynamic response have applied frequency-domain algorithms (Hjelmstad and Shin 1996; Natke 1989). Regardless of the huge amount of time-domain raw data, after the data is transformed into a frequency-domain data by a technique such as Fast Fourier Transform, the algorithm can deal with only the identified modal data of natural frequencies and mode shapes. Since most of the vibration energy is absorbed by limited low modes, only limited modal information can be usually generated in the frequency domain. It is hard to activate and measure higher modes from usual vibration on civil structures. Since local damage in a structure influences more on higher modes, however, such modal information with some of low modes may not be sufficient to assess damage depending on its location and severity (Raghavendrachar and Aktan 1992).

The applications of time-domain SI methods for structural engineering problems have been relatively limited so far by using space state models (Imai et al. 1989; Shinozuka et al. 1982) or by solving a constrained nonlinear optimization problem (Hjelmstad et al. 1995). A SI method using a space state model was also applied to identify nonlinear structural systems (Yun and Shinozuka 1980). However, we can observe some drawbacks in the methods using space state models regardless of their limited success in the simulated applications. The algorithms can be applied when the state vectors of acceleration, velocity, and displacement are completely obtained at all the degrees of freedom (DOF) in the analytical model, which is hard to be satisfied in reality. The dimension of matrices becomes twice that of system matrices due to the reduction of the second order differential system of equations to the first order one. One strong point in the approach using space state models is the well-established statistical schemes to estimate the system parameters. On the other hand, the time-domain SI method solving an optimization problem can be more reasonably applied for a complex structure. The method allows unmeasured DOF in measurements and has strong theoretical backgrounds on optimization techniques. However, the method has still some drawbacks of mathematical difficulties and computational burden in dealing with huge amount of noisy and sparse state vectors.

In the present paper, a damage assessment algorithm is proposed by modifying the time-domain SI method of Hjelmstad et al. (1995). The algorithm uses a parametric finite element model with system parameters of mass, damping, and stiffness properties, and applies an optimization technique by minimizing the error in the dynamic governing equation at each time interval rather than applying a state space model. It is assumed that histories of acceleration are measured at limited locations under forced excitations. To localize damaged members in a structure, an adaptive parameter grouping technique is applied (Shin 1994).

To evaluate damage in a statistical manner with a consideration on noise in measurements, data perturbations in conjunction with a time-windowing scheme are applied (Baner and Hjelmstad 1993; Shin 1994). A numerical simulation study has been carried with a truss structure to demonstrate the efficiency of the proposed damage assessment algorithm.
2. Damage Assessment with Transient Dynamic Response

2.1 SI Algorithm

The structural vibration under forced excitation can be described by the following equilibrium equation at time step \( t_k \).

\[
M(x_M) \ddot{u}(t_k) + C(x_C) \dot{u}(t_k) + K(x_K) u(t_k) = f(t_k)
\]

(1)

where \( x_M, x_C, x_K \) = the mass, damping, and stiffness parameters of the structural matrices, respectively. In Eq. (1), it is assumed that the structural parameters do not vary during the forced vibration and the excitation \( f \) is a known equivalent nodal force vector at every time step. The structural matrices can be decomposed as a linear combination of parameters and kernel matrices by Eq. (2).

\[
M(x_M) = \sum_{m=1}^{N_M} \sum_{l=1}^{m} x_M_{ml} G_{ml}
\]

\[
C(x_C) = \sum_{l=1}^{N_C} \sum_{l=1}^{l} x_C_{ll} G_{C l}
\]

\[
K(x_K) = \sum_{l=1}^{N_C} \sum_{l=1}^{l} \sum_{l=1}^{l} x_K_{ll} G_{K l}
\]

(2)

where \( N_M, N_C, N_K \) = the number of groups for mass, damping, and stiffness parameters, \( \Omega_M, \Omega_C, \Omega_K \) = population of elements in the corresponding mass, damping, and stiffness group, \( \Pi_{(\nu)} \) = number of different stiffness parameter types in the element, and \( G_{ml}, G_{C l}, G_{K l} \) = mass, damping, and stiffness kernel matrices, respectively.

In the current time-domain SI method, it is assumed that the time history of acceleration is measured at limited DOF and the initial conditions are known. The unmeasured state vectors of velocity and displacement at the measured DOF are computed by numerical integration. However, all the state vectors at the unmeasured DOF cannot be obtained nor condensed out so that they have to be considered as additional unknowns to the structural parameters. A possible reduction in the number of unknowns can be achieved by expressing the acceleration and velocity with displacements of consecutive time intervals using a finite difference scheme. Therefore, the error vector at time step \( k \) for estimating the structural parameters and the displacements at the unmeasured DOF can be defined by

\[
e_k(x, \tilde{U}_k) = \tilde{M}(x) \ddot{\tilde{u}}_k + \tilde{C}(x) \dot{\tilde{u}}_k
\]

\[
+ \tilde{K}(x) \tilde{u}_k - g_k + \sum_{i=1}^{N_M} Z_i(x) \tilde{u}_{i,k}
\]

(3)

where \( \tilde{M}(x), \tilde{C}(x), \tilde{K}(x) \) = part of mass, damping, and stiffness matrix corresponding to the measured DOF, \( \tilde{U}_{k(N)} = [\tilde{u}_{k,1}, \tilde{u}_k, \tilde{u}_{k+1}] \) = a matrix containing unknown displacement vectors at the consecutive time steps, \( \tilde{u}_{k,(\nu)} \), \( \tilde{\tilde{u}}_{k,(\nu)} \) = measured acceleration and computed velocity and displacement at the measured DOF, \( g_{(\nu)} \) = consistent force vector at time step \( t_k \) for Newmark method defined by Eq. (4), \( Z_{i,(\nu)} \) = coefficient matrix for the Newmark method defined by Eq. (5), and \( N_M \) = number of DOF, \( N_d \) = number of measured and unmeasured DOF, respectively.

\[
g_k = \left( \frac{1}{\Delta t^2} - \beta - \gamma \right) \Gamma_{k} + \left( \frac{1}{\Delta t} - 2 \beta + \gamma \right) \Gamma_{k} + \beta \Gamma_{k-1}
\]

(4)

\[
Z_i(x) = \frac{1}{\Delta t^2} \tilde{M}(x) - \frac{1}{\Delta t} (\gamma - 1) \tilde{C}(x) + (\frac{1}{\Delta t} + 2 \beta - \gamma) \tilde{K}(x)
\]

\[
Z_0(x) = -\frac{2}{\Delta t^2} \tilde{M}(x) - \frac{1}{\Delta t} (2 \gamma - 1) \tilde{C}(x)
\]

\[
+ (\frac{1}{\Delta t} - 2 \beta + \gamma) \tilde{K}(x)
\]

\[
Z_i(x) = \frac{1}{\Delta t^2} \tilde{M}(x) - \frac{1}{\Delta t} \tilde{C}(x) + \beta \tilde{K}(x)
\]

(5)
where $\mathbf{M}(\mathbf{x})_{(N_x N_y)}$, $\mathbf{C}(\mathbf{x})_{(N_x N_y)}$, $\mathbf{K}(\mathbf{x})_{(N_y N_y)}$ are part of mass, damping, and stiffness matrix corresponding to the unmeasured DOF, respectively, and $\beta$, $Y$ are Newmark coefficients.

With the consideration on unmeasured state vectors as additional unknowns at each time step, we can solve the following nonlinear constrained optimization problem of the following equation to estimate optimal structural parameters.

\[
\begin{align*}
\text{Minimize} & \quad J(x, \mathbf{U}) = \frac{1}{2} \sum_{k=m}^{n} \sum_{i=1}^{n_p} w_i \|e_i(x, \mathbf{U}_k)\|^2 \\
\text{subject to} & \quad x \leq x \leq \bar{x}
\end{align*}
\]  

(6)

where $\mathbf{U}_{(N_x N_y)}^{(m)} = [\mathbf{U}_1, \mathbf{U}_2, \ldots, \mathbf{U}_n]$ = population of unmeasured displacement vectors between time intervals of $[t_k, t_{k+1}]$, $n_p = N_t - n + 1$ number of time points for the current estimation, $w_i$ = weighting factor for the measured data at time step $k$, and $x$, $\bar{x}$ = lower and upper bounds for the parameters.

To solve the linearly constrained nonlinear optimization problem of Eq.(6), the Fletcher–active set strategy combined with recursive quadratic programming technique is applied (Ranjan and Hjelmstad 1993; Lueneberger 1989). Gradient vector and Gauss–Newton hessian matrix of the objective function can be computed by Eq.(7).

\[
\begin{align*}
\nabla J(x, \mathbf{U}) &= \sum_{i=1}^{n_p} w_i \nabla e_i(x, \mathbf{U}) \\
H_{x}^{g}(x, \mathbf{U}) &= \sum_{i=1}^{n_p} w_i \nabla e_i(x, \mathbf{U}) \nabla e_i(x, \mathbf{U})
\end{align*}
\]  

(7)

where $\nabla e_i(x, \mathbf{U})_{(N_y)}$ = the sensitivity matrix of the error vector with respect to the structural parameter $x_{(N_y)}$ and the unmeasured displacements $\mathbf{U}_{(N_y)}^{(m)}$ and $N_y$ = the number of unknown structural parameters of mass, damping, and stiffness. If the Newton method is applied to compute the direction vector using the gradient vector and hessian matrix of Eq.(7), the computation is basically a least squared problem so that a solution can be obtained when the following identifiability criterion is satisfied to make the system of equations squared or over-determined.

\[
q x \hat{N}_a \geq N_p
\]  

(8)

Because the identifiability criterion of Eq.(8) can be easily satisfied from usual time-domain data, theoretically the identification problem is almost free from the requirement for the quantity of measurements.

2.2 Damage Assessment

When measured responses are sparse in space with a large number of DOF in the analytical model, it is almost impossible to estimate every structural parameter accurately even if the identifiability criterion of Eq.(8) relaxes the requirement for the quantity of measurements.

To reduce the number of unknown parameters for estimation, a parameter grouping scheme can be utilized (Hjelmstad et al. 1990). The method collects several members in a group whose sectional or material properties are in common. A scheme of collecting members with different properties in a group was also proposed by using scale factors (Koh and Shin 1996).

To detect damage in a complex structure, it is required to develop a scheme of successfully locating actual damage with a defined finite element model for the structure. An adaptive parameter grouping scheme can be applied for that purpose. Starting from the known baseline parameter grouping case, parameter groups are subdivided sequentially until all the damage can be localized. During the parameter groups
are updated, the finite element model need not be modified and only the structural parameter values can vary. Some different ideas of sequential subdivision of parameter groups are available in the literature (Shin and Koh 1999; Natke and Cempel 1991). In the current paper, we have applied a simple binary subdivision technique of dividing a group with the most distant estimated parameter from the baseline value into two groups.

After localizing damage, theoretically we can determine damaged members in a structure by checking the reduction in the estimated parameter from the baseline value. However, since measurements always contain noise, the parameters should be evaluated with the consideration on noise. Since noise is random in nature, we need to evaluate parameters statistically (Crandall and Mark 1963).

Since accelerations are usually measured at a number of time steps, a small portion of the measured data can easily satisfy the identifiability criterion of Eq. (8). This small portion of measured time history is called a time-window (Hjelmstad et al. 1995). For each time-window of measured response, the adaptive parameter grouping scheme is applied and damaged members are localized. Because many time-windows can be selected from the full time history of measured data, mean and standard deviation of each structural parameter can be computed from a series of estimated values for a parameter. To yield statistically meaningful estimation of parameters, the number of collected time-windows should be large enough. From the computed mean and standard deviation, the following damage index DI and damage severity DS can be defined for each structural parameter.

$$DI_{pe} = \frac{|\bar{x}_{pe} - x^*|}{\sigma_{pe}} \times 100(\%) , \quad DS_{pe} = \frac{|\bar{x}_{pe} - x^*|}{\sigma_{pe}}$$  (9)

where $\bar{x}_{pe}, \sigma_{pe}$ are mean and standard deviation, $x^*_p$ is the baseline value of the $p$th parameter type of element($e$), respectively.

3. Numerical Simulation Study

Numerical simulation studies have been carried out with a Watchet truss to demonstrate the efficacy of the developed damage assessment algorithm using a time-domain SI method. The truss structure is composed of 32 elements with 32 degrees of freedom as shown in Fig. 1. The baseline structure is composed of four different sectional properties as summarized in Table 1. For the simulation studies, it is assumed that accelerations are measured at node 5, 8, 11, and node 15 to the vertical and horizontal directions and at node 1 and node 18 to the horizontal direction only. A multiple-damage

![Fig. 1 Geometry and topology of a Watchet truss](image)

<table>
<thead>
<tr>
<th>Table 1 Member properties</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Table" /></td>
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</tbody>
</table>

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case is simulated with 20% reduction in the sectional area of member (13) and 30% reduction in member (31).

3.1 Baseline Study

Table 2 summarizes the computed participation mass ratio of each mode from the baseline structure (SAP2000, 1996). From Table 2 and Fig. 2, we can observe that the first mode is a horizontal mode but the second and third modes are vertical ones. Table 2 also illustrates that most of the vibration energy of the structure can be absorbed by the lowest five modes.

Table 3 compares the computed natural frequencies of the damaged and undamaged baseline structures up to mode 5, respectively. From Table 3, we can observe that it is almost impossible to determine any local damage from such negligible change in natural frequencies.

The change of mode shapes is compared with two indices of MAC (modal assurance criterion) and CoMAC (Coordinate MAC) defined by the following equations.

\[
\text{MAC}(\mathbf{u}_n, \mathbf{v}_n) = \frac{(\mathbf{u}_n \cdot \mathbf{v}_n)^2}{\|\mathbf{u}_n\|^2 \|\mathbf{v}_n\|^2}
\]

\[
\text{CoMAC}(i) = \frac{\sum_{n=1}^{nmm} \sum_{n=1}^{nmm} u_{ni}^2 v_{ni}^2}{\sum_{n=1}^{nmm} u_{ni}^2 \sum_{n=1}^{nmm} v_{ni}^2}
\]

where \(nmm\) = number of measured modes, \(\mathbf{u}, \mathbf{v}\) = modal vectors from baseline and damaged structures. When each value approaches 1.0, the two vectors can be considered as almost identical. The computed MAC values are summarized in Table 3 for each mode, and CoMAC values are drawn for each DOF with five low modes in Fig. 3. From the figure and table, as the case of natural frequencies, we can observe that it is almost impossible to detect damage from such negligible changes of mode shapes.

<table>
<thead>
<tr>
<th>Mode</th>
<th>x-direction (%</th>
<th>x-sum %</th>
<th>y-direction (%)</th>
<th>y-sum %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60.25</td>
<td>60.25</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>60.25</td>
<td>36.57</td>
<td>36.57</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>60.25</td>
<td>48.51</td>
<td>85.08</td>
</tr>
<tr>
<td>4</td>
<td>33.08</td>
<td>93.33</td>
<td>0</td>
<td>85.08</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>93.33</td>
<td>5.03</td>
<td>90.11</td>
</tr>
</tbody>
</table>

Fig. 2 Comparison of mode shapes of damaged and baseline structures

<table>
<thead>
<tr>
<th>Mode</th>
<th>Natural frequency (Hz)</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baseline</td>
<td>Damaged</td>
</tr>
<tr>
<td>1</td>
<td>0.193</td>
<td>0.191</td>
</tr>
<tr>
<td>2</td>
<td>0.202</td>
<td>0.200</td>
</tr>
<tr>
<td>3</td>
<td>0.383</td>
<td>0.381</td>
</tr>
<tr>
<td>4</td>
<td>0.388</td>
<td>0.383</td>
</tr>
<tr>
<td>5</td>
<td>0.666</td>
<td>0.659</td>
</tr>
</tbody>
</table>

Table 3 Comparison of natural frequencies and computed MAC
3.2 Damage Case Study

To examine the proposed damage assessment algorithm, the measured accelerations are simulated for 10 seconds by adding randomly generated proportional error of maximum amplitude of 10% to the analytical solution. A frame model has been used to compute the analytical accelerations. For the identification, however, the structure is modeled with truss elements so that an axial stiffness parameter in each element is estimated. We assume that the damping ratio is 5% and the mass properties are the a priori knowledge in the current simulation study. Sinusoidal forces are applied to the vertical direction at node 5 or node 15 with a frequency close to the resonance to the first natural frequency of the baseline structure.

The time increment for the analysis is selected as \( dt = 0.01 \) sec to satisfy \( dt \leq \frac{2}{\omega_{\text{max}}} \), where \( \omega_{\text{max}} \) is the maximum circular frequency of the given structure. The same time increment is applied to the time step of the simulated measured accelerations. Thirty non-overlapping time-windows are selected from the time history of acceleration. For each time window, 20 data points were collected with a consistent jump because adjacent data points may contain almost identical information. Because structural parameters can be estimated from each time window, averaged parameter values and damage indices of DI and DS can be obtained from the chosen 30 non-overlapping time-windows.

To evaluate damage statistically and to determine the thresholds for the damage indices, we have applied a Monte Carlo simulation study by perturbing the measured acceleration data (Shin 1994). The estimated stiffness parameters and the computed damage indices averaged from 30 Monte Carlo trials are drawn in Fig. 4. From Fig. 4, the actually damaged member (11) and member (31) can be identified as damaged from relatively large damage indices.

However, the actually undamaged member (32) also seems to be identified as damaged.

Damage possibility of each member with respect to the current measuring and loading condition
is computed from 30 Monte Carlo trials of perturbations and drawn in Fig. 5(a) and Fig. 5(b) for the baseline structure and for the damaged structure, respectively. In the figures, the damage possibility is computed as the probability that both computed damaged indices are larger than their thresholds. The figures are drawn by varying the threshold for DS but with the fixed threshold for DI of the maximum amplitude of noise as suggested by Shin (1994). From Fig. 5(a), we can determine an optimal threshold for $DS = 1.8$ with the damage possibility of 3.3% for an actually undamaged member. By applying this value to Fig. 5(b) with the damaged structure, we can finally assess damage in member (11) and (32) statistically. From Fig. 5(b), we can observe that the defined threshold of $DS = 1.8$ is a reasonable choice for damage detection even though the actually undamaged member (32) can be identified as damaged with 60% possibility. The two actually damaged members can be identified as damaged with over 80% possibility.

The vertical components of the identified state vectors at node 15 are compared with the simulated measured acceleration and the numerically integrated velocity and displacement in Fig. 6. From the figure, it can be observed that the state vectors match well each other except some discrepancies in the velocity and displacement due to some numerical integration error.

4. Conclusions

A damage assessment algorithm using a time-domain SI method is proposed. The SI method solves a linearly constrained nonlinear optimization problem for optimal structural parameters. To localize damaged members, an adaptive parameter grouping scheme is applied. To assess damage with a consideration on noise in measurements, data perturbations in conjunction with a time-windowing scheme are applied to compute
damage indices.

The reliable efficiency of the proposed algorithm could be clearly demonstrated by comparing the results from the proposed identification method with those from conventional approaches. Simple comparisons of natural frequencies or mode shapes could not detect any damage in the actually damaged structure. Reduction in natural frequencies and modal indices were almost negligible for the simulated case study with a multiple damage. Through the simulation studies, the efficiency of the proposed damage assessment algorithm has been successfully examined. The algorithm could locate and assess actual damage successfully with sparse and noisy measured data.

A desirable aspect of the proposed algorithm is that the method can provide the information of damage possibility of each member with respect to the applied loading and measuring conditions. By comparing the figures of damage possibility for the baseline structure and that for the damaged structure, the thresholds for damage indices can be determined and the damage possibility of each member can be evaluated statistically.

Another desirable aspect of the applied time-domain SI method is the identifiability criterion of Eq.(8) that can be easily satisfied with time history of transient dynamic response. However, because the unmeasured displacements can dominate the unknowns as time step accumulates, the estimation of the structural parameters can be ruined by the estimation of the unmeasured displacements. Even though rigorous simulation studies have proved the reliable identification of the current algorithm using a time-domain SI method, the problem related to the unmeasured state vectors is still in hand for a future improvement. Another drawback of the algorithm is the computational inefficiency because the current algorithm requires a lot of iterations for each Monte Carlo trial. In spite of these difficulties, the study and application of an algorithm using a time-domain SI method for a structural system is promising with its desirable aspects illustrated in the paper.

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