Korean J. Ecol., 23(6): 435~438, 2000

Prediction of Stand Structure Dynamics for Unthinned Slash Pine Plantations

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ABSTRACT: Diameter distributions describe forest stand structure information. Prediction equations for percentiles of diameter distribution and parameter recovery procedures for the Weibull distribution function based on four percentile equations were applied to develop prediction system of even-aged slash pine stand structure development in terms of the number of stems per diameter class changes. Four percentiles of the cumulative diameter distribution were predicted as a function of stand characteristics. The predicted diameter distributions were tested against the observed diameter distributions using the Kolmogorov-Smirnov two sample test at the α=0.05 level. Statistically, no significant differences were detected based on the data from 236 evaluation data sets. This stand level diameter distribution prediction system will be useful in slash pine stand structure modeling and in updating forest inventories for the long-term forest management planning.

Key Words: Parameter recovery, Percentiles, Slash pine, Stand structure modeling, Weibull distribution

INTRODUCTION

The size-class distribution information is important because it affects the type and timing of management strategies for merchantability standards, and is needed as input elements such as construction of stand tables, estimation of the total or merchantable volumes and stand structure development for the forest management decisions. A variety of approaches providing the distribution of volume by size classes and stand structure development have been taken in the development of stand level growth models.

A number of methods have been proposed to model diameter distributions in forest stands. Many statistical distribution functions such as log-normal, exponential, Beta, Gamma, Weibull, the Johnson's SB, and Bivariate distribution have been used to describe diameter distributions in forest stands (Johnson 1949, Bliss and Reinker 1964, Lenhart 1968, Bailey and Dell 1973, Hafley and Schreuder 1977, Clutter *et al.* 1983, Knoebel and Burkhart 1991). However, most of the recent works have used the Weibull distribution to model diameter distributions since the early applications by Bailey and Dell (1973).

Weibull parameters were predicted by empirical functions of whole stand characteristics such as age, site index, and density (Smally and Bailey 1974). Subsequently, parameter recovery techniques replaced the parameter prediction approach (Bailey *et al.* 1981, Hyink and Moser 1983, Cao and Burkhart 1984, Borders *et al.* 1987, Lenhart 1988, Bailey *et al.* 1989). Several different methods for estimating the two- and three-parameters of the Weibull distributions such as the percentile, the maximum likelihood and the moment method were investigated by numerous authors (Dubey 1967, Zanakis 1979, Zarnoch *et al.* 1985, Border *et al.* 1987, Clutter *et al.* 1983). The parameter recovery technique employed in this work was first presented by Bailey *et al.* (1989). This parameter recovery procedure utilizes the expected value of the minimum observation from a sample size n from the Weibull distribution, four percentiles, and the second moment of the Weibull distribution to estimate the a, b, and c parameters.

The objective of this study was to develop dynamics of even-aged slash pine stand structure development prediction system using the long-term repeated measurement data sets in the south-western United States and an illustration is given for the practical computations for size-class distribution model.

MATERIALS AND METHODS

Data

The study area consists of 22 counties in East Texas, USA. Generally, the counties are located within the rectangle from 30°~35° north latitude and 93°~96° west longitude.

The East Texas Pine Plantation Research Proj-

Table 1. Summary statistics for unthinned slash pine stand data sets

Variables	Mean	Std Dev.	Min.	Max.
AGE	11	5.1	1	26
TPH	991	425	193	2,550
HT	10.8	5.4	0.6	33.8
D_0	4.3	3.6	0	16.8
D_{25}	9.9	5.3	0	22.6
D_{50}	11.9	6.1	0	27.9
D_{95}	16.3	7.9	0	35.6
DQMEAN	11.9	6.1	0	26.7

Where: AGE= plantation age (years), TPH= total trees per hectare, HT= average height of ten tallest trees (meters), D_0= 0th diameter percentile (cm), D_{25}= 25th diameter percentile (cm), D_{50}= 50th diameter percentile (cm), D_{95}= 95th diameter percentile (cm), DQMEAN= quadratic mean diameter (cm). A total of 722 observations from slash pine plantations were utilized for model fitting. Average age of slash pine was about 11 years. The average number of trees per hectare was 991.

ect (ETPPRP) was initiated in 1982. Measurements were made on a 3 year cycle because it takes 3 year to measure all plots. Each plot was located in a different plantation and consisted of two adjacent subplots separated by a 18.3 meter buffer zone. One subplot was designated for model development and the other for model evaluation. A subplpot was 30.5 meter by 30.5 meter in size, and all planted slash pines within a subplot were tagged and measured. Measurements taken on each tree include dbh, total height, and height to base of live crown. Other characteristics recorded include crown class, tree vigor, disease, and hardwood component. Typical site preparation methods for establishing the plantations in which ETPPRP plots were involved are various combinations of shearing, pushing down, piling and chopping, plus burning. Evaluation subplots were utilized for evaluation purposes and all subplots were combined for model fitting. The summary statistics of the observed slash pine stand data sets are depicted in Table 1.

Model development

1) Prediction of diameter distribution percentile equations

The Weibull parameter recovery method was applied in this study that required use of the 0th, 25th, 50th, and 95th diameter percentiles. The 0th (D_0), 25th (D_{25}), 50th (D_{50}), and 95th (D_{95}) percentiles were obtained for each subplot. Separate regression equations for the percentiles were developed for the planted slash pines based on the model selection criteria.

2) Weibull parameter recovery methods

The Weibull distribution parameter recovery procedure, developed by Da Silva (1986) and subsequently utilized by Bailey *et al.* (1989) and Brooks *et al.* (1992), first determines the predicted location parameter '*a*' using the predicted values for D_0 and D_{50}, and the initial assumption is that the shape parameter '*c*' is 3.0.

Assuming that c=3, the location parameter, '*a*', was obtained by using the minimum (D_0) and median (D_{50}) diameters and sample size (n):

$$\hat{a}=(n^{1/3}D_0-D_{50})/(n^{1/3}-1),$$
$$\text{if } a<0.0 \text{ then } a=0. \quad (1)$$

The shape parameter was estimated by using the estimate for the location parameter and D_{95} and D_{25}:

$$\hat{c}=2.343088/\ln\left[\frac{D_{95}-\hat{a}}{D_{25}-\hat{a}}\right], \quad (2)$$

and the scale parameter, 'b', was obtained by solving the second moment of the Weibull distribution for the positive root with the estimates for 'a', 'c', and D_q^2:

$$\hat{b}=-\frac{\hat{a}\Gamma_1}{\Gamma_2}+\sqrt{\left(\frac{\hat{a}}{\Gamma_2}\right)^2(\Gamma_1^2-\Gamma_2)+\frac{D_q^2}{\Gamma_2}}. \quad (3)$$

where: Γ = the gamma function,
$\Gamma_1=\Gamma(1+1/c)$,
$\Gamma_2=\Gamma(1+2/c)$,
D_q=quadratic mean diameter.

There were favorable advantages to this percentile-based parameter recovery procedure over other recovery procedures such as the location and shape parameters were obtained by using simultaneous solutions for two points in the distribution, and the location parameter was obtained by using an analytical relationship between two percentiles rather than an arbitrary proportion of the minimum diameter.

3) Weibull cumulative function for the stand table calculations

The Weibull function has been widely used to model diameter distributions since the early applications by Bailey and Dell (1973), Schreuder and Swank (1974), and Little (1983):

$$F(X) = 1 - \exp\left[-\left(\frac{X-a}{b}\right)^c\right] \quad (4)$$
$(a \leq X < \infty)$, 0 otherwise.

The location parameter 'a' gives the minimum value of the distribution (minimum diameter values is ≥ 0), the scale parameter 'b' is related to the range of the diameter distribution, and the shape parameter 'c' determines the skewness of the distribution. Subtracting the cumulative distribution up to the lower limit of the class from the upper limits gives the proportion of trees in that class (Avery and Burkhart 1994).

$$P_i = \left(1 - \exp\left[-\left(\frac{U_i - a}{b}\right)^c\right]\right) - \left(1 - \exp\left[-\left(\frac{U_{(i-1)} - a}{b}\right)^c\right]\right) \quad (5)$$

where: P_i = proportion of trees in diameter class i,
U_i = upper limit of diameter class i.

This equation was used for calculating diameter class frequencies with all Weibull diameter distribution models. Predicted diameter distributions were calculated and compared with observed diameter distributions. Each evaluation subplots from observed and predicted diameter distributions were tested by using the Kolmogorov-Smirnov two-sample test at the $\alpha = 0.05$ level.

RESULTS AND DISCUSSION

Diameter percentile prediction equations

The components of diameter distribution prediction system are equations to estimate certain diameter percentiles and quadratic mean diameter. A total of 722 observations from slash pine plantations were utilized for model fitting. Separate regression equations for the minimum dbh (D_0) on the plot and 25th, 50th, and 95th percentiles were developed for slash pines based on the model selection criteria. The prediction equations for the 0th, 25th, 50th and 95th percentiles plus DQMEAN are presented in equations (6) to (10). The average variation explained by these regression equations ranged from 70.0% for equation (6) to 99.2% for equation (8). The root mean square error (RMSE) is the representative of total variability within each equation.

$$D_0 = \exp(-2.27863 + 1.67425 \ln(DQMEAN))$$
$$(R^2 = 0.700 \ \ \text{RMSE} = 0.369) \quad (6)$$

$$D_{25} = \exp(-0.41643 + 1.13082 \ln(DQMEAN))$$
$$(R^2 = 0.963 \ \ \text{RMSE} = 0.089) \quad (7)$$

$$D_{50} = \exp(-0.07520 + 1.04472 \ln(DQMEAN))$$
$$(R^2 = 0.992 \ \ \text{RMSE} = 0.037) \quad (8)$$

$$D_{95} = \exp(0.49629 + 0.87400 \ln(DQMEAN))$$
$$(R^2 = 0.972 \ \ \text{RMSE} = 0.070) \quad (9)$$

Quadratic mean diameter (DQMEAN) is the most important independent variable in predicting percentile-based diameter prediction equations.

DQMEAN is estimated as:

$$DQMEAN = \exp\left(3.21454 - 28.28500\left(\frac{1}{HT}\right) + 0.188682 \ln(AGE) - 0.14931 \ln(AGE * T)\right)$$
$$(R^2 = 0.920 \ \ \text{RMSE} = 0.094) \quad (10)$$

By the second moment estimate for Weibull probability density function, we can also derive stand-level variable DQMEAN information.

Applications of stand structure development predictions

To illustrate the use of this system, we randomly selected one permanent sample plot from evaluation subplots. The solution of above equations and parameter recovery procedures give the following stand structure development computations shown in Fig. 1. The dynamics of an even-aged slash pine stand in terms of the number of stems per diameter class changes were compared with observed and predicted stand structure development over times using data from slash pine evaluation permanent

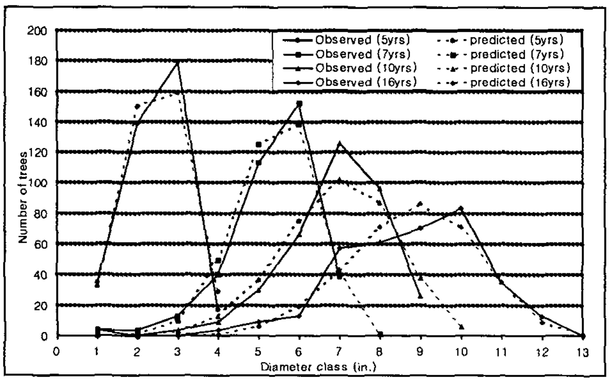

Fig. 1. Comparison of observed and predicted stand structure dynamics over times using data from slash pine evaluation permanent sample plot number 45.

sample plot number 45. A 5 year old stand has a limited range of small diameters in a bell-shaped distribution. When the stand gets older such as 16 years old, the number of trees drops and the bell-shaped distribution tends to flatten as the trees grow at variable growth rates in the stand.

The evaluation subplot data sets, which were separated from development subplots by a 18.3 meter wide buffer zone, provide an opportunity to analyze the reliability of the stand structure development prediction system. Plottings of predicted versus observed diameter distributions for each evaluation subplots were visually checked, and the predicted diameter distributions were tested against the observed diameter distributions using the Kolmogorov-Smirnov two-sample test at the $\alpha=0.05$ level. Statistically, no significant differences were detected for any of the predicted diameter distributions. This stand structure prediction model could provide estimates of the number of trees per acre unit (0.4047 hectare) by diameter classes. Therefore, the results of this study indicated that the use of percentile-based Weibull diameter distribution prediction system will be useful in slash pine stand structure modeling and in updating forest inventories for the long-term forest management planning.

ACKNOWLEDGEMENTS

This project was supported by industrial forest companies in the East Texas Pine Plantation Research Project - Champion International Corporation, International Paper Company, Louisiana - Pacific Corporation, Resource Management Services, Inc., and Temple - Inland Forest Products Corporation.

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(*Received October 27, 2000*)