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# 다중경로 페이딩 채널하에서 PLL이득에 따른 DS/SS시스템의 성능분석

(Performance Analysis of DS/SS System with PLL Gain in the Multipath Fading Channel)

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요 약

본 논문에서는 광범위한 이동통신채널환경에 적용할 수 있는 나카가미-m 페이딩 채널로 다중경로 페이딩 채널을 모델화하고, 수신신호와 수신기 내의 PLL(Phase Locked Loop)에서 발생된 참조신호와의 위상차를 위상에러로 가정하고 이러한 에러를 보정하기 위해 PLL을 이용한 새로운 RAKE수신기를 제안하였으며, 제안된 수신기로부터, RAKE수신기의 브랜치수 L, MIP(Multpath Intensity Profile)의 지수감소율  $\delta$ , PLL의 이득  $\gamma_n$ 에 따른 DS/SS(Direct Sequence/Spread Spectrum) 시스템의 성능을 분석하였다. 그 결과, 제안된 RAKE수신기의 L이 증가되고,  $\delta$ 가 감소할수록 시스템의 성능이 개선되었으며 또한 PLL이득이 30dB가 되었을 때 위상이 일치하게 되어 완전동기된 시스템과 동일한 성능을 나타냈다. 따라서 제안된 RAKE수신기로 위상에러를 보정할 수 있고, 수신기 내의 PLL에서 요구되는 이득의 상한이 30dB임을 입증하였다.

# Abstract

In this paper, we modelized the multipath fading to Nakagami-m distribution fading channel which can be applied to the extended mobile communication channel environment. We assumed that the phase difference with reference signal happened in the received signal and in the receiver PLL(Phase Locked Loop) is the phase error. To correct the error we propose new RAKE receiver using PLL. In addition, we analyze the performance of DS/SS(Direct Sequence/Spread Spectrum) system according to the gain of PLL,  $\gamma_n$ , the number of RAKE receiver branch L and MIP(Multipath Intensity Profile)'s exponential decay  $\delta$ . As a result, when the proposed RAKE receiver L is increased and the  $\delta$  is decreased the performance of the system gets better. Futhermore when PLL gain was 30dB, phase is identified. That is when the PLL gain is 30dB, the performance equals with the perfect coherent system's. Therefore, we can correct the phase error by using the proposed RAKE receiver and we proved that the PLL's requested limit gain should be 30dB.

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## I. Introduction

In the mobile communication channels, performance of the receiver is influened by environments which are delay, fading, and phase-error of signals. In the environments, if we used the PSK(Phase Shift

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Keying) modulation, it must be coincided with phase of the received signal and reference signal which are generated by PLL in the receiver. But when the transmitter or the receiver was moved a settled time, it is occured of phase difference between received signal and reference signal caused by doppler shifts, and it makes many problems when the receiver demodulates the received signal. So we must consider the phase error to improve the performance of the system.

Recently, it has been analyzed the limited performance of the receiver's BER(Bit Error Rate) assuming that the system was coherently perfect, or the phase difference has been reference signal generated in receiver's PLL and receiving signal was the phase error. multipath fading channel by Rayleigh fading channel was modeled.[1] However, in this paper, it is analyzed BER performance of DS/SS system in the frequency selective multipath fading channel. The multipath fading channel is modeled by Nakagami-m distribution fading channel and an enhanced RAKE receiver to correct the phase error which is generated by the difference between received signal and reference signal. From the proposed RAKE receiver, it is analyzed BER of DS/SS system with various PLL gains, numbers of RAKE receiver branch and MIP  $\delta$ .

# II. DS/SS System Model

### 1. Transmitter Model

Fig. 1 shows the DS/SS transmitter model. Data signal with period minimum T, d(t) which was modulated with PSK is transmitted to the receiver after spread by s(t), spread code with chip period Tc. The transmission signal, u(t) is transmitted through L paths. Transmitted signals are corrupted by multipath fading and added AWGN(Additive White Gaussian Noise).

In Fig. 1 let s(t) denote the code sequence waveform, and be multiplied by data signal d(t), then

the final transmitted signal u(t) received from the receiver is the following

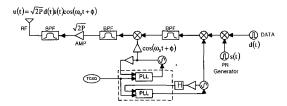


Fig. 1. DS/SS Transmitter model. 그림 1. DS/SS 송신기 모델

$$u(t) = \sqrt{2P} d(t) s(t) \cos(\omega_0 t + \mathbf{\Phi})$$
 (1)

where P means the average transmitted power,  $\omega_0$  is the carrier frequency, and  $\boldsymbol{\sigma}$  means phase delay of carrier. Without loss, we may assume  $\boldsymbol{\sigma}$  is zero radians. Therefore, the final transmission signal is the following.

$$U(t) = Re\left[\sqrt{2P} d(t) s(t) e^{j(\omega_0 t + \Phi)}\right]$$
 (2)

#### 2. Channel Model

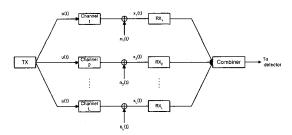


Fig. 2. Channel Model. 그림 2. 채널 모델

In Fig. 2  $x_L(t)$ , transmitted signals which are transmitted through each path are demodulated by RAKE receiver employing phase locked loop for phase recovery after passing the matched filter. The phase error means the difference between reference phase generated by receiver's PLL and transmitted signal phase. It is assumed that spread code or PN(Pseudo-Noise) code has a recursive period much longer than the value of a processing gain N.

The general model of frequency selective multipath fading channel is a finite-length tapped delay line. <sup>[2]</sup> The tap weight  $\beta_L(t)$  for L receiver branches affects the transmission signal to the n-th branch. In this paper, it is supposed that the tap weight is time-invariable for the convince. Therefore it is possible to express the tap weight  $\beta_L$  and  $\beta_L$  is random variable which has Nakagami probability density function<sup>[3]</sup>.

$$p(\beta_L) = \frac{2}{I(m)} \left(\frac{m}{\Omega_L}\right) \beta_L^{2m-1} \exp\left(\frac{-m \beta_L^2}{\Omega_L}\right)$$
(3)

where m is a fading index,  $Q_L$  is average received power, and  $\Pi$ () is Gamma function.

Generally, m value exists in the interval,  $0.5 \le m \le \infty$  and as the m value is increasing, the fading is decreasing in the channel.

### 3. Receiver Model

The phase  $\theta_L$  is ideally distributed random variable in the interval  $0 < \theta \le 2\pi$  and is independent for  $\beta_L$ . The function expressed by  $n_L(t)$  is complex-valued low-pass-equivalent AWGN with noise spectral density  $\eta_0$ . Therefore,  $n_L(t)$  is zero mean Gaussian random process with covariance function  $E[n_L(t) \ n_L^*(t)] = 2\eta_0 \delta(t-t)^{[4]}$ .

Fig. 3 shows a block diagram of a proposed RAKE receiver.

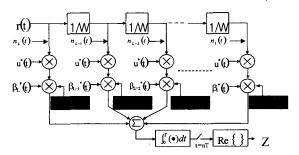


Fig. 3. Block diagram of a proposed RAKE receiver.

그림 3. 제안된 RAKE수신기의 블록도

In Fig. 3 r(t) is received signal, n(t) is the AWGN of each path, u\*(t) is a complex signal of a received

signal estimated by receiver and  $\beta^*(t)$  is the estimated complex channel gain.  $(\theta_L - \overline{\theta_L})$  is the phase difference between received signal and reference signal at the Lth path.

Generally the RAKE receiver can remove the interference between a symbol and a symbol by an ability of anti-multipath in the frequency selective fading channel. In addition, decreasing the fading by diversity effect makes the performance of system improve<sup>[5,6]</sup>. The proposed RAKE receiver was added PLL to correct the phase error at each branch.

To analyze the system easily, we assume that the proposed RAKE a receiver in Fig. 3 is partly coherent and it has L tap delay lines and the matched filter of receiver be matched with transmitted spread code. Assume that the system can achieve time synchronization with the initial path of the signal. Because the period of the spread code is larger than the bit interval, the matched filter sub-sequence needs to be updated every T seconds. Also, in order to demodulate the multipath, the filter must remain matched to each sub-sequence for  $(L-1)T_c$ .

Fig. 4 shows a physical block diagram of proposed RAKE receiver. We add a PLL to recover the phase error.

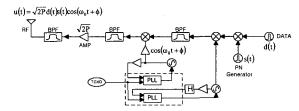


Fig. 4. Physical block diagram of proposed RAKE receiver.

그림 4. 제안된 RAKE수신기의 물리적인 구조

# III. Performance Analysis

When the input of the channel is the same as (2),

$$\begin{split} \gamma(t) &= \sum_{n=1}^{L} \beta_n e^{i(\theta_n + \phi)} \, U(t - n T_c) + \sum_{n=1}^{L} n_L(t) \\ &= \sqrt{2P} \sum_{n=1}^{L} \beta_n e^{i(\theta_n + \phi)} \, d(t - n T_c) \, s(t - n T_c) + \sum_{n=1}^{L} n_L(t) \end{split}$$

(4)

the received signal, r(t), in the receiver is following. After the r(t) passes through the matched filter in the receiver, the signal comes to be narrow band signal. The signals coming from all paths are sampled at the  $t=T+(L-1)T_c$ , and decide the signal from  $Z_T$ .  $Z_T$  is divided signal term  $Z_\tau$  and noise terms  $Z_{mb}$  and  $Z_n$ .  $Z_{mb}$  means multipath fading noise and  $Z_n$  means zero mean AWGN noise. So,  $Z_T$  is following.

$$Z_T = Z_r + Z_{mb} + Z_n \tag{5}$$

Assume that the noise and fading processes are stationary, it is clear that the error probability at only one sampling instant need to be evaluated.

$$Z_n = Re\left\{\sqrt{2P} \sum_{n=1}^{L} \beta_n e^{-j\overline{\theta_n}} \int_0^T s(t) n(t + nT_c) dt\right\}$$
 (6)

where  $Z_n$  is a complex Gaussian random variable with zero mean and its variance is following.

$$\sigma_n^2 = E[Z_n Z_n^* | \{\beta_n\}]$$

$$= Re\left\{\sqrt{2P} \sum_{n=1}^{L} \beta_n e^{-j\overline{\theta_n}} \int_0^T s(t) n(t+nT_c) dt \cdot \sqrt{2P}\right\}$$

$$= \sum_{n=1}^{L} \beta_n e^{j\overline{\theta_n}} \int_0^T s(t) n(t+nT_c) dt$$

$$= 2P \sum_{n=1}^{L} \beta_n^2 (2\eta_0 T)$$

$$= 4 \eta_0 PT \sum_{n=1}^{L} \beta_n^2$$
(7)

Consider the response to  $r_0(t) = r(t) - n(t)$ . Assume that the receiver could achieve in time synchronization with the initial path r1(t) of and sampling time of demodulation is  $T + (L-1)T_c$ . Then,  $Z_T$  from the channel model and the receiver model, is calculated by correlation of  $Z_r$ . Therefore,  $Z_r$  is following.

$$Z_r = Re \left\{ 4d_0 PT e^{j\phi} \sum_{n=1}^{L} \beta_n^2 e^{j(\theta_n - \overline{\theta_n})} \right\}$$
 (8)

where  $e^{j\Phi}$  is phase delay of the carrier frequency. multipath fading noise,  $Z_{m\phi}$  is more complicated and is dependent on the adjacent bits  $d_{-1}$  and  $d_1$  as well as on the spread code and its partial auto-correlation properties. Fortunately, one of the properties of spread code is that its partial auto-correlation is small compared to its peak at zero shift. In order to minimize of multipath fading, we use a gold code to spread the code. A subset of the gold code can be constructed having their desirable correlation properties. So auto-correlation by multipath can be ignored in BER. Therefore, total response of the receiver is following.

$$Z_T = Z_r + Z_n$$

$$= Re \left\{ 4d_0 PT e^{j\theta} \sum_{n=1}^{L} \beta_n^2 e^{j(\theta_n - \overline{\theta_n})} \right\} + Re\{n_L(t)\}$$
(9)

As a result, the test statistic is approximated as following.

$$Re[Z_T] = \left\{ 4d_0 PT \cos(\mathbf{\Phi}) \sum_{n=1}^{L} \beta_n^2 \cos(\theta_n - \overline{\theta_n}) \right\} + Re\{n_L(t)\}$$
(10)

where  $n_L(t)$  is a zero mean Gaussian variable in which variance is  $1/2\sigma_L^2$ . Where  $\cos(\theta_n - \overline{\theta}_n)$  can be replaced by  $I(\gamma_n)$ . Where  $\gamma_n$  is SNR value in receiver's PLL and it follows Tikhonov probability density function. [1]

$$E_{\mathcal{A}\theta}\left[\cos\left(\theta_{n} - \overline{\theta_{n}}\right)\right] = E_{\mathcal{A}\theta}\left[\cos\left(\mathcal{A}\theta_{n}\right)\right]$$

$$= \int_{0}^{2\pi} \cos\left(\mathcal{A}\theta_{n}\right) \frac{\exp\left(\gamma_{n}\cos\mathcal{A}\theta_{n}\right)}{2\pi I_{0}(\gamma_{n})} d\mathcal{A}\theta_{n}$$

$$= \frac{I_{1}(\gamma_{n})}{I_{0}(\gamma_{n})}$$

$$= I(\gamma_{n})$$
(11)

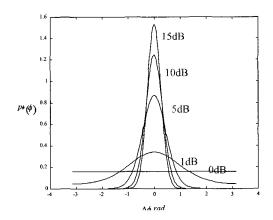


Fig. 5. Tikonov probability density function. 그림 5. Tikonov 확률밀도함수

We can use the function, *I*() to calculate the probability density function of phase errors, if the phase errors were made in the PLL of the receiver. Then the phase errors will be followed with Tikonov probability density function. Fig. 5 is a general Tikonov pdf according to the PLL gain. Therefore, the final decision in the receiver can be expressed as following.

$$Re[Z_T] = \left\{ 4d_0 PT\cos\left(\mathbf{\Phi}\right) \sum_{n=1}^{L} \beta_n^2 I(\gamma_n) + \right\} + Re[Z_n] \qquad (12)$$

Supposing that the gain of PLL added in each branch of the RAKE receiver is higher than any critical value  $\rho$ , the probability of the event is the following.

$$P_{L}(\rho) = \prod_{n=1}^{L} \int_{\rho}^{\infty} p(\gamma_{n}) d\gamma_{n}$$

$$= \exp\left(\frac{-\rho L}{\sigma_{L}}\right)$$
(13)

The final decision of a receiver is following.

$$Re[Z_T] = 4d_0PT\cos(\mathbf{\Phi})I(\rho)\sum_{n=1}^{L}\beta_n^2 + Re[Z_n]$$
 (14)

In equation (14), supposing that the data is 1,  $F = (1/\sigma^2) \sum_{n=1}^{L} \beta_n^2$ , the conditional probability is following.

$$P_o(\rho, F) = Q(\sqrt{2}\lambda F) \tag{15}$$

$$\lambda = \frac{E\sigma^2}{\eta_0} I^2(\rho) \frac{\Omega}{m} \tag{16}$$

The average received power of the received signal through the multipath channel is related to the receiving power of the initial received signal. In other words, supposing that the average received power of received signal through the initial path is  $\Omega_1^2$ , the average received power through n-th path is closely related to  $\Omega_1^2$ . That can be expressed as following

$$\Omega_n^2 = \Omega_1^2 e^{-\delta n}, \quad n = 2, \dots, L$$
 (17)

where the average received power in the receiver for each path is also related to  $\delta$ (rate of exponential decay of the MIP). By using Nakagami probability density function, the probability density function of F is following.

$$p(F) = \frac{2}{\Gamma(m_F)} \left(\frac{m_F}{\Omega_F}\right) F^{2m_F - 1} \exp\left(\frac{-m_F F^2}{\Omega_F}\right) \quad (18)$$

where fading index  $m_F$  and received power of each path  $\Omega_F$  are following.

$$m_F = \frac{\left(\sum_{n=1}^{L} \sigma_n^2\right)^2}{\sum_{n=1}^{L} \frac{(\sigma_n^2)^2}{m}} = \frac{\left(\sum_{n=1}^{L} e^{-\delta n}\right)^2}{\sum_{n=1}^{L} \frac{(e^{-\delta n})^2}{m}}$$
(19)

$$Q_F = \frac{1}{\sigma_n^2} \sum_{n=1}^{L} \sigma_n^2 = \sum_{n=1}^{L} e^{-\delta n}$$
 (20)

Equations (19) and (20) are applied in case all PLL gains higher than critical value  $\rho$ . Therefore, we can obtain the average BER from conditional Probability in case instant SNR of PLL in the receiver is higher than  $\rho$  using the (22).

$$P_{e,L}(\rho) = \int_0^\infty P_e(\rho, F) p(F) dF$$

$$= \int_0^\infty (Q(\sqrt{2}\lambda_F)) \frac{2}{I(m_F)} \left(\frac{m_F}{Q_F}\right) F^{2m_F - 1}$$

$$\exp\left(\frac{-m_F F^2}{Q_F}\right) dF$$

$$= \sqrt{\frac{\lambda_F}{1 + \lambda_F}} \frac{(1 + \lambda_F)^{-m_F} I(m_F + \frac{1}{2})}{2\sqrt{\pi} I(m_F + 1)}$$

$$\cdot {}_2F_1\left(1, m_F + \frac{1}{2}; m_F + 1; \frac{1}{1 + \lambda_F}\right)$$
(21)

$$\lambda_F = \frac{E \sigma_0^2}{\eta_0} I^2(\rho) \frac{\Omega_F}{m_F} \tag{22}$$

where, 2F1(., ;. ;. ) is hypergeometric function.

### IV. Performance Calculation

Figs. 6 and 9 were worked out with fixing the number of channel path, L=5 and the rate of exponential decay of MIP,  $\delta = 0.1$  and with the fading numer of Nakagami-m fading channel, m= $\frac{1}{2}$ , 1, 2, 3

in the several loop gain in PLL.

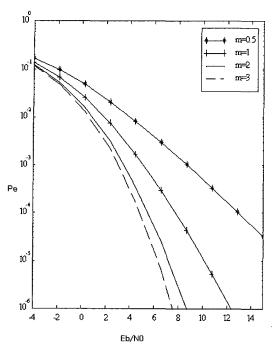


Fig. 6. Error Probability with L=5,  $\delta$ =0.1, and PLL gain 5dB.

그림.6. L=5, δ=0.1, PLL Gain=5dB인 경우 P(e)

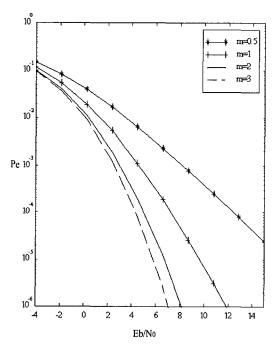


Fig. 7. Error Probability with L=5,  $\delta$ =0.1, and PLL gain 10dB.

그림 7. L=5, δ=0.1, PLL Gain=경우 P(e)

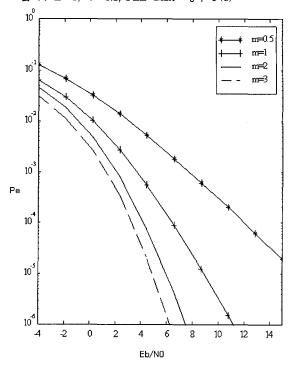


Fig. 8. Error Probability with L=5,  $\delta$ =0.1, and PLL gain 20dB.

그림 8. L=5, δ=0.1, PLL Gain=20dB인 경우 P(e)

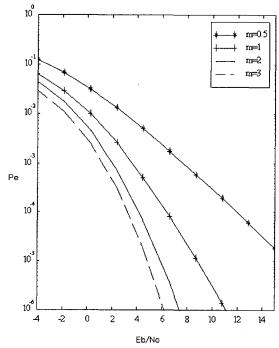


Fig. 9. Error Probability with L=5,  $\delta$ =0.1, and PLL gain 30dB.

그림 9. L=5, δ=0.1, PLL Gain=30dB인 경우 P(e)

From the figure versus, where is defined as, for L=5, and PLL gain is 5dB with fading index's of 0.5, 1, 2 and 3. In this paper, we simulated for L=5, and PLL gain 5dB, 10dB, 20dB, 30dB. As increasing the PLL gain from 5dB to 10dB, 10dB to 20dB, and 20dB to 30dB, performance of the system was increased about each 0.9dB, 0.8dB, and 0.2dB. Even though the PLL gain was higher than 30dB, it couldn't influence to performance of the system.

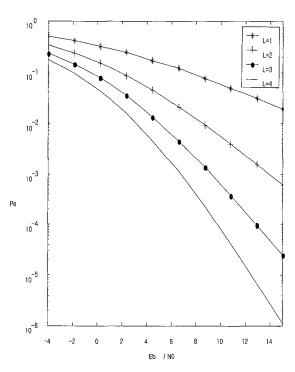


Fig. 10. Error Probability with m=1,  $\delta$ =0.1, and PLL gain 5dB. 그림 10. m=5,  $\delta$ =0.1, PLL Gain=5dB인 경우 P(e)

In Fig. 10 as the number of branch increased with L=1 to 2, 2 to 3, and 3 to 4, the performance was increased 5.2dB, 2.6dB, and 2dB in the 10-3 BER. When the L=2, the performance was 15dB at the 10-3 BER and the L=3, the performance was 9.8dB and the L=4, the performance was 7.2dB. So we found out that performance was improved by increase of the number of branch.

In Fig. 11 as the rate of exponential decay of MIP( $\delta$ ) was increased with  $\delta$  =0 to 0.2, 0.2 to 0.4,

and 0.4 to 0.8, the performance was increased 1.6dB, 1.6dB and 3.6dB in the 10-3 BER. When the  $\delta$ =0, the performance was 6dB at the 10-3 BER and the  $\delta$ =0.2, the performance was 7.6dB and the  $\delta$ =0.4, the performance was 9.2dB, and finally the  $\delta$ =0.8, the performance was 12.8dB. So it was found that performance of DS/SS system was influenced by the rate of exponential decay of MIP( $\delta$ ).

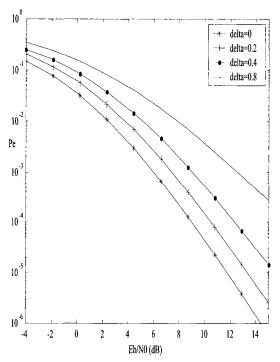


Fig. 11. Error Probability with m=1, L=4, and PLL gain 5dB.

그림 11. m=5, L=4, PLL Gain=5dB인 경우 P(e)

# V. Conclusions

In this paper, we proposed the new RAKE receiver by using PLL to correct the phase error in DS/SS system. From the proposed receiver we analyzed the parameter which is the number of branches (L), exponential decay ( $\delta$ ) and PLL gain( $\gamma_n$ ), through the computer simulation. As a result, when  $\delta$  decreases, L value increases and the sysem performance gets better. when the PLL gain increases by 5dB, 10dB,

20dB and 30dB, the differences of performance takes 0.9dB, 0.8dB, and 0.2dB each. In addition, when PLL gain is 30dB, the phase becomes identifed. Therefore, it shows that the performance equals with the perfect coherent system's. The proposed RAKE receiver can correct the phase error according to the PLL gain. It means that PLL gain can make the performance of the system be better. From the result of this research, the proposed RAKE receiver can be applied to the mobile communication system so that the system's performances get better. In the future, we can expect to invent low power RAKE receiver.

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