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불확실성을 갖는 퍼지 시스템의 출력궤환 견실 H^{∞} 제어
(Output Feedback Robust H^{∞} Control for Uncertain Fuzzy Dynamic Systems)

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요 약

본 논문에서는 불확실성을 갖는 비선형 시스템의 출력 궤환 퍼지 H^{∞} 제어 문제를 고려한다. 비선형 시스템은 Takagi-Sugeno(T-S) 퍼지모델로 나타내고 제어기 설계는 퍼지모델을 이용하여 설계한다. Lyapunov 함수를 이용하여 퍼지모델에 대한 폐-루프 시스템의 안정성뿐만 아니라 외란감쇠에 대한 L_2 이득 성능을 보장하는 충분조건을 유도한다. 유도된 조건식 으로부터 퍼지 H^{∞} 제어기가 존재할 충분조건을 선형 행렬부등식으로 나타내고, 이 선형 행렬부등식의 해로부터 제어기를 설계하는 알고리듬을 제시한다. 설계된 제어기는 비선형이며 퍼지 동작에 의해 자동적으로 조정된다.

Abstract

This paper presents an output feedback robust H^{∞} control problem for a class of uncertain nonlinear systems, which can be represented by an fuzzy dynamic model. The nonlinear system is represented by Takagi-Sugeno fuzzy model, and the control design is carried out on the basis of the fuzzy model. Using a single quadratic Lyapunov function, the globally exponential stability and disturance attenuation of the closed-loop fuzzy control system are discussed. Sufficient conditions for the existence of robust H^{∞} controllers are given in terms of linear matrix inequalities(LMIs). Constructive algorithm for design of robust H^{∞} controller is also developed. The resulting controller is nonlinear and automatically tuned based on fuzzy operation.

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I. INTRODUCTION

Recently, stability analysis and systematic design are among the most important issues for fuzzy control systems. There have been significant research efforts on these issues. With the development of fuzzy systems, it is known that the qualitative knowledge of a system can also be represented in nonlinear functional form. On the basis of this idea, some fuzzy models based control system design methods have appeared in the fuzzy control field[1]-[6]. These methods are conceptually simple and straightforward. The nonlinear system is represented by a Takagi-Sugeno (T-S)-type fuzzy model. And then, the control design is carried out on the basis of the fuzzy model via the so-called parallel distributed compensation scheme. Since uncertainties are frequently a source of instability, Tanaka *et al.*[5]-[6] presented stability analysis for a class of uncertain nonlinear systems and method for designing robust fuzzy controllers to stabilize the uncertain nonlinear systems. However, all the above design method has to predetermine the state feedback gains before checking the stability condition of the closed-loop system. In real control problems, all of the states are not available, thus it is necessary to design output feedback controller. Ma *et al.*[8] presented the analysis and design of the fuzzy controller and fuzzy observer on the basis of T-S fuzzy model using separation property. Tanaka *et al.*[9] also presented systematic design method of the fuzzy regulator and fuzzy observer on the basis of T-S fuzzy model.

In the last decade, H^∞ control theory for linear systems has been well developed and found extensive applications to efficiently treat the robust stabilization and disturbance rejection problem[10]-[11]. Also, there have been a lot of interests on the problem of robust H^∞ control for the systems with parameter uncertainty. The robust H^∞ control approach is concerned with the design of controller which stabilizes an uncertain system while satisfying an H^∞-norm bound constraint on disturbance attenuation for all admissible uncertainties[12]-[14].

In this paper, we design an output feedback robust H^∞ controller for uncertain nonlinear system on the basis of fuzzy model. To design a robust H^∞ controller, the nonlinear systems are represented by T-S fuzzy model. The control design is carried out on the basis of the fuzzy model and the resulting controller is tuned on-line based on fuzzy operations. A sufficient condition is derived such that the closed-loop system is globally exponentially stable and L_2 gain of the input-output map is bounded. Based on the derivation, LMI-based robust H^∞ controller are constructed.

Ⅱ. PROBLEM FORMULATION

The continuous fuzzy dynamic model, proposed by Takagi and Sugeno, is described by fuzzy IF-THEN rules which represented local linear input-output relations of nonlinear system. Consider an uncertain nonlinear system that can be described by the following uncertain T-S fuzzy model:

Plant Rule i :

IF $z_1(t)$ is M_{i1} and $\cdots$ and $z_g(t)$ is M_{ig}

THEN $\dot{x}(t)=(A_i+\Delta A_i(t))x(t)+B_{1_i}w_1(t)+B_{2_i}u(t)$

$e_1(t)=C_i x(t), \quad e_2(t)=u(t)$

$y(t)=C_{y_i}x(t)+w_2(t), \quad i=1,2,\cdots,r$

$x(t)=0, \quad t\le 0$

where M_{ij} is the fuzzy set, $x(t)\in \mathbf{R}^n$ is the state vector, $u(t)\in \mathbf{R}^{k_1}$ is the control input vector, $[w_1^T(t)\ w_2^T(t)]^T\in \mathbf{R}^p$ is the square-integrable disturbance input vector, $y(t)\in \mathbf{R}^{k_2}$ is the measured output, $[e_1^T(t)\ e_2^T(t)]^T\in \mathbf{R}^q$ is the controlled output vector, r is the number of **IF-THEN** rules, $z_1\sim z_g$ are some neasurable system variables, i.e., the premise variables, and all matrices are constant matrices with appropriate dimensions. And the time-varying parameter uncertainties $\Delta A_i(t)$ are defined as follows :

$$\Delta A_i(t)=HF(t)E_{x_i},\ \ i=1,2,\cdots,r \qquad (2)$$

where H and E_{x_i}, $i=1,2,\cdots,r$, are known constant real matrices with appropriate dimensions, and $F(t)$ is unknown matrix function which is bounded by

$$F(t) \in \Omega := \{F(t) | F^T(t) F(t) \leq I, \text{ the elements of } F(t) \text{ are Lebesgue measurable}\} \quad (3)$$

Let $\mu_i(z(t))$ be the normalized membership function of the inferred fuzzy set $h_i(z(t))$, i.e.,

$$\mu_i(z(t)) = h_i(z(t)) / \sum_{i=1}^{r} h_i(z(t)) \quad (4)$$

where

$$h_i(z(t)) = \prod_{j=1}^{g} M_{ij}(z_j(t))$$
$$z(t) = [z_{1(t)} \; z_{2(t)} \; \cdots \; z_g(t)]^T. \quad (5)$$

$M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij}. It is assumed that

$$h_i(z(t)) \geq 0, \; i = 1, 2, \cdots, r$$
$$\sum_{i=1}^{r} h_i(z(t)) > 0 \quad (6)$$

for all t. Then we can obtain the following conditions:

$$\mu_i(z(t)) \geq 0, \; i = 1, 2, \cdots, r$$
$$\sum_{i=1}^{r} \mu_i(z(t)) = 1 \quad (7)$$

for all t. Let $\rho \in \mathbf{R}^r$ be the set of membership function satisfying (7). Given a pair of $(y(t), u(t))$, by using a center average defuzzifer, product inference, and singleton fuzzifier, the dynamic fuzzy model (1) can be expressed by the following global model

$$\dot{x}(t) = (A(\mu) + HF(t)E_x(\mu))x(t) + B_1(\mu) w_1(t) + B_2(\mu) u(t)$$
$$e_1(t) = C(\mu) x(t), \; e_2(t) = u(t) \quad (8)$$
$$y(t) = C_y(\mu) x(t) + w_2(t)$$

where $\mu = [\mu_1, \mu_2, \cdots, \mu_r] \in P$,

$$A(\mu) = \sum_{i=1}^{r} \mu_i(z(t)) A_i, \; B_1(\mu) = \sum_{i=1}^{r} \mu_i(z(t)) B_{1_i},$$
$$B_2(\mu) = \sum_{i=1}^{r} \mu_i(z(t)) B_{2_i}, \; C(\mu) = \sum_{i=1}^{r} \mu_i(z(t)) C_i,$$
$$C_y(\mu) = \sum_{i=1}^{r} \mu_i(z(t)) C_{y_i}, \; E_x(\mu) = \sum_{i=1}^{r} \mu_i(z(t)) E_{x_i}. \quad (9)$$

As a robust H^∞ controller of the fuzzy system (1), we consider the following output feedback controller:

$$\dot{\hat{x}}(t) = A_k(\mu) \hat{x}(t) + B_k(\mu) y(t); \qquad \hat{x}(0) = 0 \quad (10)$$
$$u(t) = C_k(\mu) \hat{x}(t)$$

where $A_k(\mu)$, $B_k(\mu)$, and $C_k(\mu)$ are to be determined. From (8) and (10), we obtain the following closed-loop system

$$\dot{\zeta}(t) = [\hat{A}(\mu) + \hat{H}F(t)\hat{E}(\mu)]\zeta(t) + \hat{B}(\mu) w(t)$$
$$e(t) = \hat{C}(\mu) \zeta(t)$$
$$\zeta(t) = 0, \quad t \leq 0 \quad (11)$$

where $\zeta(t) = [x^T(t), \hat{x}^T(t)]^T$, $w(t) = [w_1^T(t), w_2^T(t)]^T$, and $e(t) = [e_1^T(t), e_2^T(t)]^T$,

$$\hat{A}(\mu) = \begin{bmatrix} A(\mu) & B_2(\mu) C_K(\mu) \\ B_k(\mu) C_y(\mu) & A_K(\mu) \end{bmatrix},$$
$$\hat{B}(\mu) = \begin{bmatrix} B_1(\mu) & 0 \\ 0 & B_k(\mu) \end{bmatrix}, \; \hat{C}(\mu) = \begin{bmatrix} C(\mu) & 0 \\ 0 & C(\mu) \end{bmatrix}$$
$$\hat{E}(\mu) = [E_x(\mu) \; 0], \; \hat{H} = [H^T \; 0]^T. \quad (12)$$

For a given γ, we define L_2 gain $\gamma-$performance of the system (11) as the quantity

$$\int_0^T \|e(t)\|^2 dt \leq \gamma^2 [\int_0^T \|w(t)\|^2 dt] \quad (13)$$

for all $T > 0$ and all $w \in L_2[0 \; T]$, where $\|\cdot\|$ denotes the Euclidean norm.

This paper addresses designing a output feedback robust H^∞ controller (10) for the system (8) such that the closed-loop system is globally exponentially stable and achieves L_2 gain $\gamma-$performance.

Ⅲ. STABILITY and L_2 NORM ANALYSIS

We can rewrite the state space representation of the closed-loop system (12) as

$$\dot{\zeta}(t)=\hat{A}(\mu)\zeta(t)+\hat{H}\,p(t)+\hat{B}(\mu)w(t)$$
$$q(t)=\hat{E}(\mu)\zeta(t)\quad e(t)=\hat{C}(\mu)\zeta(t)$$
$$p(t)=F(t)q(t),\quad \|F(t)\|\le 1.$$

Lemma 3.1: Consider the unforced system of (11). If there exist matrices $P>0$, and positive scalars λ and α satisfying the following inequalities

$$\begin{bmatrix} \hat{A}(\mu)^TP+P\hat{A}(\mu)+\hat{S}_1 & P\hat{H} & \lambda\hat{E}^T(\mu) \\ \hat{H}^TP & \lambda I & 0 \\ \lambda\hat{E}(\mu) & 0 & -\lambda I \end{bmatrix}\le 0, \quad (15)$$

for all $\mu\in\rho$, where

$$\hat{S}_1=\begin{bmatrix} \alpha I & 0 \\ 0 & 0 \end{bmatrix} \quad (16)$$

then the equilibrium of the unforced system of (11) is globally exponentially stable.

Proof: Define a Lyapunov functional

$$V(\zeta,t)=\zeta^T(t)P\zeta(t), \quad (17)$$

where $P>0$. Then there exist scalars $\delta_1>0$ and $\delta_2>0$ such that $\delta_1\|\zeta\|^2\le V(\zeta,t)\le\delta_2\|\zeta\|^2$. If there exists scalar $\alpha>0$ such that $\dot{V}(\zeta,t)\le-\alpha\|x\|$, then the unforced system of (11) is globally exponentially stable cite[15]. Assuming the zero input, the condition $\dot{V}(\zeta,t)\le-\alpha\|x\|$ for all μ is equivalent to

$$\zeta^T(t)\{\hat{A}^T(\mu)P+P\hat{A}(\mu)+\hat{S}_1\}\zeta(t)+p^T(t)\hat{H}^TP\zeta(t) + \zeta^T(t)P\hat{H}p(t)\le 0 \quad (18)$$

for all $\zeta(t)$ and $p(t)$ satisfying

$$p^T(t)p(t)\le\zeta^T(t)\hat{E}^T(\mu)\hat{E}(\mu)\zeta(t). \quad (19)$$

Applying the S-procedure cite[16], this is equivalent to the existence of $P>0$ and $\lambda\ge 0$ satisfying

$$\begin{bmatrix} \hat{A}^T(\mu)P+P\hat{A}(\mu)+\hat{S}_1+\lambda\hat{E}^T(\mu)\hat{E}(z) & P\hat{H} \\ \hat{H}^TP & -\lambda I \end{bmatrix}\le 0. \quad (20)$$

By using the Schur complement[16], it can be easily shown that the inequality (20) is equivalent to (15). □

Lemma 3.2: Consider the system (11) and let $\gamma>0$ be a given scalar. If there exist matrices $P>0$, and positive scalars λ and α satisfying the following inequalities for all $\mu\in\rho$

$$\begin{bmatrix} \hat{A}(\mu)^TP+P\hat{A}(\mu)+\hat{S}_1 & P\hat{H} & P\hat{B}(\mu) & \hat{C}^T(\mu) & \lambda\hat{E}^T(\mu) \\ \hat{H}^TP & -\lambda I & 0 & 0 & 0 \\ \hat{B}^T(\mu)P & 0 & -\gamma^2 I & 0 & 0 \\ \hat{C}(\mu) & 0 & 0 & -I & 0 \\ \lambda\hat{E}(\mu) & 0 & 0 & 0 & -\lambda I \end{bmatrix}\le 0 \quad (21)$$

where $\hat{S}_1$ is given by (16), then the corresponding closed-loop system (11) is globally exponentially stable and achieves L_2 gain γ-performance for all $w(t)$ and all admissible uncertainty.

Proof: The matrices $P>0$ and positive scalars λ and α satisfying (21) also satisfy the inequalities (15). Using

$$V(\zeta,t)=\zeta^{T(t)}P\zeta(t), \quad (22)$$

We introduce the following condition

$$J(t):=\dot{V}(\zeta,t)+e^T(t)e(t)-\gamma^2w^T(t)w(t)\le 0. \quad (23)$$

Along the trajectory of (11), $J(t)\le 0$ is equivalent to

$$\zeta^T(t)[\hat{A}^T(\mu)P+P\hat{A}(\mu)+\hat{C}^T(\mu)\hat{C}(\mu)]\zeta(t) + \zeta^T(t)P\hat{H}p(t)+\zeta^T(t)P\hat{B}(\mu)w(t)+p^T(t)\hat{H}^TP\zeta(t) + w^T(t)\hat{B}^T(\mu)P\zeta(t)-\gamma^2w^T(t)w(t)\le 0, \quad (24)$$

for all $\zeta(t)$ and $p(t)$ satisfying

$$p^T(t)p(t)\le\zeta^T(t)\hat{E}^T(\mu)\hat{E}(\mu)\zeta(t). \quad (25)$$

Using the S-procedure, this is true if and only if there exist $P>0$ and $\lambda\ge 0$ such that

$$\begin{bmatrix} A_{11}(\mu) & P\hat{H} & P\hat{B}(\mu) \\ \hat{H}^TP & -\lambda I & 0 \\ \hat{B}^T(\mu)P & 0 & -\gamma^2 I \end{bmatrix}\le 0, \quad (26)$$

where $A_{11}(\mu) = \hat{A}^T(\mu)P + P\hat{A}(\mu) + \hat{C}^T(\mu)\hat{C}(\mu) + \lambda\hat{E}^T(\mu)\hat{E}(\mu)$. By using the Schur complement, it can be shown that the matrices $P>0$, and positive scalars λ and α satisfying (21) also satisfy the inequalities (26). Thus if (21) is satisfied, then (23) is satisfied. We integrate (23) from 0 to T, with the initial condition $x(0)=0$, to get

$$V(x(T)) + \int_0^T (z^T(t)z(t) - \gamma^2 w^T(t)w(t)dt \le 0. \tag{27}$$

Since $V(x(T)) \ge 0$, this implies $\int_0^T \|z(t)\|^2 \le \gamma^2 \int_0^T \|w(t)\|^2 dt$. □

IV. FUZZY MODEL-BASED ROBUST H^∞ CONTROLLER DESIGN

In this section, we present a solution to the robust H^∞ control problem for the T-S fuzzy model in terms of LMIs.

Definition 4.1: Given the system (8) and $\gamma>0$. The **Globally ES-γ Problem** is solvable if there exist a finite dimensional controller (10) and matrices $P>0$, and positive scalars $\lambda$and α satisfying the inequality (21) for all $\mu\in\rho$.

The inequality (21) includes the unknown parameter of controller. We define a variable including all parameters of controller

$$K(\mu) = \begin{bmatrix} 0 & C_k(\mu) \\ B_k(\mu) & A_k(\mu) \end{bmatrix}, \tag{28}$$

and we introduce the abbreviations:

$$A_o(\mu) = \begin{bmatrix} A(\mu) & 0 \\ 0 & 0 \end{bmatrix}, \quad B_{1o}(\mu) = \begin{bmatrix} B_1(\mu) & 0 \\ 0 & 0 \end{bmatrix},$$
$$B_{00}(\mu) = \begin{bmatrix} B_2(\mu) & 0 \\ 0 & I \end{bmatrix}, \quad C_{00}(\mu) = \begin{bmatrix} C_y(\mu) & 0 \\ 0 & I \end{bmatrix}, \tag{29}$$
$$D_{yo}(\mu) = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, \quad C_{1o}(\mu) = \begin{bmatrix} C(\mu) & 0 \\ 0 & 0 \end{bmatrix}, \quad D_{20}(\mu) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

then the closed-loop matrices of (11) can be written as

$$\begin{aligned} \hat{A}_{(\mu)} &= A_{o(\mu)} + B_{00}(\mu)K(\mu)C_{00}(\mu), \\ \hat{B}(\mu) &= B_{1o}(\mu) + B_{00}(\mu)K(\mu)D_{yo}(\mu), \\ \hat{C}(\mu) &= C_{1o}(\mu) + D_{20}(\mu)K(\mu)C_{00}(\mu). \end{aligned} \tag{30}$$

Note that (29) involves only plant data and all matrices of (30) are affine form of the controller data $K(\mu)$. Using the notation of (29), define the left-hand side of inequalities (21) as

$$G(\mu) := \Phi(\mu) + \Sigma\Pi(\mu)K(\mu)\Theta^T(\mu) + \Theta(\mu)K^T(\mu)\Pi^T(\mu)\Sigma^T \tag{31}$$

where

$$\begin{aligned} \Sigma &= Diag(P, I, I, I, I), \\ \Pi(\mu) &= [B_{00}^{T(\mu)} \ \ 0 \ \ 0 \ \ D_{20}^T(\mu) \ \ 0]^T, \\ \Theta(\mu) &= [C_{00}(\mu) \ \ 0 \ \ D_{yo}(\mu) \ \ 0 \ \ 0]^T, \end{aligned} \tag{32}$$

and

$$\Phi(\mu) = \begin{bmatrix} A_o^T(\mu)P + PA_o(\mu) + \hat{S}_1 & P\hat{H} & PB_{1o}(\mu) & C_{10}^T(\mu) & \lambda\hat{E}^T(\mu) \\ \hat{H}^T P & -\lambda I & 0 & 0 & 0 \\ B_{1o}^T(\mu)P & 0 & -\gamma^2 I & 0 & 0 \\ C_{1o}(\mu) & 0 & 0 & -I & 0 \\ \lambda\hat{E}(\mu) & 0 & 0 & 0 & -\lambda I \end{bmatrix}. \tag{33}$$

Theorem 4.1: Given the system (8) and $\gamma>0$, **Globally ES-γ Problem** is solvable if and only if there exist matrices $X(\in \mathbf{R}^n)>0$ and $Y(\in \mathbf{R}^n)>0$ satisfying following inequalities for all $\mu\in\rho$,

$$\begin{bmatrix} \Lambda(\mu) & H & B_1(\mu) & YC^T(\mu) & E_x^{T(\mu)} & Y \\ H^T & -\lambda I & 0 & 0 & 0 & 0 \\ B_1^{T(\mu)} & 0 & -\gamma^2 I & 0 & 0 & 0 \\ C(\mu)Y & 0 & 0 & -I & 0 & 0 \\ E_x(\mu)Y & 0 & 0 & 0 & -\lambda^{-1}I & 0 \\ Y & 0 & 0 & 0 & -\lambda^{-1}I & -(\alpha I)^{-1} \end{bmatrix} < 0, \tag{34}$$

$$\begin{bmatrix} \Gamma(\mu) & XH & XB_1(\mu) & C^T(\mu) & \lambda E_x^T(\mu) \\ H^T X & -\lambda I & 0 & 0 & 0 \\ B_1^T(\mu)X & 0 & -\gamma^2 I & 0 & 0 \\ C(\mu) & 0 & 0 & -I & 0 \\ \lambda E_x(\mu) & 0 & 0 & 0 & -\lambda I \end{bmatrix} < 0, \tag{35}$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \le 0, \tag{36}$$

for some positive scalars λ and α, where

$$\Lambda(\mu)=YA^T(\mu)+A(\mu)Y-B_2(\mu)B^{T_2}(\mu),$$
$$\Gamma(\mu)=A(\mu)^TX+XA(\mu)+\alpha I-\gamma^2C_y^T(\mu)C_y(\mu). \quad (37)$$

Furthermore, one n-dimensional, strictly proper controller that solves the feedback problem is defined as:

$$A_k(\mu):=A(\mu)-\gamma^2Z^{-1}C_y^T(\mu)C_y(\mu)-B_2(\mu)B_2^T(\mu)Y^{-1}$$
$$+\gamma^{-2}B_1(\mu)B_1(\mu)^TY^{-1}+\lambda^{-1}HH^TY^{-1}-Z^{-1}H(\mu)$$
$$B_k(\mu):=\gamma^2Z^{-1}C_y^T(\mu),\ C_k(\mu):=-B_2^T(\mu)Y^{-1}, \quad (38)$$

where $Z:=X-Y^{-1}$ and

$$H(\mu)=-[Y^{-1}A(\mu)+A^T(\mu)Y^{-1}-Y^{-1}B_2(\mu)B_2^T(\mu)Y^{-1}$$
$$+\gamma^{-2}Y^{-1}B_1(\mu)B_1^T(\mu)Y^{-1}+\lambda^{-1}Y^{-1}HH^TY^{-1}$$
$$+C^T(\mu)C(\mu)+\alpha I+\lambda E_x^T(\mu)E_x(\mu)]. \quad (39)$$

Proof: ($\Rightarrow$) Let $P\in\mathbf{R}^{(n+m)\times(n+m)}$ be the positive definite matrix that satisfies the Lemma 3.2. Define $Q:=P^{-1}$. Clearly, Q is also positive definite. Partition P as

$$P=\begin{bmatrix} X & X_2 \\ X_2^T & X_3 \end{bmatrix},\quad Q=\begin{bmatrix} Y & Y_2 \\ Y_2^T & Y_3 \end{bmatrix}, \quad (40)$$

where $X, Y\in\mathbf{R}^{n\times n}$ and $X_3, Y_3\in\mathbf{R}^{m\times m}$. By the matrix inversion lemma, it follows that $X-Y^{-1}\geq 0$, which is LMI in equation (36). Now, define matrices $\Pi_\perp(\mu)$ and $\Theta_\perp(\mu)$ such that for all μ, $\Pi_\perp^T(\mu)\Pi(\mu)=0$, $\Theta_\perp^T(\mu)\Theta(\mu)=0$, and $[\Pi(\mu),\Pi_\perp(\mu)]$, $[\Theta(\mu),\Theta_\perp(\mu)]$ are full column rank. Then

$$\Pi_\perp^T(\mu)=\begin{bmatrix} I&0&0&0&0&0&-B_2(\mu)&0\\ 0&0&I&0&0&0&0&0\\ 0&0&0&I&0&0&0&0\\ 0&0&0&0&I&0&0&0\\ 0&0&0&0&0&I&0&0\\ 0&0&0&0&0&0&0&I \end{bmatrix},$$
$$\Theta_\perp^T(\mu)=\begin{bmatrix} I&0&0&0&-C_y^T(\mu)&0&0&0\\ 0&0&I&0&0&0&0&0\\ 0&0&0&I&0&0&0&0\\ 0&0&0&0&0&I&0&0\\ 0&0&0&0&0&0&I&0\\ 0&0&0&0&0&0&0&I \end{bmatrix}, \quad (41)$$

Since both $\Pi_\perp(\mu)$ and $\Theta_\perp(\mu)$ are full col rank for all $\mu\in P$, it is clear that if $G(\mu)<0$, then $\Pi_\perp^T(\mu)\Sigma^{-1}G(\mu)\Sigma^{-1}\Pi_\perp(\mu)<0$ and $\Theta_\perp^T(\mu)G(\mu)\Theta_\perp(\mu)<0$, which is equivalent to

$$\Pi_\perp^T(\mu)\Sigma^{-1}\Phi(\mu)\Sigma^{-1}\Pi_\perp(\mu)<0, \quad (42)$$
$$\Theta_\perp^T(\mu)\Phi(\mu)\Theta_\perp(\mu)<0 \quad (43)$$

for all $\mu\in P$. Carrying out the algebraic manipulations, it can readily be seen by taking Schur complements of the resulting expressions in (42) and (43) that the conditions (34) and (35) are satisfied.

($\Leftarrow$) For sufficiency, we verify that the controller given in (38) satisfies the **Globally ES$-\gamma$ Problem** using

$$P:=\begin{bmatrix} X & -(X-Y^{-1}) \\ -(X-Y^{-1})^T & X-Y^{-1} \end{bmatrix}. \quad (44)$$

First, note by the Schur complements, $P>0$. We redefine the left-hand side of inequality (21) as

$$Y(\mu):=\hat{A}^T(\mu)P+P\hat{A}(\mu)+\hat{S}_1+\hat{C}^T(\mu)\hat{C}(\mu)$$
$$+\lambda\hat{E}^T(\mu)\hat{E}(\mu)+\gamma^{-2}P\hat{B}(\mu)\hat{B}^T(\mu)P$$
$$+\lambda^{-1}P\hat{H}\hat{H}^TP \quad (45)$$

where the closed-loop matrices $\hat{A}$, $\hat{B}$, $\hat{C}$, $\hat{H}$ and $\hat{E}$ are defined in (12). We perform the following matrix manipulations to verify $Y(\mu)<0$. Partition Y inton $n\times n$ blocks Y_{11}, Y_{12}, and Y_{22}. Define a transformation

$$T:=\begin{bmatrix} I & 0 \\ I & I \end{bmatrix} \quad (46)$$

note $Y(\mu)<0$ if and only if $T^TY(\mu)T<0$. Define the transformed state-space data

$$\tilde{A}(\mu):=T^{-1}\hat{A}(\mu)T,\ \tilde{B}(\mu):=T^{-1}\hat{B}(\mu),$$
$$\tilde{C}(\mu):=\hat{C}(\mu)T,\ \tilde{E}(\mu):=\hat{E}(\mu)T,\ \tilde{H}:=T^{-1}\hat{H}. \quad (47)$$

Let $\tilde{P}(\mu):=T^TP(\mu)T$ and $\tilde{S}_1:=T^T\hat{S}_1T$. Then $T^TY(\mu)T<0$ can be written as

$$\widetilde{A}^T(\mu)\widetilde{P}+\widetilde{P}\widetilde{A}(\mu)+\widetilde{S}_1+\widetilde{C}^T(\mu)\widetilde{C}(\mu)+\lambda\widetilde{E}^{T(\mu)}\widetilde{E}(\mu)$$
$$+\gamma^{-2}\widetilde{P}\widetilde{B}(\mu)\widetilde{B}^T(\mu)\widetilde{P}+\lambda^{-1}\widetilde{P}\widetilde{H}\widetilde{H}^T\widetilde{P}<0. \tag{48}$$

Denote the left-hand side of (48) as $\widetilde{Y}$ and partition it into blocks $\widetilde{Y}_{11}$, $\widetilde{Y}_{12}$, $\widetilde{Y}_{22}\in R^{n\times n}$. Using the controller (38), it can be shown that

$$\widetilde{Y}(\mu)=\begin{bmatrix} -H(\mu) & -H(\mu) \\ -H(\mu) & Y_{11}-H(\mu)\end{bmatrix}. \tag{49}$$

Using the Schur complement, $\widetilde{Y}<0$ if and only if

$$-H(\mu)<0,\quad Y_{11}(\mu)<0. \tag{50}$$

These conditions are equivalent to (34) and (35).

Theorem 4.2: Consider the system (9) with assumption (2). Then the **Globally ES-** γ problem is solvable, if there exist common matrices $\widetilde{X}>0$, $\widetilde{Y}>0$ and positive scalars $\widetilde{\gamma}$, $\widetilde{\alpha}$, λ satisfying the following LMIs:

$$\Psi_{ii}<0, i=1,2,\ldots,r,\quad \Psi_{ij}+\Psi_{ji}<0, i<j<r, \tag{51}$$

$$\Omega_{ii}<0,\quad i=1,2,\ldots,r,\ \Omega_{ij}+\Omega_{ji}<0,\quad i<j<r, \tag{52}$$

$$\begin{bmatrix} \widetilde{X} & I \\ I & \widetilde{Y}\end{bmatrix}\geq 0. \tag{53}$$

In here

$$\Psi_{ij}=\begin{bmatrix} \Lambda_{ij} & B_{1_i} & \widetilde{Y}C_i^T & \widetilde{Y}E_{x_i}^T & \widetilde{Y} \\ B_{1_i}^T & -\widetilde{\gamma}I & 0 & 0 & 0 \\ C_i\widetilde{Y} & 0 & -\lambda I & 0 & 0 \\ E_{x_i}\widetilde{Y} & 0 & 0 & -I & 0 \\ \widetilde{Y} & 0 & 0 & 0 & -\widetilde{\alpha}^{-1}I \end{bmatrix}, \tag{54}$$

$$\Omega_{ij}=\begin{bmatrix} \Gamma_{ij} & I & \widetilde{X}H & \widetilde{X}B_{1_i} & C_i^T \\ I & -\widetilde{\alpha}^{-1}I & 0 & 0 & 0 \\ & 0 & -I & 0 & 0 \\ H^T\widetilde{X} & 0 & 0 & -\widetilde{\gamma}I & 0 \\ C_i & 0 & 0 & 0 & -\lambda I \end{bmatrix}, \tag{55}$$

where

$$\Lambda_{ij}=\widetilde{Y}A_i^T+A_i\widetilde{Y}-\lambda B_{2_i}B_{2_j}^T+HH^T,$$
$$\Gamma_{ij}=\widetilde{X}A_i+A_i^T\widetilde{X}-\widetilde{\gamma}C_{y_i}^TC_{y_i}+E_{x_i}^TE_{x_i}, \tag{56}$$

Furthermore, positive definite matrices X, Y and positive scalars α, γ are given by

$$X=\lambda\widetilde{X},\quad Y=\lambda^{-1}\widetilde{Y},$$
$$\alpha=\lambda\widetilde{\alpha},\quad \gamma^2=\lambda\widetilde{\gamma} \tag{57}$$

and a suitable controller is expressed in (38).

Proof: The inequality (34) and (35) of Theorem 4.1 are equivalent to

$$YA^T(\mu)+A(\mu)Y-B_2(\mu)B^{T_2}(\mu)+\lambda^{-1}HH^T+\gamma^{-2}B_1(\mu)B^{T_1}(\mu)$$
$$+YC^T(\mu)C(\mu)Y+\lambda YE^{T_x}(\mu)E_x(\mu)Y+\alpha YY<0, \tag{58}$$
$$XA(\mu)+A^T(\mu)X+\alpha I-\gamma^2C^{T_y}(\mu)C_y(\mu)+\lambda^{-1}XHH^TX$$
$$+\gamma^{-2}XB_1(\mu)B^{T_1}(\mu)X+C^T(\mu)C(\mu)+\lambda E^{T_x}(\mu)E_x(\mu)<0. \tag{59}$$

First, multiply λ to (58) and λ^{-1} to (59) and let $\widetilde{X}=\lambda^{-1}X$, $\widetilde{Y}=\lambda Y$, $\widetilde{\alpha}=\lambda^{-1}\alpha$, and $\widetilde{\gamma}=\lambda^{-1}\gamma^2$. Then, using Schur complement and notation (9), inequalities (58) and (59) are equivalent

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(z(t))\mu_j(z(t))\Psi_{ij}<0,$$
$$\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(z(t))\mu_j(z(t))\Omega_{ij}<0. \tag{60}$$

Inequality (60) can be also written as

$$\sum_{i=1}^{r}\mu_i(z(t))\mu_i(z(t))\Psi_{ii}+\sum_{i<j}\mu_i(z(t))\mu_j(z(t))(\Psi_{ij}+\Psi_{ji})<0,$$
$$\sum_{i=1}^{r}\mu_i(z(t))\mu_i(z(t))\Omega_{ii}+\sum_{i<j}\mu_i(z(t))\mu_j(z(t))(\Omega_{ij}+\Omega_{ji})<0. \tag{61}$$

From (61), we get (51) and (52). □

It has been seen that the controller design problem of the fuzzy system (8) can be transformed into a linear algebra problem. This set of LMIs constitutes a finite-dimensional convex feasibility problem. There are several efficient algorithms to solve the above convex LMIs problem[17]-[19].

V. DESIGN EXAMPLE

We will design a fuzzy H^∞ filter for the following nonlinear system;

$$\dot{x}_{1(t)}=-0.1x_1^3(t)-0.02x_2(t)-0.67x_2^3(t)+w(t)+u(t)$$
$$y(t)=x_2(t)$$
$$e_1(t)=x_2(T),\quad e_2(t)=u(t).$$

The purpose of control is to achieve closed loop stability and to attenuate the influence of the exogenous input disturbance $w(t)$ on the penalty variable $[e_1^T(t)\ e_2^T(t)]^T$. It is also assumed that $x_2(t)$ is measurable and $x_1(t)\in[-1.5\ 1.5]$, $x_2(t)\in[-1.5\ 1.5]$. Since $x_1(t)$ is not observable, let assume that $x_1^3(t)=c(t)x_1(t)$ where $c(t)$ is the uncertain term and $c(t)\in[0\ 2.25]$. Using the same procedure as in cite[6], the nonlinear term can be represented as

$$-0.67x_2^3(t)=M_{11}\cdot 0\cdot x_2(t)-(1-M_{11})\cdot 1.5075x_2(t).$$

By solving the equation, M_{11} is obtained as follows:

$$M_{11}(x_2(t))=1-\frac{x_2^2(t)}{2.25},$$

$$M_{12}(x_2(t)):=1-M_{11}(x_2(t))=\frac{x_2^2}{2.25}.$$

M_{11} and M_{12} can be interpreted as membership functions of fuzzy set. By using these fuzzy sets, the nonlinear system can be presented by the following uncertain T-S fuzzy model

Plant 1:
IF $x_2(t)$ is M_{11} THEN

$$\dot{x}(t)=(A_1+\Delta A_1(t))x(t)+B_{1_1}w(t)+B_{2_1}u(t)$$
$$y(t)=C_{y_1}x(t)$$
$$e_1(t)=C_1x(t),\quad e_2(t)=u(t)$$

Plant 2: IF $x_{2(t)}$ is M_{12} THEN

$$\dot{x}(t)=(A_2+\Delta A_2(t))x(t)+B_{1_2}w(t)+B_{2_2}u(t)$$
$$y(t)=C_{y_2}x(t)$$
$$e_1(t)=C_2x(t),\quad e_2(t)=u(t)$$

where $x(t)=[x_1(t)\quad x_2(t)]^T$,

$$A_1=\begin{bmatrix}-0.1125 & -0.02\\ 1 & 0\end{bmatrix},\ A_2=\begin{bmatrix}-0.1125 & -1.527\\ 1 & 0\end{bmatrix},$$

$B_{1_1}=B_{1_2}=[1\quad 0]^T$, $B_{2_1}=B_{2_2}=[1\quad 0]^T$,

$C_{y_1}=C_{y_2}=[0\quad 1]$, $C_1=C_2=[0\quad 1]$,

$\Delta A_1=HF(t)E_{x_1}$, $\Delta A_2=HF(t)E_{x_2}$,

$H=[-0.11125\quad 0]^T$, $E_{x_1}=E_{x_2}=[1\quad 0]$, $F(t)\in\Omega$.

Then solutions satisfying (51)-(53) are

$$X=\begin{bmatrix}9.0256 & -5.2199\\ -5.2199 & 11.6603\end{bmatrix},\quad Y=\begin{bmatrix}0.3600 & -0.0932\\ -0.0932 & 0.2603\end{bmatrix},$$

$\lambda=3.3649$, $\alpha=0.3894\cdot 10^{-6}$, $\gamma=4.625$.

The simulation results for the nonlinear systems are shown in Fig. 1. For these simulations, the initial value of the states are $x_1(0)=-1$ and $x_2(0)=-1.2$. And the disturbance signal $w(t)$ is defined by

$$w(t)=\begin{cases}0.3, & 2\text{ sec}\le t\le 3\text{ sec}\\ 3, & \text{otherwise}\end{cases}$$

The designed robust fuzzy H^∞ controller stabilizes the nonlinear system and attains disturbance attenuation effect.

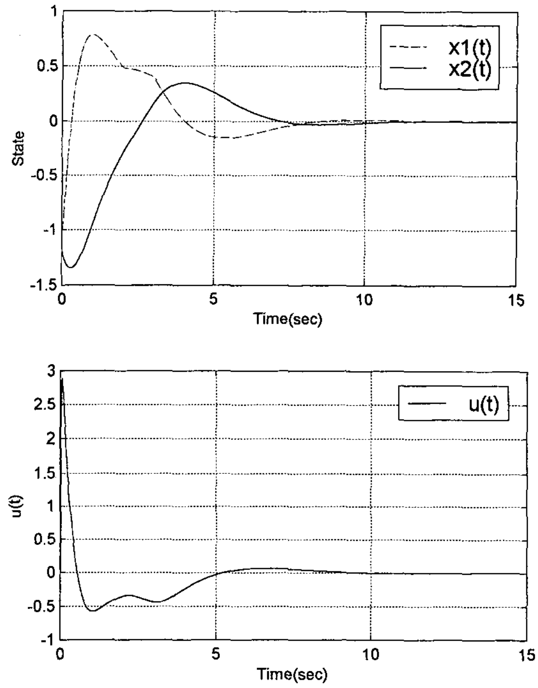

그림 1. 비선형 시스템에 대한 시뮬레이션 결과
Fig. 1. The simulation results of nonlinear system.

II. CONCLUSION

In this paper, we have developed output

feedback robust H^∞ controller design method for uncertain nonlinear systems described by Takagi and Sugeno's fuzzy model. We have obtained sufficient conditions for the existence of robust H^∞ controller such that the closed-loop fuzzy control system is globally exponentially stable and achieves a prescribed level of disturbance attenuation. Based on the derivation, an LMI-based robust H^∞ controller is constructed. The control design was carried out on the basis of the fuzzy model and the resulting controller is tuned based on fuzzy operation.

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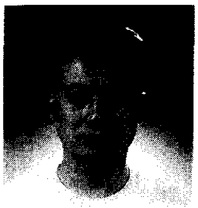

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