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Melt spinning dynamics of Phan-Thien Tanner fluids

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Abstract

Employing the Phan-Thien Tanner (PTT) fluids model, dynamic behavior of the non-isothermal melt spinning has been investigated. Subjects such as draw resonance instability, the effects of spinline cooling and of the fluid viscoelasticity on the spinning dynamics have been studied using the governing equations of the system. In particular, the draw resonance criterion based on the traveling times of various kinematic waves in the spinline has been confirmed, the reason why the spinline cooling is stabilizing is analyzed, and the effect of fluid viscoelasticity on the spinline stability is summarized. It is believed that the same method as in this study can be applied with equal ease to other extension deformation processes like film casting and film blowing.

Keywords **:** cooling effect, draw resonance, Phan-Thien Tanner model, spinning stability, tension sensitivity

1. Introduction

The dynamics of melt spinning processes can be analyzed using proper governing equations comprised of continuity, motion, constitutive, and energy equations. Based on the several assumptions like one-dimensional modeling, ignoring extrudate swell, and neglecting secondary forces on spinline, i.e., inertia, gravity, air drag and surface tension forces, we can come up with relatively simple equations governing the melt spinning process which portray the basic dynamic behavior of the process fairly well. (Denn *et al.*, 1975; Ishihara and Kase, 1975; Hyun and Ballman, 1978; Bechtel *et al.*, 1992) In this study, we have tried the Phan-Thien Tanner fluid model (Phan-Thien and Tanner, 1977; Phan-Thien, 1978) as the constitutive equation, because it has been proved that this PTT model is more robust and accurate in describing extensional deformation processes as compared with other frequently used constitutive equations like White-Metzner model which quite often results in the blow up of the spinline stress. (Khan and Larson, 1987; Souvaliotis and Beris, 1992)

The governing equations of non-isothermal melt spinning of PTT fluids are as follows.

Continuity equation:

$$\frac{\partial A}{\partial t'}+\frac{\partial (AV)}{\partial z}=0 \tag{1}$$

Equation of motion:

$$\frac{\partial (A\sigma)}{\partial z}=0 \tag{2}$$

Constitutive equation:

$$K\sigma+\lambda\left[\frac{\partial \sigma}{\partial t'}+V\frac{\partial \sigma}{\partial z}-2(1-\xi)\sigma\frac{\partial V}{\partial z}\right]=2G\lambda\frac{\partial V}{\partial z},$$

$$K=\exp\left(\frac{\varepsilon}{G}\mathrm{tr}\underline{\underline{\sigma}}\right),\quad \lambda=\lambda_0\exp\left[\frac{E}{RT_0}\left(\frac{T_0}{T}-1\right)\right] \tag{3}$$

Energy equation:

$$\frac{\partial T}{\partial t'}+V\frac{\partial T}{\partial z}=\frac{2\sqrt{\pi}h}{\rho C_p\sqrt{A}}(T-T_a),$$

$$h=0.473\times 10^{-4}\left(\frac{V}{A}\right)^{1/3}\left(1+\left(8\frac{V_y}{V}\right)^2\right)^{1/6} \tag{4}$$

Boundary conditions:

$$A=A_0,\ V=V_0,\ T=T_0,\ \sigma=\sigma_0 \quad \text{at } z=0 \text{ for all } t', \tag{5}$$
$$V=V_L=rV_0 \quad \text{at } z=L \text{ for all } t'. \tag{6}$$

where, A = spinline cross-sectional area, V = spinline velocity, T = spinline temperature, T_a = cooling air temperature, σ = spinline axial stress, z = distance from the spinneret, L = spinning distance, t' = time, G = modulus, h = heat transfer coefficient between the cooling air and spinline, C_p = heat capacity, ρ = density, V_y = cooling air velocity, E = material activation energy, R = gas constant, r = drawdown ratio, ε and ξ are the parameters of PTT model. Subscripts 0 and L denote spinneret and take-up conditions, respectively. At time $t=0^+$, disturbances are

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introduced to the system with all other conditions maintained the same.

Several assumptions have been incorporated here. One-dimensional model ignoring the variation across the cross-section of the spinline, the position of maximum extrudate swell being chosen as the origin of the space coordinate, no secondary forces on the spinline other than rheological extensional force, and no crystallization arising inside the spinline.

The above equations are then rendered into dimensionless form as follows.

Dimensionless continuity equation:

$$\frac{\partial a}{\partial t}+\frac{\partial(av)}{\partial x}=0 \tag{7}$$

Dimensionless equation of motion:

$$\frac{\partial(a\tau)}{\partial x}=0 \tag{8}$$

Dimensionless constitutive equation:

$$\exp[2\varepsilon\, De_0\, \tau]\tau+De\left[\frac{\partial\tau}{\partial t}+v\frac{\partial\tau}{\partial x}-2(1-\xi)\tau\frac{\partial v}{\partial x}\right]=\frac{De}{De_0}\frac{\partial v}{\partial x},$$
$$De=De_0\exp\left[k\left(\frac{1}{\theta}-1\right)\right] \tag{9}$$

Dimensionless energy equation:

$$\frac{\partial\theta}{\partial t}+v\frac{\partial\theta}{\partial x}=-St\, v^{1/3}a^{-5/6}(\theta-\theta_a)\left[1+\left(8\frac{v_y}{v}\right)^2\right]^{1/6},$$
$$St=\frac{1.67\times10^{-4}L}{\rho C_p A_0^{5/6}V_0^{2/3}} \tag{10}$$

Dimensionless boundary conditions:

$$a=1,\ v=1,\ \theta=1,\ \tau=\tau_o \quad \text{at } x=0 \text{ for all } t, \tag{11}$$
$$v=r \quad \text{at } x=1 \text{ for all } t. \tag{12}$$

where the dimensionless variables and numbers are defined below.

$$t=\frac{t'V_0}{L},\ x=\frac{z}{L},\ a=\frac{A}{A_0},\ v=\frac{V}{V_0},\ v_y=\frac{V_y}{V_0},$$
$$\theta=\frac{T}{T_0},\ \theta_a=\frac{T_a}{T_0},\ k=\frac{E}{RT_0},\ \tau=\frac{\sigma L}{2G\lambda_0 V_0},\ De_0=\frac{\lambda_0 V_0}{L} \tag{13}$$

2. Results and Discussions

In this section we deal with three different dynamic aspects of the non-isothermal spinning of PTT fluids: the draw resonance criterion, the stabilizing effect of spinline cooling and the effect of fluid viscoelasticity on the spinning stability.

The first subject of draw resonance has been studied and reported several times by Hyun and coworkers in recent years (Hyun, 1978; Kim *et al.*, 1996; Jung *et al.*, 2000a), thus we rewrite here without derivation the draw resonance criterion equation.

$$(t_L-\Delta t-\delta)_1+\mathbf{T}/4+(t_L-\Delta t-\delta)_2+\mathbf{T}/4\geq[\theta_L-\Delta t-\delta]_1$$
$$+[\theta_L-\Delta t-\delta]_2=\mathbf{T} \tag{14}$$

$$\text{or } (t_L)_1+(t_L)_2+\mathbf{T}/2\geq[\theta_L]_1+[\theta_L]_2 \text{ for } r\leq r_{rc}$$

where, t_L is dimensionless traveling time of unity-throughput waves from the spinneret to the take-up, θ_L is dimensionless traveling time of A_{max} or A_{min} waves from the node 1 to the take-up, Δt is dimensionless time (phase) differences between the F (spinline tension) curve and A curve at the take-up, δ is time delay between the F curve and corresponding A_{max} or A_{min} waves at node 1, $\mathbf{T}$ is the dimensionless period of draw resonance, r is the drawdown ratio and r_c is the critical drawdown ratio.

The left hand side of Eq. (14) is the required time for two successive unity throughput waves (with a pause time of $\mathbf{T}$/4 in between) to be able to travel from the spinneret to the take-up, whereas the right hand side is the time for two successive peak and trough cross-sectional area waves (A_{max} and A_{min}) to travel the same spinning distance. The right hand side is also equal to the period of draw resonance oscillation. In stable situations, the required time for waves on the left side becomes greater than the allowed time on the right side, which makes sustained oscillation impossible. If the drawdown ratio (r) becomes larger than its critical value (r_c), the required time gets smaller than the allowed time so that steady sustained oscillation of draw resonance becomes possible.

Figs. 1 and 2 show the simulation results in x-t grid at the critical drawdown ratios for the two representative cases of PTT fluids spinning ($\varepsilon=0.015$, $\xi=0.1$ and $\varepsilon=0.015$, $\xi=0.7$). The transient curves of the spinline cross-sectional area (A), velocity (V), throughput (AV), and tension (F) are plotted at five spinline positions, when sufficient time has elapsed after an initial step disturbance introduced at take-up velocity. The node 1 was chosen instead of the spinneret because the solutions except for tension data are constant at the spinneret due to boundary conditions at x=0.

As the results show, the draw resonance criterion is exactly satisfied in the PTT spinning for both extension thickening (Fig. 1) and extension thinning (Fig. 2) cases. This is actually, however, not surprising, because the criterion was derived based on the comparison of the traveling times of kinematic waves on the spinline, which is independent of the constitutive equation. In other words, the draw resonance in spinning is the result of dynamic interaction within the governing equations of the hyperbolic system, i.e., a hydrodynamic phenomenon, and is not

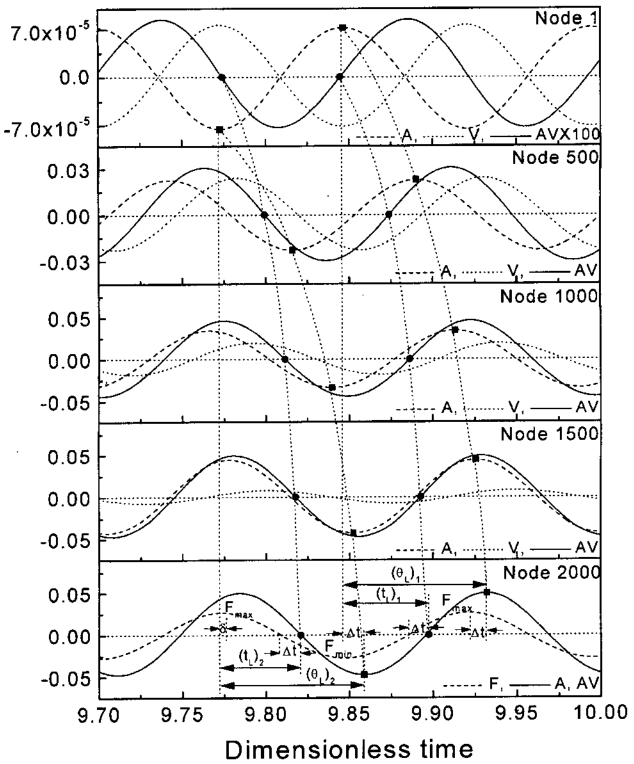

Fig. 1. Transient curves of spinline variables at five different spatial positions of spinline, when $\varepsilon = 0.015, \xi = 0.1$ $De_o = 0.002$, $St = 0.1$, $k = 5.8$, $\theta_a = 0.543$, $v_y = 1.66$ and $r = r_c = 50$.

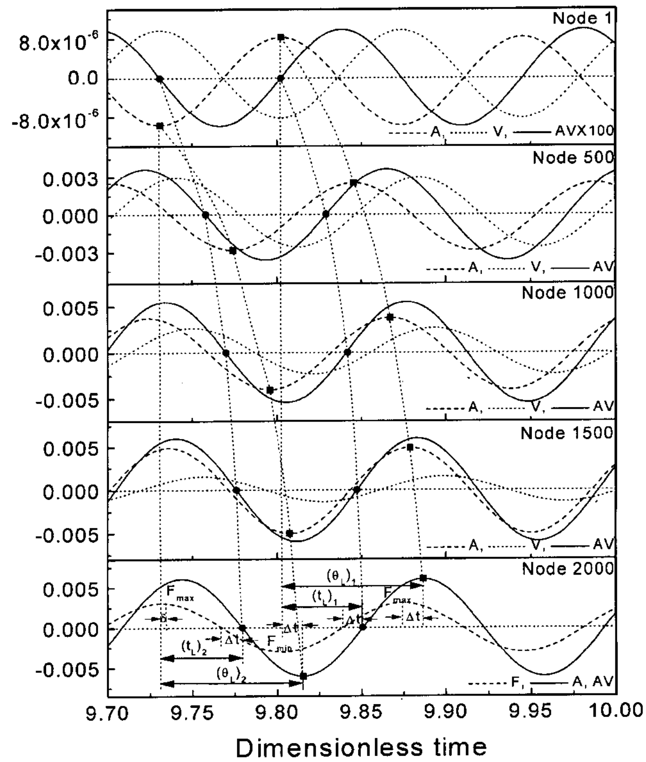

Fig. 2. Transient curves of spinline variables at five different spatial positions of spinline, when $\varepsilon = 0.015$, $\xi = 0.7$, $De_o = 0.002$, $St = 0.1$, $k = 5.8$, $\theta_a = 0.543$, $v_y = 14.12$ and $r = r_c = 50$.

caused by the fluid viscoelasticity, albeit being altered by it.

Second, we have studied the effect of spinline cooling on the spinning stability. As Jung *et al.* (1999) explained, the spinline cooling is stabilizing in Newtonian and Maxwell fluids spinning. Adopting the same methodology as in there, we performed a sensitivity analysis as regards how system disturbances are transmitted to the final take-up cross-sectional area in the PTT spinning case when the magnitude of spinline cooling changes. As shown in Tables 1 and 2 with Figs. 3 and 4, the spinline cooling has been found to have a stabilizing effect on spinline stability.

The crux of this stabilization study is whether the effect of disturbances on the system can be reduced by the spinline cooling. In other words, whether or not the sensitivity of the spinline take-up cross-sectional area to the system disturbances is decreased by the spinline cooling, is the question. (Kase and Araki, 1982)

As Jung *et al.* (1999) explained, there are two consecutive links between the system disturbances and the spinline take-up cross-sectional area: the first link is from the disturbances to the spinline tension and the second from the spinline tension to the spinline cross-sectional area. It was found that the first link is dominant in that the tension sensitivity in the first link is overriding the second sensitivity. The key to this sensitivity study is thus, whether or not spinline cooling decreases this tension sensitivity in the first link. As Tables 1 and 2 with Figs. 3 and 4 show, the tension sensitivity always decreases with spinline cooling

Table 1. Sensitivity of each PTT fluids of spinline tension and spinline cross-sectional area at take-up to a disturbance (5%) in take-up velocity for nonisothermal spinning process ($r = 60$, $De_o = 0.001$, $St = 0.1$, $k = 5.8$, $\theta_a = 0.543$).

(a) Extension thickening behavior ($\varepsilon = 0.015$, $\xi = 0.1$)

v_y		F	$\lvert\Delta \ln F\rvert$	$\frac{\lvert\Delta \ln a_L\rvert}{\lvert\Delta \ln F\rvert}$	$\lvert\Delta \ln a_L\rvert$
0	Unstable	5.0340	0.08079	1.5175	0.1226
5	Stable	6.0116	0.06976	1.6270	0.1135
10	Stable	6.5070	0.06312	1.6904	0.1067

(b) Extension thinning behavior ($\varepsilon = 0.015$, $\xi = 0.7$)

v_y		F	$\lvert\Delta \ln F\rvert$	$\frac{\lvert\Delta \ln a_L\rvert}{\lvert\Delta \ln F\rvert}$	$\lvert\Delta \ln a_L\rvert$
0	Unstable	4.6358	0.07231	1.5862	0.1147
10	Unstable	5.9063	0.05736	1.8026	0.1034
20	Stable	6.4539	0.05141	1.9004	0.0977

Table 2. Sensitivity of PTT fluid ($\varepsilon = 0.015, \xi = 0.1$) of spinline tension and spinline cross-sectional area at take-up when (a) a disturbance in cooling air velocity is introduced and (b) a disturbance in cooling air temperature is introduced ($r = 60$, $De_o = 0.001$, $St = 0.1$, $k = 5.8$, $\theta_a = 0.543$).

(a) Cooling air velocity (a 5% decrease)

v_y		F	$\lvert\Delta \ln F\rvert$	$\frac{\lvert\Delta \ln a_L\rvert}{\lvert\Delta \ln F\rvert}$	$\lvert\Delta \ln a_L\rvert$
2	Unstable	5.6545	0.004001	2.3459	0.009386
5	Stable	6.1293	0.003890	2.2139	0.008612
10	Stable	6.6415	0.003885	2.0373	0.007915

(b) Cooling air temperature (a 5% increase)

v_y		F	$\lvert\Delta \ln F\rvert$	$\frac{\lvert\Delta \ln a_L\rvert}{\lvert\Delta \ln F\rvert}$	$\lvert\Delta \ln a_L\rvert$
0	Unstable	5.1649	0.02376	1.7138	0.04072
5	Stable	6.2205	0.01820	1.9610	0.03569
10	Stable	6.7611	0.01531	2.1195	0.03245

in this PTT fluids spinning as was the case in Newtonian and Maxwell fluids.

One important thing to notice here is the fact that as the tension sensitivity is reduced by the spinline cooling, the spinline tension itself increases. That is, the increasing spinline tension and decreasing tension sensitivity go together to increase the spinning stability. Actually this is the very concept behind the success of the ingenious device called draw resonance eliminator developed by Lucchesi *et al.* (1985) for film casting process. By blowing a maximum amount of cooling air onto the film coming out of the die, they successfully maximized tension and minimized the tension sensitivity so as to increase the film casting stability and its productivity to a maximum extent.

Out of many possible disturbances to the system, three different kinds of disturbances were tried in this sensitivity study: disturbances in take-up velocity, cooling air velocity and cooling temperature. As Table 1 and 2 shows, all three cases exhibit the stabilizing effect of the spinline cooling (with decreased tension sensitivity and increased tension) while the second sensitivity can either increase or decrease with the spinline cooling. As mentioned before, though, this second sensitivity doesn't change the whole picture because the first sensitivity is always dominant in determining the spinning stability.

Finally, the effect of fluid viscoelasticity on the spinning dynamics has also been studied in this PTT fluid case. As Jung *et al.* (2000b) show, the viscoelasticity represented by Deborah number has either stabilizing or destabilizing effect according as the spinning fluid is extension thickening ($\xi = 0.1$) or extension thinning ($\xi = 0.7$), respectively. This seemingly opposite finding is due to the contrasting behavior of the extensional viscosity on the spinline depending on the value of ξ, as shown in Fig. 5. In other words, extension thickening fluids exhibit the spinline tension getting larger with increased viscoelasticity whereas extension thinning fluids does the opposite, which then leads to decreased and increased tension sensitivity, respectively. The dichotomy in extensional behavior of viscoelastic fluids is nothing new but has been well documented. (Chang and Denn, 1980; Jung and Hyun, 1999; Jung *et al.*, 2000b)

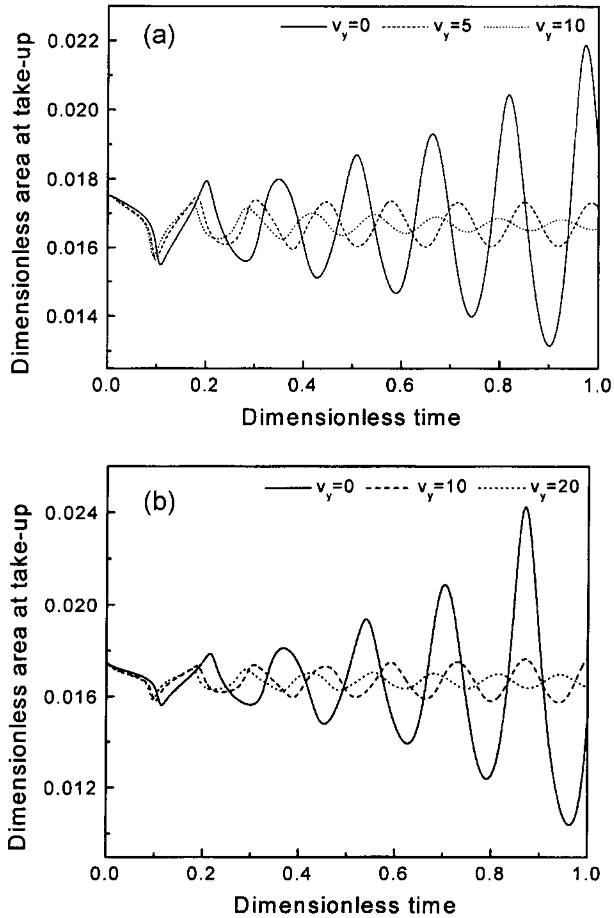

Fig. 3. Transient behavior of spinline cross-sectional area at take-up for (a) extension thickening PTT fluid ($\varepsilon = 0.015$, $\xi = 0.1$), and (b) extension thinning PTT fluid ($\varepsilon = 0.015$, $\xi = 0.7$), when a disturbance in take-up velocity is introduced (a 5% increase).

In this study of PTT fluids spinning, the parameter, ξ, thus plays the key role in deciding the dichotomous behavior of extension deformation. Fig. 6 shows how two different kinds of stability behavior are exhibited by PTT fluids ($\xi \approx 0.485$ is the demarcating value for these two different behaviors). As with the case of White-Metzner fluids (Jung and Hyun, 1999), increasing elasticity imbedded

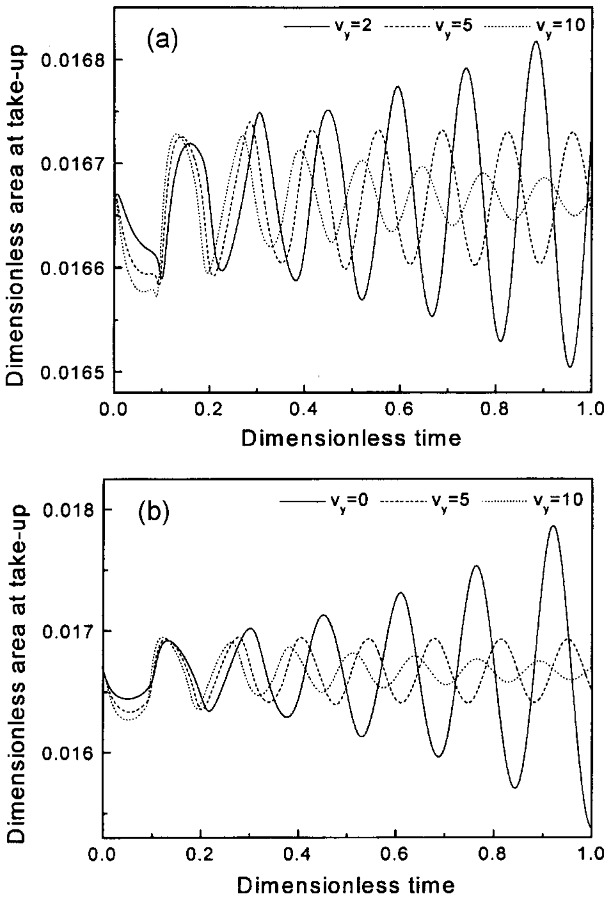

Fig. 4. Transient behavior of spinline cross-sectional area at take-up for PTT fluid ($\varepsilon = 0.015$, $\xi = 0.1$), when (a) a disturbance in cooling air velocity is introduced (a 5% decrease) and (b) a disturbance in cooling air temperature is introduced (a 5% increase).

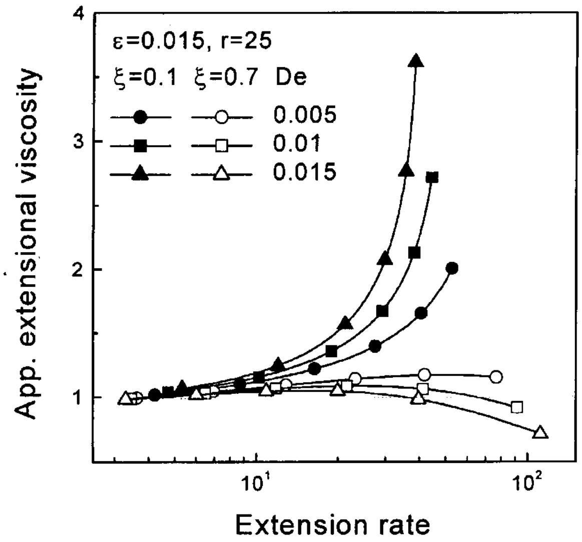

Fig. 5. Dichotomous behavior of the spinline extensional viscosity of PTT fluids spinning.

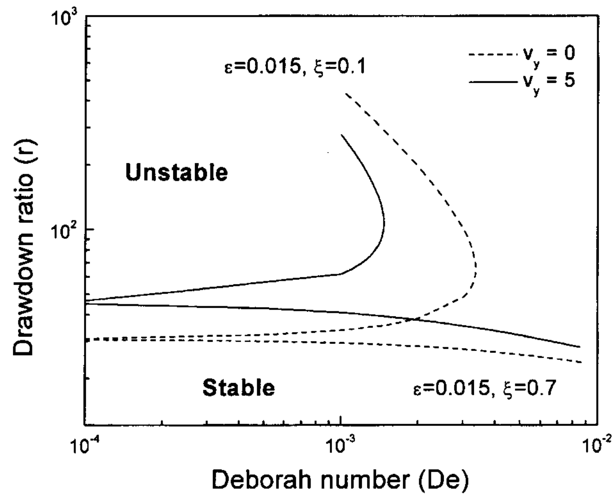

Fig. 6. Stability diagrams of PTT fluids spinning for extension thickening ($\xi = 0.1$) and extension thinning ($\xi = 0.7$) with two different cooling air conditions ($v_y = 0$, $v_y = 5$)

in Deborah number stabilizes spinning for extension thickening fluids whereas it destabilizes for extension thinning fluids.

There are still other ways to change Deborah number than viscoelasticity, i.e., changing the extrusion velocity of the spinning or changing the spinning distance. In these cases, however, increasing De does not necessarily produce consistent stability results as also mentioned by Jung *et al.* (2000b). The reason for these interesting results lies in the fact that increasing extrusion velocity (or spinning distance) inevitably introduces inertia effect (or gravity effect) into the system, and then, the governing equations of the system should have all the secondary forces including the inertia and gravity terms. Detailed results along this line will be presented elsewhere while in this study it suffices to say that the Deborah number is not always changed by the fluid viscoelasticity alone, but also by other process conditions like the extrusion velocity and the spinning distance. But in these cases the effect of Deborah number on the spinning stability varies depending on what speed ranges we are operating in, because the inertia effect becomes dominant in high speed spinning.

3. Conclusions

Fundamental aspects of the melt spinning dynamics of PTT fluids are presented in this study with focus on three subjects: draw resonance criterion, the stabilizing effect of spinline cooling and the fluid viscoelasticity effect on spinning stability. The draw resonance criterion based on interaction of the traveling times of kinematic waves on spinline was confirmed to hold in this PTT case. The stabilizing effect of the spinline cooling was confirmed performing sensitivity analysis which shows increased cooling leads to increased spinline tension and reduced tension sensitivity to system disturbances. The fluid viscoelasticity shows both stabilizing and destabilizing effects depending on whether fluids are extension thickening or thinning as was the case of White-Metzner fluids. Changing Deborah number through changing extrusion velocity can have either stabilizing or destabilizing effect on spinning stability depending on the spinning speed ranges because in this case inertia effect becomes important in high speed spinning, whereas it is negligible in low speed spinning.

Acknowledgments

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