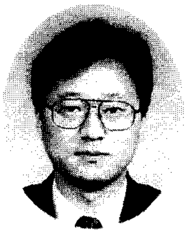
Estimation of the Fundamental Period for Residential Buildings with Shear-Wall System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ABSTRACT

This study focused on evaluating the reliability of code formulas such as those of the current Korean Building Code(KBC 1988), UBC 1997, NBCC 1995, and BSLJ 1994 for estimating the fundamental period of RC apartment buildings with shear-wall dominant systems, representative of typical residential buildings in Korea. For this purpose, full-scale measurements were carried out on fifty RC apartment buildings, and these results were compared to those obtained by code formulas and also by dynamic analysis. Although these code formulas are based on the measured periods of buildings during various earthquakes and building period varies with the amplitude of structural deflection or strain level, ambient surveys should provide an effective tool for experimentally verifying the design period to the completed building. This comparison shows that comparatively large errors are likely to occure when the code formula of KBC 1988 is used, and all the other code formulas are not sufficient to estimate the fundamental period of apartment buildings with shear-wall dominant systems. An improved formula is proposed by regression analysis on the basis of the measured period data. The proposal is for the servicebility stress level, but it can also be applied for seismic code in the regions of low seismicity similar to Korea.

Keywords : Fundamental period, Code formulas, Apartment building, full-scale measurements

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1. Introduction

Most Semi-empirical building codes including the current Korean Building Code(KBC 1988)(1) use the building period directly proportion to the magnitude of force which should be sustained by buildings at a specific stress level and provide the empirical formulas to determine the lower bound fundamental period in order to establish the proper design force level. However, such codes have not settled on a uniform method to determine the period, because the required design force level and characteristics of buildings constructed in each region are different. To determine the design base-shear for seismic design, the formulas of the period specified in the current KBC are derived from those of the 1988 Uniform Building Code(UBC 1988)(2) which were based on the measured periods of buildings from strong motion records during the 1971 San Fernando earthquake. In case of the Apartment building with the Shear-wall Dominant System(ASDS), it has long been realized that comparatively large errors are likely to occure when this formula is used, because it gives a period much shorter in the longitudinal direction and longer in the transverse direction than that were obtained from dynamic analysis.

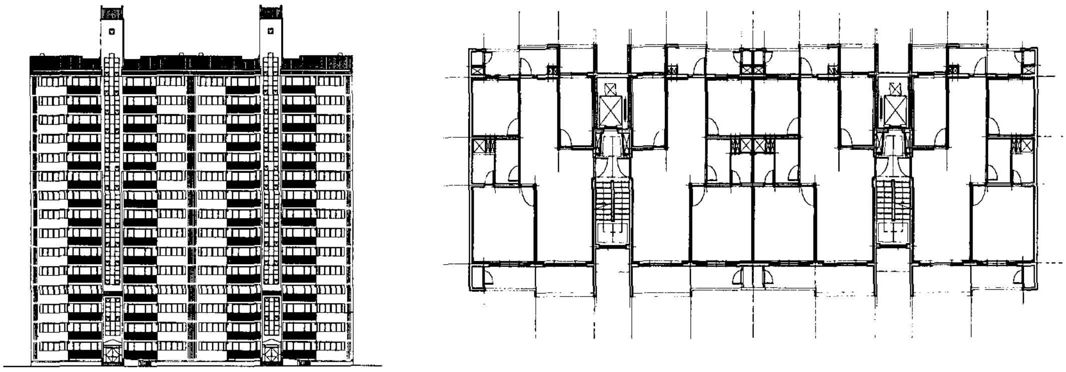
Approximately over 85% of total housing stocks constructed today in Korea are ASDS. But, in spite of its common use, little research has been done concerning this problem, and there has not been a solution to correct this formula as yet. Therefore, more research must be done in this field. Futhermore, since the current trend for these buildings which are to be more slender and lightweight has resulted in problems associated with their dynamic behaviour, accurate asessment of the fundamental period of vibration is also important for serviceability condition as well as for seismic condition.

The objective of this study is to provide updated information on the period of ASDS and to evaluate the reliability of the period formula of KBC 1988 and the availability of the period formulas such as those of UBC 1997(3), NBCC 1995(4), and BSLJ 1994(5). For this purpose, full-scale measurements were carried out for fifty RC ASDS, and these results were compared to those computed by code formulas. Although building period varies with the amplitude of structural deflection or strain level, ambient surveys should provide an effective tool for experimentally verifying the design period with the completed building.(6,7,8) This comparison shows that these code formulas for estimating the fundamental period of RC ASDS are grossly inadequate. Subsequently, an improved empirical formula is proposed from a analysis with the measured period data.

2. Description of the Buildings

ASDS are representative of residential buildings constructed in Korea. These buildings are almost reinforced concrete structures consisting of walls and regularly shaped flat plate slabs without columns and beams, and a centrally located rectangular core or cores spaced by 2 housing units(Fig. 1). In general, the thickness of walls and slabs are almost equal(about 200mm), and the walls in units and cores which are the primary lateral force resisting elements are continuous throughout the height of the building.

The measured buildings are fifty apartment with 10 to 25 stories having various size of plan shape (3.0~7.0). The story height is about 2.6m in all stories. Each building has a mat or a pile foundation. Details regarding the



(a) Elevation

(b) Typical floor plan

Fig. 1 Apartment building with cores spaced by 2 housing units

building plan dimension, the number of story and height, and the ratio of the sum of the length of walls aligned in the direction the periods were measured to the plan area of a typical floor are presented in Table 1.

3. Instrumentation and Response Measurements

The acceleration data for fifty apartment buildings was recorded from March 1996 to April 1997. Buildings were completed and unoccupied during the measurements. The dynamic responses were measured using two couples of accelerometers perpendicular to each other and accelerometers set on the highest floor of each building. Some references [6,7,9,10] give details of the measurement procedures.

The accelerometers selected are B&K Model 8318 with 1.0kHz natural frequency, 3.16 V/g sensitivity, and very low cross axis sensitivity. Because the range of acceleration is within a few milli-g, the application of some degree of signal amplification to the basic output from the accelerometer is required. Also, in order to obtain a clean signal, whereby background noise is eliminated, it is necessary to filter out the higher frequency background component. The amplification and filtering functions are carried out by a purpose-built unit containing a sixth order 12.5Hz low-pass filter and an amplifier with a fixed gain of 100. Conditioned signals could be stored on tape via a Sony PC204ADAT tape recorder (4- channels 16-bit resolution, 80dB or more dynamic range, 78dB or more S/N ratio, 0.02 % or less distortion within the band width) and digitized using data transfer system PCIF 200 A (16-bit resolution, programmable global gains ±20V, 192 Kwords/second conversion rate).

4. Identification of Natural Frequencies

In order to identify natural frequencies, the Averaged Normalized Power Spectrum (ANPS) was computed with the NPS program which was developed by the Korea National Housing Corporation.

NPS employs the maximum entropy method[11] which gives a very high resolution for short time series in the low-frequency domain.

Table 1 Period data for apartment building

Building number	Number of stories	Dimesion,m			L_w	Measured period,sec		Predicted period,sec	
		Height	Length	Width		Longitudinal	Transverse	Longitudinal	Transverse
1	15	40.0	38.98	11.26	0.15/0.28	1.92	0.71	0.58(2.42)	1.07(0.81)
2	15	40.0	27.22	12.83	0.15/0.26	N.A	1.08	0.69	1.01
3	20	53.5	30.94	12.38	0.13/0.23	1.89	1.19	0.87(2.16)	1.37(1.29)
4	20	53.5	31.66	12.02	0.14/0.21	1.90	1.44	0.86(2.36)	1.39(1.59)
5	20	53.5	30.94	10.88	0.14/0.27	1.93	N.A	0.87	1.46
6	15	40.0	49.22	11.61	0.07/0.20	N.A	1.27	0.51	1.06
7	15	40.0	27.22	12.83	0.12/0.26	2.22	N.A	0.69	1.01
8	15	40.0	56.28	12.47	0.13/0.25	1.86	1.16	0.48	1.02
9	15	40.0	28.14	12.47	0.13/0.27	1.66	1.09	0.68	1.02
10	15	40.0	34.46	12.47	0.13/0.26	1.93	N.A	0.61(2.06)	1.02(0.90)
11	20	53.5	42.20	12.14	0.13/0.24	2.11	N.A	0.74(3.09)	1.38(1.35)
12	15	40.0	38.98	11.28	0.15/0.25	1.63	N.A	0.58	1.07
13	15	40.0	27.22	12.83	0.12/0.28	2.05	0.91	0.69(2.58)	1.01(1.09)
14	20	53.5	41.80	11.18	0.16/0.23	1.82	1.16	0.74(2.06)	1.44(1.21)
15	20	53.5	37.20	12.36	0.16/0.21	1.95	N.A	0.79(2.66)	1.37(1.05)
16	20	53.5	45.40	11.94	0.16/0.21	1.88	N.A	0.71(2.28)	1.39(1.15)
17	20	53.5	45.40	11.94	0.17/0.21	1.82	1.50	0.71	1.39
18	20	53.5	32.00	11.94	0.17/0.22	1.76	N.A	0.85(2.60)	1.39(1.14)
19	15	40.0	51.90	10.36	0.15/0.29	1.91	0.90	0.50(2.36)	1.12(1.10)
20	15	40.0	34.60	10.36	0.15/0.30	N.A	0.86	0.61(1.89)	1.12(0.91)
21	15	40.0	61.80	11.80	0.15/0.25	1.89	1.28	0.46	1.05
22	15	40.0	41.60	11.80	0.14/0.26	N.A	0.99	0.56	1.05
23	15	40.0	53.40	10.80	0.13/0.28	N.A	1.16	0.49	1.10
24	15	40.0	36.60	11.90	0.15/0.27	1.92	1.27	0.60	1.04
25	15	40.0	35.60	10.80	0.17/0.29	1.79	N.A	0.60	1.10
26	15	40.0	42.90	11.00	0.17/0.24	1.65	N.A	0.55	1.09
27	18	48.1	43.40	11.62	0.11/0.24	1.81	N.A	0.66(2.74)	1.27(1.05)
28	20	53.5	34.64	10.73	0.16/0.28	1.85	1.17	0.82	1.47
29	18	48.1	34.60	12.50	0.15/0.23	1.88	1.23	0.74	1.22
30	20	53.0	53.60	11.40	0.14/0.19	1.88	1.12	0.65(2.85)	1.41(1.22)
31	20	53.5	29.44	11.40	0.14/0.20	1.83	N.A	0.89(2.75)	1.43(1.53)
32	20	53.5	35.48	11.40	0.15/0.20	1.92	1.31	0.81(2.96)	1.43(1.44)
33	20	53.5	52.50	10.92	0.16/0.28	1.79	1.06	0.66(2.88)	1.46(1.32)
34	22	58.9	52.50	10.92	0.16/0.29	1.89	1.04	0.73	1.60
35	25	67.0	43.40	12.12	0.12/0.23	2.33	1.79	0.92(3.23)	1.73(1.93)
36	25	67.0	35.00	10.92	0.15/0.30	N.A	1.33	1.02	1.82
37	25	67.9	38.10	12.30	0.12/0.24	2.56	1.39	0.99(3.65)	1.74(1.41)
38	25	67.9	20.80	11.50	0.16/0.28	2.04	1.59	1.34(2.67)	1.80(1.73)
39	25	67.9	27.30	12.00	0.11/0.25	2.17	1.61	1.17(3.06)	1.76(1.87)
40	25	68.0	63.90	11.50	0.10/0.23	2.50	N.A	0.77(3.66)	1.80(2.20)
41	25	68.0	51.84	12.60	0.13/0.21	2.13	1.69	0.85(2.68)	1.72(1.90)
42	19	51.1	36.80	11.20	0.16/0.28	1.89	N.A	0.76(2.09)	1.37(1.20)
43	20	53.9	36.80	11.20	0.16/0.26	1.79	1.25	0.80(2.12)	1.45(1.42)
44	15	40.0	18.30	10.70	0.11/0.30	1.69	0.90	0.84(1.88)	1.10(0.92)
45	20	55.6	35.60	11.40	0.19/0.17	1.79	N.A	0.84(2.65)	1.48(1.18)
46	20	55.6	53.40	11.40	0.16/0.17	1.72	1.25	0.68(2.32)	1.48(1.29)
47	20	55.6	41.60	12.00	0.14/0.25	1.82	1.27	0.78(2.63)	1.44(1.29)
48	20	54.0	31.80	10.00	0.13/0.28	N.A	1.25	0.86(2.06)	1.54(1.30)
49	20	54.0	51.20	11.60	0.11/0.22	1.96	1.39	0.68(3.61)	1.43(1.68)
50	20	54.0	50.40	12.30	0.09/0.27	2.13	1.20	0.68(3.70	1.39(1.53)

Note : L_w implies the wall length, in meters, per unit plan area, in square meters, in the longitudinal/transverse direction : N.A indicates date not available : Predicted period implies the period given by the code formula, $0.09H/\sqrt{B}$, and () indicates the period obtained from computer based ananlysis

ANPS was obtained by first normalizing the power spectral density amplitudes of each record with respect to the sum of all the amplitudes within the frequency range of interest of the record, and then averaging all the normalized spectral densities of selected sets of records. NPS sampled each record at a sampling frequency of 46.7Hz in conjunction with the earlier low-pass filtering at 12.5Hz and filtered by the high-pass filter at 0.1Hz in order to remove any D.C. offset from record. The peaks in the ANPS represented the natural frequencies for various modes of vibration. Since there were pairs of sensors oriented to both the longitudinal and transverse directions, translational and torsional motions could be enhanced by adding and subtracting the signals from each pair of sensors in the time domain. But in this study, only the lateral behavior of buildings is of interest. In terms of dynamic response of the building, the fundamental mode of vibration is usually the most important. The fundamental frequencies for tested buildings range from 0.43Hz (2.33sec) to 0.62Hz (1.63sec) for the longitudinal direction, and from 0.56Hz (1.79sec) to 1.16Hz (0.86sec) for the transverse direction. The fundamental frequencies of vibration for tested buildings are summarized in Table 1.

5. Comparison with Code-Prescribed Periods

The current Korean code(KBC 1988)(1) specifies that the fundamental period of a multistory RC ASDS to determine the design base shear for earthquake resistant design can be estimated by

$$T = 0.09H/\sqrt{B} \qquad (1)$$

H=the height of the building in meters,

B=the full plan dimension of the building, in meters, in the direction parallel to the applied forces without regard to shear wall dimensions. This formula is derived from the U.S. building code, UBC 19882, because Korea has insufficient experience for earthquakes. To show its relative accuracy, the measured periods are compared to the periods obtained from formula (1) in Fig. 2(a). It is observed for a majority of buildings to which formula (1) gives a period much shorter in the longitudinal direction and longer in the transverse direction than the one measured in this study. In general, the period measured from an earthquake is longer than that measured from ambient vibration because of stiffness degradation. But considering the fact that the code formula is really not meant to provide the real period but to determine the lower bound fundamental period for finding the proper design force level and Korea is located in the regions of low seismicity, the periods from the code formula, to be conservative, should be less than the periods measured under ambient conditions. In this respect, the longer period from formula (1) in the transverse direction underestimates the seismic coefficient ; otherwise, the shorter period from formula (1) in the longitudinal direction overestimates the seismic coefficient. Plotted are in Fig. 2(b) the seismic coefficients based on the measured periods and the seismic coefficient spectrum, with A=0.12 for Zone 2; I=1.2 for Importance level 2; S=1.2 for soil profile type 2 with stiff soil coditions, where the soil depth exceeds 60 meters; R=3.5 for concrete bearing wall system with column type reinforcements at both ends of the wall. Fig. 2(b) illustrates this fact. In addition, the major problem in formula (1) is that the difference of stiffness due to shear-wall layouts in both directions is ignored. Fig. 2(a) always shows a period much longer in the longitudinal direction than in the transverse direction. It means that the stiffness of tested buildings is different from what the code would tend to imply. These observations clearly indicate that formula (1) is grossly inadequate and that it is difficult to estimate the fundamental period of RC ASDS only with simple variables such as the height or dimension of the building, which can not incorporate the stiffness of the building. The analogous result can be found in Fig. 3 which shows the relationship between the measured periods and the values given by various code empirical formulas such as those of the Building Standard Law in Japan of 19945, the Australian Wind Loading Code AS 1170.2 of 198912, and the Uniform Building Code of 19973.

The Canadian building code NBCC 1995(4) and the U.S. building codes UBC 1997(3) and SEAOC 1996(13) permit alternative formulas including the amounts of wall to estimate the fundamental period for the shear-wall buildings. The alternative formula of NBCC 1995 is of the form

$$T = 0.09H/\sqrt{D_s} \tag{2}$$

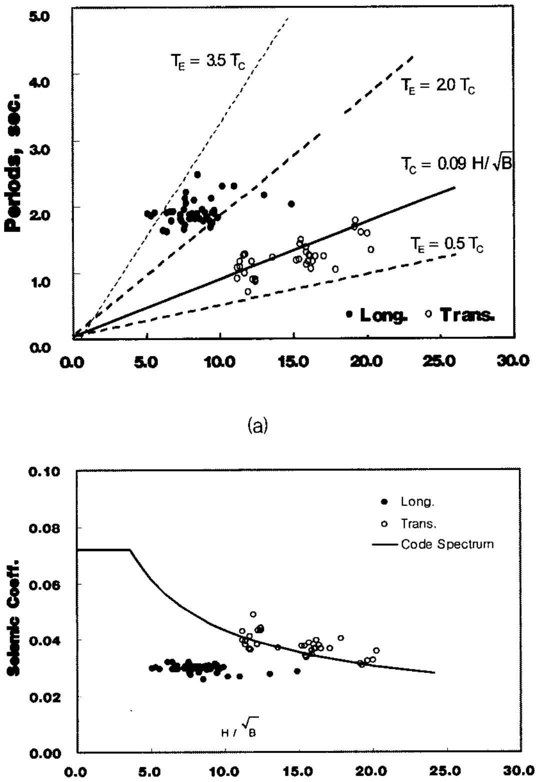
where D_s=length of wall or braced frame, in meters, which constitutes the main lateral load resisting system in the direction parallel to the applied forces. The alternative formula of UBC 1997 and SEAOC 1996 is of the form

$$T = 0.1/\sqrt{A_c}\, h^{3/4} \tag{3}$$

where A_c=the combined effective area, in square feet, of shear walls is defined as

$$A_c = \sum A_e[0.2 + (D_e/h_n)^2] \tag{4}$$

in which A_e=the minimum cross-sectional shear area, in square feet, of a shear wall; D_e=the length, in feet, of a shear wall in the direction parallel to the applied forces; h_n=the height of the building, in feet, above the base. The value of D_e/h_n in(4) should not exceed 0.9. The relationship between the periods obtained from these two formulas and the measured periods are plotted against each of the predictors in Fig. 4 and Fig. 5, respectively. From the observation of figures, for all the buildings, formulas (2) and (3) give a period much shorter, by two or three times, than the measured period and a seismic coefficient is much larger than the value based on the measured period. Since the poor correlation between the predictor and the measured period is also apparent, it is clear that formulas (2) and (3) can not incorporate the stiffness of the tested apartment buildings.



(a)

(b)

Fig. 2 Comparison of : (a) measured and code periods ; (b) KBC 1988 seismic coefficient from measured and code periods

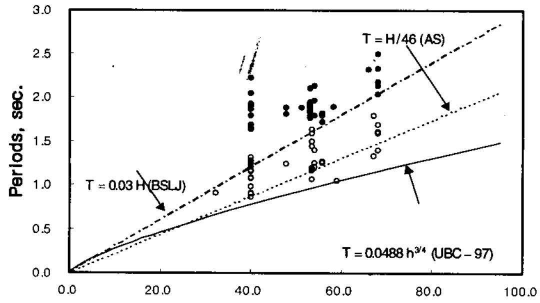


Fig. 3 Comparison of measured and valuious code periods

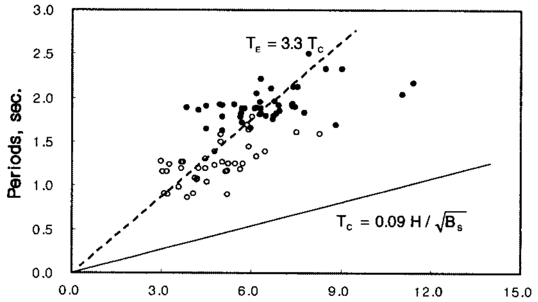


Fig. 4 Comparison of measured and NBCC(1995) code periods

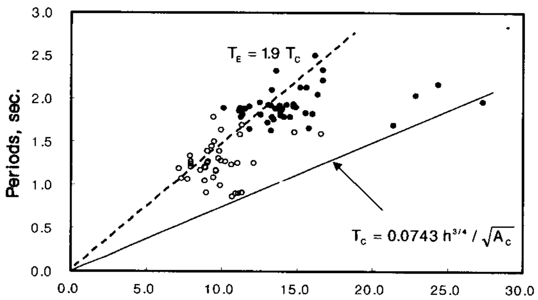


Fig. 5 Comparisons of measured and UBC(1997) code periods

It is therefore inappropriate to use these formulas to estimate the fundamental period of apartment buildings as well.

6. Simple Empirical Formula for Estimating the Funda-mental Period

To obtain a simple but more reliable formula for estimating the fundamental period of RC ASDS, regression analysis was carried out on the basis of the measured period data for 50 buildings(78 data points) listed in Table 1. The following regression model was used for these correlations :

$$T = C_1 \frac{1}{\sqrt{L_w}} H^{\beta} + C_2 \qquad (5)$$

where C_i=the constants determined by regression analysis; L_w=the values defined as the wall length per unit plan area; H=the height of the building in meters. This model was derived from the fundamental period of a uniform cantilever beam, considering flexural and shear deformations. Details this model obtained are found in many books and papers(14,15,16). L_w substitutes the wall length for the shear area of the wall, considering the thickness of the walls are almost equal and not determined accurately in the preliminary design stage. For regression with ease, the above relation (5) was transformed to :

$$\log T = \log C_1 + \beta \log H - \frac{1}{2} \log L_w + \log C_2 \qquad (6)$$

Using the regression method, the following best-fit line was determined :

$$T = b_1 \frac{1}{\sqrt{L_w}} H^{\beta_1} + b_2 \qquad (7)$$

Table 2 Result of regression analysis

Regression analysis type	Period Formula		
	Best-fit	Correlation	Standard error
Unconstrained	$T = 0.33H^{0.25}/\sqrt{L_w} - 0.5$	0.902	0.178
Constrained with β_1=0.2	$T = 0.4H^{0.2}/\sqrt{L_w} - 0.5$	0.900	0.180
Constrained with β_1=0.2	$T = 0.27H^{0.3}/\sqrt{L_w} - 0.45$	0.890	0.182

Table 3 Calibration of computer-based model of building No.37

Model complexity	Natural period, second	
	Longitudinal	Transverse
1. Bared frame with Fixed base	3.05	1.83
2. 1 + Soil spring	3.27	1.96
3. 2 + Opening walls and lintel beams	2.72	1.68
4. 3 + Others	2.36	1.68
Experimental	2.17	1.61

in which b_1, b_2, β_1 are unbiased estimaters of C_1, C_2, β, respectively. For regression curve fits, error was assessed using the standard error and correlation coefficient. These tools are not perfect, but they do give helpful evaluation of the performance of the curve fit. After relation (7) has been determined, constrained regression analysis was executed in order to simplify relation (7) with β_1 fixed at 0.2, to which β_1 in (7) is rounded off. This formula is proposed for estimating the fundamental period of RC ASDS because it is simpler, although it has a slightly larger standard error. The results of these regression analyses are summarized in Table 2, and the formula obtained from the second regression analysis is presented in Fig. 6 together with the measured periods shown in circles. Although the proposal is for serviceability stress levels, it can be applied for seismic code in the regions of low seismicity, considering the fact that the code formula must be consulted to determine the lower bound fundamental period in order to establish the proper design force level. It is verified in the following clause that this proposal is not overly conservative for a low seismicity region like Korea.

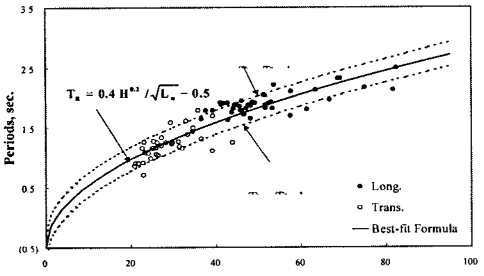


Fig. 6 Proposed period formula

7. Comparison with Analytical Predictions

If a dynamic analysis is used in the design process, building periods are normally computed with a mathematical model rather than from the code formulas. Unless care is exercised on the part of the designer to include all the effective stiffness and mass of the structural and non-structural elements, the computed period could result in a less conservative design. It is theoretical idealization which is generally responsible for the major errors in this final result. Therefore, to obtain more accurate predictions, a better understanding of the overall behaviour of buildings is necessary, and this will be achieved only by comparing theoretical predictions with experimental measurements. The natural frequencies are often used as one of the basic criteria for more accurate structural idealization. For the serviceability conditions, it should be possible to calibrate the mathematical model to obtain a good correlation with the measured values for any one building. This type of study has been completed for some buildings. Table 3 presents the results of a study on one building. Vertical soil stiffness was modeled by providing an additional story, and lateral and torsional soil springs were added to the base level. Such a model is employed by several researchers(6,9).

The difference between the real period of the building from the period obtained from the computer based analysis, using ETABS v6.13, is presented in Table 1. It is shown by comparison that the ratio of measured periods to those obtained from the computer based method is about 0.75 for the longitudinal direction and about 0.9 for the transverse direction, which means that the elements providing additional stiffness were much more in the longitudinal direction than in the transverse direction. It is generally known that the earthquake period is longer than the ambient period because of cracks and a loss of bond between the structural members and non-structural members, and the period of the building during an earthquake approaches the natural period computed from a theoretical model of the pure structural system, neglecting all non-structural elements. In this respect, the proposed formula is not overly conservative and can give a period in reasonable agreement with what the code would tend to imply, considering the averaged value of the ratio of the periods obtained from the proposed formula to those obtained from the computer based analysis is 0.75.

8. Conclusions

In order to provide the updated information on the fundamental period of RC ASDS and to evaluate the reliability of the period formulas such as those of KBC 1988, UBC 1997, NBCC 1995, and BSLJ 1994, full-scale measurements were carried out on fifty RC ASDS and measured periods were compared to those obtained from these code formulas. As was expected, the measured periods in the longitudinal direction were longer than those in the transverse direction, which shows that the stiffness of such buildings is inversely propotional to what KBC would tend to imply. Also, any code formulas examined in this study are not sufficient to estimate the fundamental period of RC ASDS, because the poor correlations between the predictors in these code formulas and the measured periods are apparent.

An improved formula was proposed by regression analysis on the basis of the ambient vibration measurements. This formula is for the servicebility stress level, but can also be applied for seismic code in the regions of low seismicity similar to Korea.

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