論文

# Elastic Shear Buckling of Transversely Stiffened Orthotropic Web Plates

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# 수직보강된 직교이방성 복부판의 전단탄성좌굴

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## 초 록

본 연구는 면내 전단력을 받으며 수직보강재가 설치된 직교이방성판의 탄성좌굴거동해석에 관한 것으로서, 판의 네 변은 단순지지 되었다고 가정하였으며, 등간격으로 배치된 보강재는 비틀림강성을 무시한 보요소로 간주하였다. 수직보강된 직교이방성판의 좌굴해석식은 Rayleigh-Ritz법을 사용하여 유도하였다. 유도된 좌굴해석식을 사용하여 수직보강된 직교이방성판의 좌굴응력의 한계값을 구해 그래프로 제시하였으며, 이 결과를 사용하여 수직보강재가 한 개 또는 두 개 설치된 판의 전단좌굴응력이 최대가 되도록 하기 위해 보강재에 요구되는 휨강성을 구하여 그래프로 제시하였다.

## ABSTRACT

In this paper an analytical investigation pertaining to the elastic shear buckling behavior of transversely stiffened orthotropic plate under in-plane shear forces is presented. All edges of plate are assumed to be simply supported and the evenly placed stiffener is considered as a beam element neglecting its torsional rigidity. For the solution of the problem Rayleigh-Ritz method is employed. Using the derived equation, the limit of buckling stress of transversely stiffened plate is suggested as a graphical form. Based on the limit of buckling stress of stiffened plate, graphical form of results for finding the required stiffener rigidity is presented when one and two stiffeners are located, respectively.

## 1. INTRODUCTION

Advanced materials, mainly fiber reinforced plastic (FRP) composites, have broadly being used in many engineering fields due to their attractive physical and mechanical properties.

Although their advantageous properties, FRP structures have been around for forty years, mostly as showcases with limited applications in civil engineering construction. One of the reasons inhibiting the general use of composites in civil engineering construction is the lack of design

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criteria of structural members [1].

In recent years, numerous theoretical and experimental researches have been performed for FRP structural members to establish design criteria.

Most of structural members consist of plate elements. In order to design such a thin walled member composed of plate elements, a knowledge of the critical load is always desirable.

In the design of flexural members, one of the stability problem to be considered is the web buckling. In general, web may be reinforced by longitudinal and transverse stiffeners to obtain the higher buckling stress. Thus the buckling behavior of stiffened plate needs to be investigated to establish design criteria for the stiffened web plate, and proper location and size of stiffener.

In this paper, the analytical solution for the buckling behavior of transversely stiffened orthotropic web plate subjecting the in-plane shear forces is presented. All edges of the plate are assumed to be simply supported and the stiffener is considered as a narrow rectangular section beam element. Since the cross section of beam is assumed to be narrow rectangular section, the torsional rigidity of stiffener is neglected. For the equation of buckling analysis Rayleigh-Ritz method is employed.

Using the derived equation, the upper limit of the buckling stress of transversely stiffened plate is predicted approximately and the required stiffener rigidity for which the buckling stress of plate becomes the maximum is suggested as a graphical form when one or two stiffeners are located.

## 2. THEORETICAL DERIVATIONS

### 2.1 Basic Equations

In this paper, we solved the buckling problem of transversely stiffened orthotropic plate under in-plane shear forces as shown in Fig. 1 by employing the Rayleigh-Ritz method. The method adopted in this study was originally applied by Way (1936) [2] to the isotropic web plates.

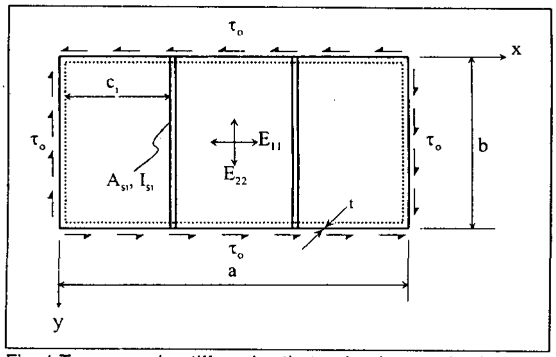


Fig. 1 Transversely stiffened orthotropic plate under in-plane shear forces

In Fig. 1, $E_{11}$ and $E_{22}$ are the elastic modulus in 1-1 and 2-2 material property directions, respectively. $A_s$ and $I_s$ are the cross sectional area and the moment of inertia of stiffener with respect to the axis along the mid-plane of web plate, respectively. The transverse stiffeners are attached equally spaced along the x-axis of plate and $c_i$ represents the distance of i-th stiffener from the origin (y-axis).

In order to derive the buckling equation, the following assumptions are introduced following Way (1936) in addition to the assumptions for the classical orthotropic plate theory [3].

(1) The deflection of stiffener is equal to the out-of-plane deflection of plate at the common junction of plate and stiffener.

(2) The torsional rigidity of stiffener is negligibly small.

(3) The stiffener is attached symmetrically at the mid-plane of plate.

Considering the transversely stiffened orthotropic plate as shown in Fig. 1, the strain energy of bending of plate and stiffener is given by, respectively [3, 4, 5]:

$$U_P = \frac{1}{2}\int_0^a\int_0^b\{D_{11}\left(\frac{\partial^2 w_P}{\partial x^2}\right)^2$$

$$+ 2\nu_{21}D_{11}(\frac{\partial^2 w_p}{\partial x^2})(\frac{\partial^2 w_p}{\partial y^2}) + D_{22}(\frac{\partial^2 w_p}{\partial y^2})^2 + 4D_{66}(\frac{\partial^2 w_p}{\partial x \partial y})^2\}dxdy \quad (1)$$

$$U_s = \sum_{i=1}^{N_s} \frac{E_{11s}I_s}{2} \int_0^b (\frac{\partial^2 w_{si}}{\partial y^2})^2 dy \quad (2)$$

In Eqs. (1) and (2) $w_p$ and $w_{si}$ represent the out-of-plane deflection of plate and the deflection of stiffener, respectively. $N_s$ in Eq. (2) is a number of stiffeners. $D_{11}$ and $D_{22}$ are the flexural rigidities in 1-1 and 2-2 material property directions, respectively, and $D_{66}$ is the twisting rigidity of plate in 1-2 direction. They are defined as follows:

$$D_{11} = \frac{E_{11}t^3}{12(1-\nu_{12}\nu_{21})} \quad (3a)$$

$$D_{22} = \frac{E_{22}t^3}{12(1-\nu_{12}\nu_{21})} \quad (3b)$$

$$D_{66} = \frac{G_{12}t^3}{12} \quad (3c)$$

The work done by in-plane shear forces on the plate (T) may be given:

$$T = -\tau_0 t \int_0^a \int_0^b (\frac{\partial w_p}{\partial x})(\frac{\partial w_p}{\partial y}) dxdy \quad (4)$$

The total potential energy, then, can be found as:

$$\Pi = U_p + U_s + T \quad (5)$$

## 2.2 Derivation of Buckling Equation

The deflection $w_p$ of simply supported plate can be taken in the form of double trigonometric sine-series following Navier's approach [2, 5, 6].

$$w_p = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \quad (6)$$

In Eq. (6) $A_{mn}$ is the amplitude of deflection. The deflection curve of i-th transverse stiffener can be found by replacing x with $c_i$.

Substituting the deflection curves of plate and stiffener into Eqs. (1), (2), and (4), and integrating as indicated, the strain energy and work done can be found as follows:

$$U_p = \frac{ab\pi^4}{8} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn}^2 \{D_{11}\frac{m^4}{a^4} + 2\nu_{21}D_{11}\frac{m^2n^2}{a^2b^2} + D_{22}\frac{n^4}{b^4} + 4D_{66}\frac{m^2n^2}{a^2b^2}\} \quad (7)$$

$$U_s = \frac{bE_{11s}I_s\pi^4}{2} \sum_{i=1}^{N_s} \sum_{n=1}^{\infty} \frac{n^4}{b^4} \cdot \{\sum_{m=1}^{\infty} A_{mn}^2 \sin\frac{m\pi i}{N_b}\} \quad (8)$$

$$T = \tau_0 t \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} A_{mn}A_{pq} \cdot \frac{mnpq}{(p^2-m^2)(n^2-q^2)\pi^2} \quad (9)$$

In Eq. (9) $m \pm p$ and $n \pm q$ are always odd numbers.

Buckling stress of orthotropic plate is defined by Eq. (10) following Timoshenko (1961) by defining $k_s$ the shear buckling coefficient.

$$\tau_{cr} = k_s \frac{\pi^2 \sqrt{D_{11}D_{22}}}{b^2 t} \quad (10)$$

Substituting Eq. (10) into Eq. (9) and again substituting Eqs. (7), (8), and (9) into Eq. (5), the total potential energy of transversely stiffened plate can be found as a function of aspect ratio $\phi$, rigidity ratio of stiffener and plate $\gamma$, and amplitude of deflection $A_{mn}$.

According to the principle of stationary potential energy, the coefficients $A_{mn}$ must be chosen to make the total potential energy being stationary [6, 7].

Employing Rayleigh-Ritz method, the minimization of total potential energy with respect to $A_{mn}$ results in a set of simultaneous homogeneous linear equations represented by following equation:

$$A_{mn}k_{\beta}Q_{mn}+2\gamma k_{\beta}\sum_{i=1}^{N_s}\phi^4 n^4 \sin\frac{m\pi x}{a}\cdot\sum_{r=1}^{\infty}\sin\frac{r\pi i}{N_b}$$
$$+\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}A_{pq}\frac{mnpq}{(p^2-m^2)(n^2-q^2)}=0 \qquad (11)$$

In Eq. (11) m=1, 2, 3, ···, n=1, 2, 3, ···, and $m\pm p$ and $n\pm q$ are always odd numbers. $N_b$ represents the number of panel (=$N_s$+1), $\gamma$ is the rigidity ratio of stiffener and plate. Parameters $k_\beta$, $Q_{mn}$, and $\gamma$ are given in appendix, respectively.

The set of homogeneous Eqs. (11) can be divided into two independent equations which can be solved separately, one group containing terms with (i+j) even and the other group containing terms with (i+j) odd.

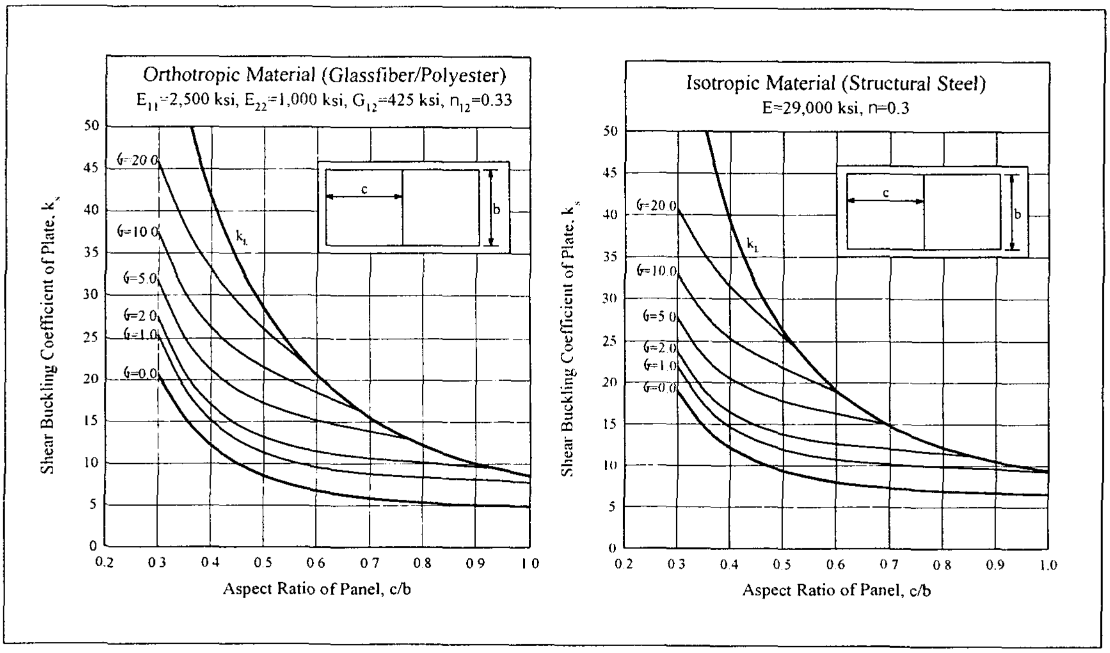
In this paper, each set of equations is expanded into ten terms. Ten simultaneous homogeneous equations may be written in matrix form.

Since the equations are homogeneous, the determinant of coefficient matrix of $A_{mn}$ must be vanished to get the solution other than the trivial one. By equating the determinant equal to zero and using the numerical analysis technique such as the secant method, the buckling coefficient of plate at each value of $\phi$ with a certain value of $\gamma$ can be found. Of the two systems, the least value of $k_s$ should be chosen.

## 3. BUCKLING OF TRANSVERSELY STIFFENED ORTHOTROPIC PLATE

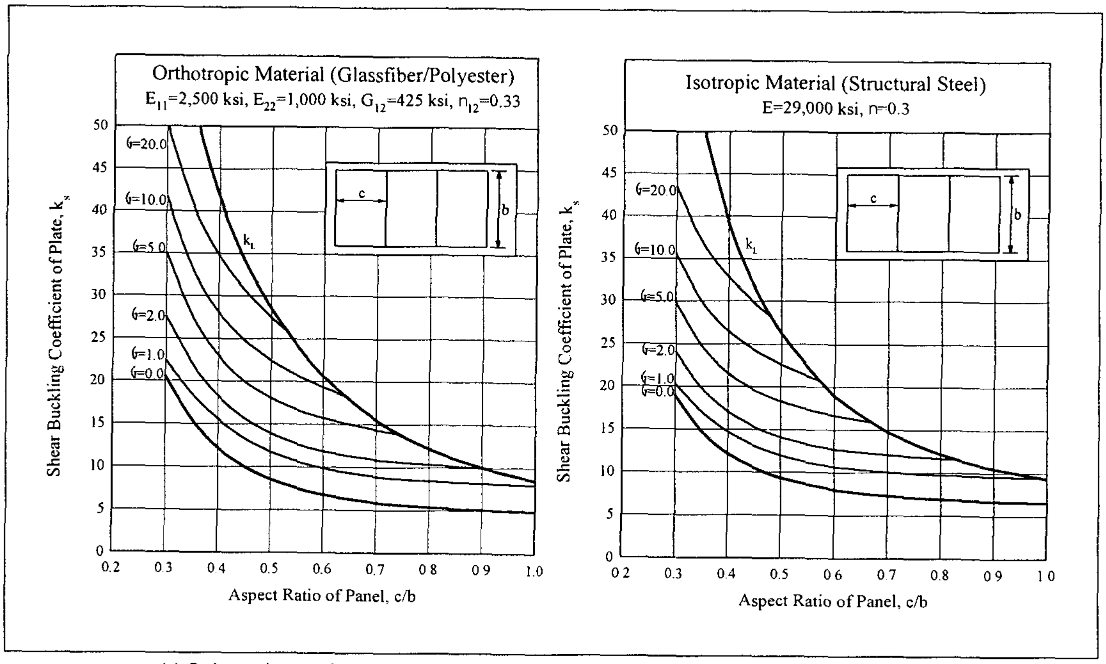
### 3.1 Buckling Stress of Transversely Stiffened Orthotropic Plate

The buckling stress of orthotropic plate can be easily found when the buckling coefficient of plate is known as shown in Eq. (10). Using the derived equation, the buckling coefficient of




(a) Orthotropic material (b) Isotropic material

Fig. 2 The shear buckling coefficient of plate with a transverse stiffener




(a) Orthotropic material (b) Isotropic material

Fig. 3 The shear buckling coefficient of plate with two transverse stiffeners

transversely stiffened plate with respect to aspect ratio of plate can be found.

As previously mentioned, the derived equations (11) are divided into two independent equations in which i+j is odd and i+j is even. For the plate with single transverse stiffener, the system with (i+j) even leads to the least values of $k_S$, and for the plate with two transverse stiffener, the system with (i+j) odd leads to the least values of $k_S$.

For various values of rigidity ratio $\gamma$, the shear buckling coefficient of plate with one or two transverse stiffeners was calculated and plotted in Fig. 2 and 3, respectively. As shown in Fig. 2 and 3, increasing the aspect ratio of plate decreases the buckling coefficient of plate. Thus, in order to obtain the minimum buckling coefficient of stiffened plate, the buckling analysis of longitudinally stiffened infinite long plate need to be investigated.

To verify the accuracy of results orthotropic material properties were replaced with isotropic ones, and the results were plotted in Fig. 2 (b) and Fig. 3 (b). This results were coincided with published ones (Way, 1936) [2].

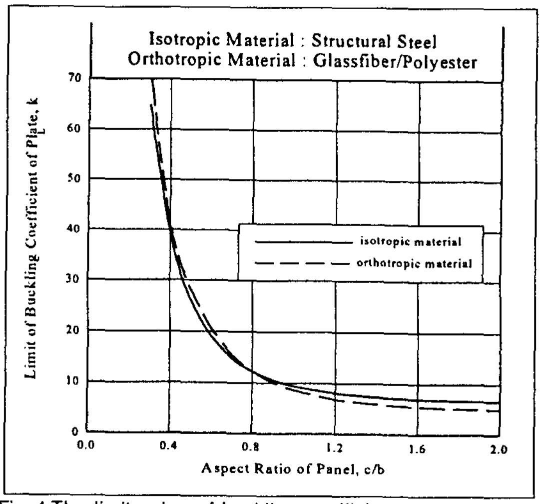



Fig. 4 The limit value of buckling coefficient of transversely stiffened plate

### 3.2 The Limit of Buckling Stress

As shown in Fig. 2 and 3, increasing the stiffener rigidity increases the load necessary to buckle the plate. If the stiffeners are made very rigidly, they will remain straight and only the panels of the plate will buckle. The critical load will then be very nearly the same as that for a simply supported plate of the dimensions of one panel since the torsional rigidity of stiffener is neglected. In this paper, it is assumed that the limit value of buckling stress for one panel of the plate is the same as buckling stress for a simply supported plate having the same dimensions of the panel.

Since only one panel of the plate is considered, rigidity ratio $\gamma$ can be vanished in Eq. (11). Using this equation, the limit value of shear buckling coefficient of transversely stiffened plate $k_L$ was found as shown in Fig. 4.

### 3.3 Required Stiffener Rigidity

Transverse stiffener must be stiff enough to preserve the straight boundaries that are assumed in computing the limit value of buckling stress of plate [8].

In this paper, the limit values of rigidity of stiffener were calculated in the form of rigidity ratio as follows:

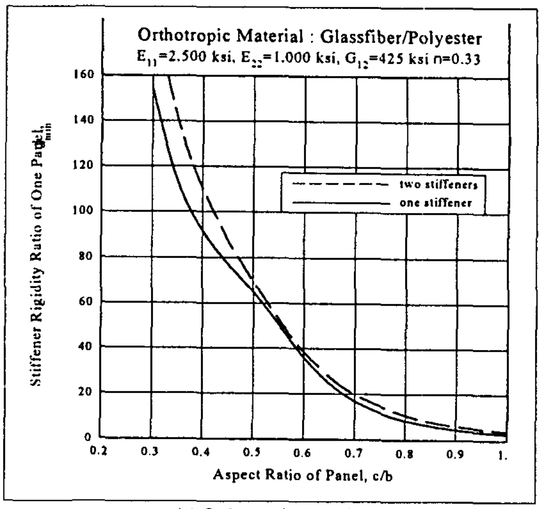
$$\gamma=\frac{E_{11s}I_s}{c\sqrt{D_{11}D_{22}}} \tag{12}$$

Eq. (12) is referred to the stiffener rigidity to the rigidity of one panel. It is convenient to compare the required stiffener rigidity when one or two stiffeners are located.

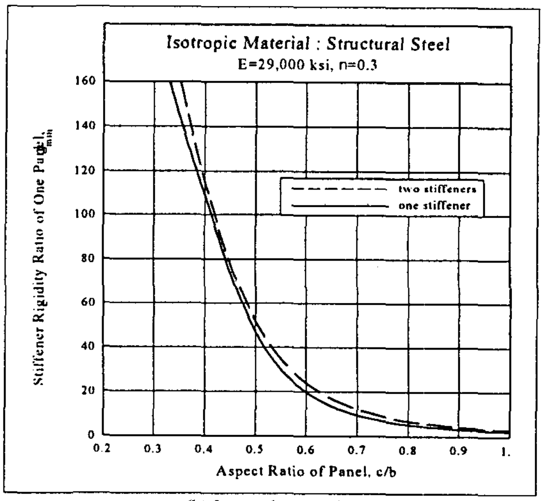
Using the limit value of shear buckling coefficient as previously calculated, the required stiffener rigidity was found with respect to the aspect ratio of one panel and plotted in Fig. 5.

As can be seen in Fig. 5, the required stiffener rigidity for the case of three panels is only slightly increased than the one for the case of two panels, the aspect ratio of one panel being the same in both cases.

Thus, the predicted results can be assumed to an approximation of the correct value.




(a) Orthotropic material




(b) Isotropic material

Fig. 5 Required stiffener rigidity of given panel

## 4. DISCUSSION AND CONCLUSION

In this paper the results of an analytical elastic buckling analysis for the orthotropic plate with transverse stiffeners subjected to in-plane shear forces were presented. Rayleigh-Ritz method was employed in the solution of the problems. To find the required stiffener rigidity it was assumed that the limit value of buckling stress for one panel of the plate is the same as the buckling stress for a simply supported plate of the same dimensions of the panel. This is not strictly true since each panel tends to stiffen the adjacent one but gives conservative values.

The required stiffener rigidity was calculated based on the assumed maximum buckling stress of plate when one and two stiffeners were located, respectively. Comparing both results, those results can give an approximation of the correct value.

For the exact value of required stiffener rigidity, the theoretical solution for the critical shear stress of infinitely long, simply supported orthotropic plate with equally spaced transverse stiffeners needs to be further investigated.

## ACKNOWLEDGEMENT

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## APPENDIX

$$k_\beta = \frac{\pi^2}{32 k_s \phi^3}, \quad \gamma = \frac{E_{11s} I_s}{a\sqrt{D_{11} D_{22}}}$$

$$Q_{mn} = m^4 \sqrt{\frac{E_{11}}{E_{22}}} + 2\phi^2 m^2 n^2 \nu_{21} \sqrt{\frac{E_{11}}{E_{22}}} + \phi^4 n^4 \sqrt{\frac{E_{22}}{E_{11}}} + 4\phi^2 m^2 n^2 \frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11} E_{22}}}$$

## UNIT CONVERSION

1ksi = 1kip/in$^2$ = 70.31kg/cm$^2$
1ksi = 1,000lb = 453.6kg
1in = 2.54cm