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Determining of an Optimal Spares Stocking Policy with Reliability Improvement

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Abstract

We present in this paper an optimal stocking policy for a repairable inventory system under reliability improvement. For this purpose we illustrate commercial flight lines with a large number of planes. This model is supported by a central repair facility. For modeling the nonstationary M/M/s system we implemented SIMAN for computing the time dependent number of units in the repair facility with any number of units. In this model we provide the required inventory level at each location, ly month, for various levels of associated stock-out risk.

1. Introduction

The problem of reliability improvement was initially introduced by Daune,J.T.[3], through the concept of learning curve. His aim was to determine the manner in which reliability performance of complex electro-mechanical and mechanical systems change the develop and design improvement stages. In this paper Duane's formula is used for determining of an optimal spares stocking policy in a logistical support system. Repairable systems are those systems that are repaired when failure occur, such as cars, airplanes, and computers. A large number of mathematical models have been developed relating to various aspects of handling repairable inventory systems[7,10] and the common assumption of these models is that failures in the field are generated by a stationary Poisson process[11]. In the situation where the failure process is continually changing, existing models are not directly applicable.

Many repairable systems have a long life in service and undergo further evolutionary development; therefore, a continuing reliability improvement program is needed for the effective follow-on development. The concept of reliability improvement is that increased usage will reveal product deficiencies through failures, all failures are analyzed fully, and corrective actions are taken in design or production to ensure that such a failure cannot occur on products

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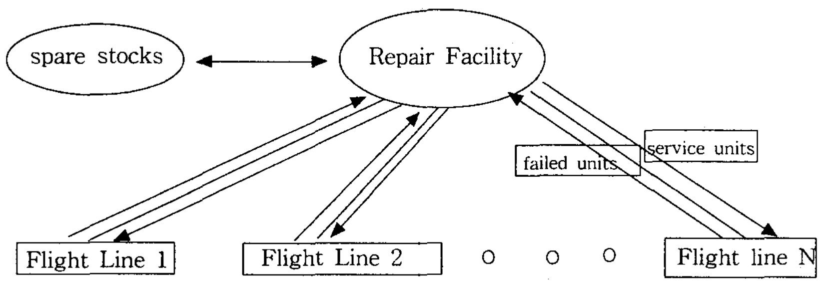
in the next service. Under reliability improvement, MTBF of products is seen to be an increasing function of the total operational time which means that the number of failures in successive periods tends to decrease.

In this paper a methodology for a repairable inventory system under reliability improvement is presented. Two basic problems are of importance. The first is determining the number of spares needed at the repair location to replace failed units and the second determining the number of spares needed at the repair facility to provide replaceable units at a designated service level. This situation is interesting in that reliability improvement is in effect, thus the failure rate is continually changing.

We consider commercial flight lines with a large number of planes in their fleets. The flight lines are supported by a central repair facility. At random times, a unit malfunction is discovered and the unit is removed from the plane for repair. If an identical spare unit is available it is used to replace the failure and the plane is returned to service. If no spares are available, the plane is grounded until a serviceable unit is provided by the repair facility. <Figure 1> is a depiction of the structure of a repairable inventory system.



<Figure 1 : Structure of a recoverable inventory system>

It is desirable to have low probability of an aircraft being grounded waiting for parts, therefore inventories are carried at various points in the system. Spare inventories also must be considered at the repair facility because contractually this facility guarantees that a received unit will be replaced within a certain period of time. However, care must be taken when determining these inventory levels, since typical flight hardware is very costly. The complexity of this problem lies in ascertaining how to control inventories for repairable items with time varying demand rates, and how to allocate spare units to maximize the availability of serviceable units.

2. Model assumption and notations

Since the concern in this inventory modeling situation is the improvement in reliability of systems for handling repairable parts, we need to design a model for the failure process to represent successive time between failures of the system that tend to increase. In other words, the expected number of failures in any initial interval is no less than the expected number of failures in any interval of the same length occurring latei. One popular stochastic process to represent this situation is the nonhomogeneous Poisson Process(NHPP). The property of the NHPP is similar to the HPP with the exception that the failure rate is time dependent. Within the class of NHPP models, the Duane model [3] is most commonly discussed in the literature [1]. Duane postulated that the system instantaneous failure rate at cumulative operating time t is

$$\rho(t) = \lambda\beta t^{\beta-1} \tag{1}$$

where

$\rho(t)$ = instantaneous failure rate at time t

λ = constant

β = growth rate

t = total operational time.

The future failure rate in equation (1) is a function of the operating time length and the growth rate parameters. Estimation of the parameters of a failure rate is presented in Cow[2] and is strongly dependent on the number and times of occurrence of failures. The parameters will also rely on the progress of the product improvement program and the available time and resources to meet the end specified reliability target. We introduce the counting process N(t) that presents the total number of events that have occurred up to time t. Then, the probability of having n failures in the interval[t_1, t_2] is

$$P[N(t_2) - N(t_1)] = n] = \frac{e^{-\int_{t_1}^{t_2}\rho(t)dt}\left[\int_{t_1}^{t_2}\rho(t)dt\right]^n}{n!} \tag{2}$$

and, the expected number of failures over the time interval [t_1, t_2] is

$$E[N(t_1)-N(t_2)] = \int_{t_1}^{t_2}\rho(t)dt = \lambda t_2^{\beta} - \lambda t_1^{\beta} \tag{3}$$

The reliability function of the NHPP for the interval [t1,t2] is

$$R(t_1, t_2) = e^{-\int_{t_1}^{t_2}\rho(t)dt} \tag{4}$$

Under the NHPP assumption, the probability that exactly n units will fail in any interval [t_1, t_2], P(n) has a Poisson distribution with mean $E[N(t_1)-N(t_2)]$.

That is, for all $0 \le t_1 \le t_2$

$$P(n) = \frac{[E[N(t_1) - N(t_2)]^n e^{E[N(t_1)-N(t_2)]}}{n!} \tag{5}$$

To illustrate how this distribution might be used in a typical situation, suppose we would expect our hardware to initially demonstrate reliability improvement with the following parameter values:

λ = 0.00145

$\beta = 0.86$.

Then the mean value function is found by substitution into equation (2) and is:

$$E[N(t_1)-N(t_2)] = 0.00145(t_2^{0.86} - t_1^{0.86})$$.

Consider the case where there are two flight lines with the forecasted flight hours over the first three years as given in Table 1. The flight hours for each flight line in the table are given in years, and also the aggregated flight monthly average in hours. Then the expected number of failures during the first calendar month of the program would be:

E(0, 8000) = 3.30 .

Also, we would expect, the number of failures in the second calendar month to be:

E(8000, 16000) = 2.68 .

The expected number of failure in $[t_1, t_2]$, $E[N(t_1)-N(t_2)]$ will be used in a later section to develop an approach to spare requirements at the flight lines.

year	Flight/Year		Aggregated Flight Hours/Month
	Flight Line 1	Flight Line 2	
1	46,000	50,000	8,000
2	48,000	60,000	9,000
3	48,000	60,000	9,000

< Table 1: Flight Statistics >

3. Transit and repair processes

Turn-around time for a repair request is based on the time in the three processes as follows:

(a) transit time from the flight line to the repair facility,

(b) delay time at the repair facility, and

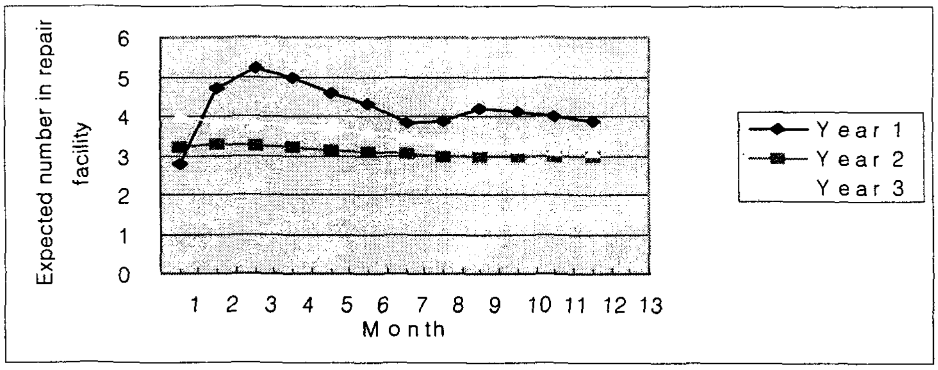
(c) shipment time from the repair facility to the flight line.

With an NHPP failure process and general shipment times, the transit process for the flight line to repair facility can be modeled as an $M/G/\infty$ system. Mirasol [4] showed that the output of an $M/G/\infty$ system is a Poisson process regardless of the distribution of service times. Hence, the repair process at the repair facility can be modeled as a nonstationary M/M/s system that is independent of the number of units in-transit. Under a similar assumption, we can model the shipment time for the repair facility to flight line as an $M/G/\infty$ system.

For modeling the nonstationary M/M/s system we implemented SIMAN [7] for computing the time dependent number of units in the repair facility (units in queue plus units in repair) with any number of units presented at time zero. In SIMAN, the Runge-Kutta-Fehlberg (RKF) procedure is used to integrate differential equations numerically. This procedure provides results for a direct numerical integration of the Chapman-Kolmogorov equations. These integrated values are the exact probability distributions of the system state at any time t that are found by tracking the time dependent arrival and/or service rates.

The following example demonstrates the effect of time-varying demand on the distribution of the expected number of items in the repair depot. In this application, we will use the associated demand data discussed in the previous section. Let us assume that the transit service time distribution is exponential and we consider two flight lines that take an average transit time from the flight lines to the repair facility of 10 and 7 days respectively. Further, in the repair facility we assume that five service personnel provide identical, exponential service with mean service time of 10 days. For solving the differential equations in a nonstationary M/M/s system, we formulated the problem using a double precision version of SIMAN, implemented on a Pentium II Microcomputer. The differential equations are coded in FORTRAN and the derivative values of each variable are passed between SIMAN and FORTRAN. In Figure 2, dynamic results are presented for the expected number of failed units at the repair facility through the planning horizon.

The expected number of failed units in the system at time t was computed by sampling the process at each integrated value of time and the values were plotted through the end of year 3. The run started in the empty and idle condition in year 1. Year 2 and year 3 are continuations of the situation in year 1.



<Figure 2: Day-by-day occupancy level of failed units at the repair facility>

4. Allocation of spare parts

We will now illustrate the methodology to establish the spare requirements both at the flight line and repair facility. The stock levels are estimated for any desired service level.

4.1. Spare Requirements at the Flight Lines

If we wish to cover failures with spare units to keep the aircraft operational then we cannot plan to the average, but we must provide some safety stock above and beyond the average. In this case, The assessment of provisional level for a risk versus spare is analogous to a situation where demand is driven by product failure and the level of product failure follows an NHPP function. To assess risk, we will use the Poisson distribution function given in equation (5).

and the expected number of failure, $E[N(t_1)-N(t_2)]$, as the mean of a Poisson distributed random variable.

Hence, for example, when $E[N(t_1)-N(t_2)] = 2.68$ failures for a certain calendar month, the probability that the expected number of failure is n is described as:

$$P(n) = \frac{2.68^n e^{-2.68}}{n!}, \ n = 0, 1, 2, \cdots \ .$$

In Table 2 the probability of this number of failures occurring is shown. We can find that if we provide five spares at the start of this calendar month to cover failed units, there would be a risk of stock-out of 5.5%, while stocking three would give a risk of 28.1%. We define the chance of an out-of-stock condition occurring as the risk level.

n	*P(n)*	*Cumulative Probability*	*Risk Level*
0	0.069	0.069	0.931
1	0.184	0.253	0.747
2	0.246	0.499	0.501
3	0.220	0.719	0.281
4	0.147	0.866	0.134
5	0.079	0.945	0.055

< Table 2: Poisson Probability values >

4.2. Spare Requirements at the Repair Facility

The distribution of outstanding orders for repair is the convolution of the following two distributions:

(i) the number of failed units in-transit from the flight lines to the repair facility.

(ii) the number of failed units either in repair at the facility or in the repair queue.

For (i), the number of failed units in-transit is the occupancy level in an $M/G/\infty$ system, and is independent of the number of units in the repair process. The occupancy level in an $M/G/\infty$ system has a Poisson distribution [6,8], and the expected value of the sum of several Poisson random variables is the sum of the expected values. The aggregated number of failed units in-transit to the repair facility is therefore Poisson distributed. The time between successive departures from the transit service to the repair facility creates the process of the time between consecutive arrivals at the repair facility. The arrival process at the repair facility for repair is modeled as a Markovian system since the output of an $M/G/\infty$ system is a Poisson process [6,8]. Hence, the number of failed units in (ii) is the occupancy level of an M/M/s system that is independent of the numbers in (i). We define :

$M_1(t)$ = aggregated number of failed units in-transit to the repair facility at time t

$M_2(t)$ = aggregated number of failed units either in repair or in the repair queue at the repair facility at time t

s_0 = planned stock level at the repair facility at time t

$B(t \mid s_0)$ = aggregated backorders at time t at the repair facility given the stock level s_0

α = risk level of the spare stock at the repair facility.

Once the distributions of $M_1(t)$ and $M_2(t)$ are estimated, the number of failed units in the system at time t can be represented as $Q(t) = M_1(t) + M_2(t)$.

The convolution of the two distributions can be solved by computer and is the aggregated outstanding orders that are either in-transit to the repair facility or in the repair facility.

The orders occurring when the repair facility is out-of-stock are backordered. For a given stock level at the repair facility, the backorder is given by:

$$B(t \mid s_0) = [Q(t) - s_0]^+ \qquad (6)$$

where $[A]^+$ denotes the nonnegative part of A .

Hence, the net inventory level at the repair facility is $s_0 - Q(t)$. The spares requirement at the repair facility must be planned corresponding to this level. A positive safety stock above the average outstanding order provides a buffer against larger-than-average outstanding orders. Therefore, we can choose the safety stock level with a designated service level ($1 - \alpha$), that is

$$P\{ Q(t) \geq s_0 \} \leq \alpha \ . \qquad (7)$$

where $P\{\bullet\}$ is the probability of stockout given the stock level s_0 at the repair facility.

Table 3 is a depiction of the required stock level for the repair facility by month. In this example, the spare stock for the repair facility is constrained to be empty not more than 10 % of the time. The increased stock levels in the beginning of year 2 are due to the increment of aggregate flight hours.

Month	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
Year 1	9	10	10	9	9	8	8	8	7	7	7	7
Year 2	8	8	8	7	7	7	7	6	6	6	6	6
Year 3	6	6	6	6	6	5	5	5	5	5	5	5

< Table 3. Stock Level Analysis for the Repair Facility per Month >

5. Concluding remarks

Throughout the paper, we assumed a one-for-one inventory policy since repairable items are typically expensive and their individual demand rates are usually relatively low. Therefore, whenever an item fails, an order is immediately placed for a replacement part of the same type with an order quantity of one. we illustrated some of the considerations when integrating reliability improvement phenomena to an inventory system. Two practical implementation issues were addressed in developing the model. The first issue is concerned with determining spare requirements at flight lines under the condition of time varying demand. The nonstationary failure process is recognized by the presented model to determine the demand rate of a specific hardware under consideration. The second issue is concerned with inventory requirements at the repair facility to supply replaceable units to the flight lines.

Since the demand is a probabilistic phenomena, one is always taking the risk of an out-of-stock occurrence. In this presented model, we provide the required inventory level at each location, by month, for various levels of associated stock-out risk. We believe that this model could be used by many companies that design and develop a hardware for commercial and military aircraft.

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