⊠연구논문

Using Minimal Path Sets for the Evaluation of the Reliability of DRDT Interconnection Networks[+]

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Abstract

In this paper, we consider an interconnection network, DRDT (Dual Receive Dual Transmit), that is a double-loop ring topology and adopts the concept of multiple packets transmission. For three types of DRDT configurations, we investigate some properties related to path sets and discuss the method for finding minimal path sets. Using the concept of the terminal reliability and the path sets approach, we evaluate the reliability of the DRDT networks and compare them with a single ring network and a unidirectional double-loop ring network.

1. Introduction

In the future information society, a number of systems are connected based on communication networks to provide various information services. The reliability of communication environment becomes more and more important, and it mainly depends on the reliability of communication networks. Ring network configurations are designed for improving reliability and performance. The totologies of ring networks are (i) a star shaped ring with all ring interface hardwares housed in a

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central equipment [13], (ii) a single ring with bypass switched in which failed nodes are isolated [2, 3], (iii) a dual ring network where a pair of nodes have several alternate paths for communication [6, 7, 8, 14].

In other study, authors have introduced the concept of multiple packets in which duplicated packets are transmitted through multiple links and nodes and one of them is selected at the receiver [5, 15, 16]. The types of ring network topologies and their performance are summarized in details in [4, 12].

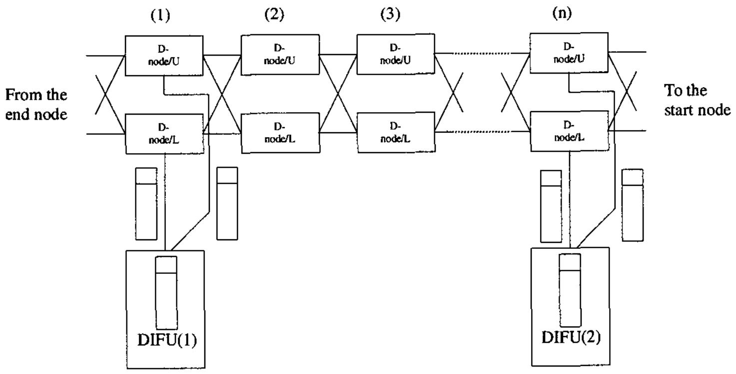
Recently, ETRI (Electronics & Telecommunications Research Institute) of Korea proposes a new architecture and a transmission method of interconnection network which provides real time processing. It is called DRDT (Dual Receive Dual Transmit) interconnection network. DRDT is designed for improving the reliability of SMX-1, which is also developed by ETRI and is a core component of signaling network [1].

In this paper, we consider three configurations of DRDT network and evaluate their reliabilities. We also consider a single ring (hereafter SR) network and a unidirectional double-loop ring (hereafter UDLR) network and compare them with DRDT networks in terms of reliability and the MTTF(Mean Time To the Failure). In Section 2, we describe the basic configuration of DRDT network, and, in Section 3, several characteristics related to path sets are investigated and a method for obtaining minimal path sets is discussed. Section 4 is devoted to evaluate reliability of DRDT networks and to compare them with a SR network and a UDLR network.

2. DRDT configuration and its operation

As shown in Figure 1, DRDT network is composed of DIFUs (DRDT Interface Units) which reside in the host system connected to the interconnection network and multiple D-Nodes. As the D-Nodes composing DRDT network are physically separate, DRDT is not affected by local faults, and if any path is available, the time to get to the destination is not affected by the fault of D-Node. But the durability against the fault and packet transfer performance is different from each other according to the network configuration. The more durable structure against the fault than the basic DRDT structure will be described at the end of this section.

Each node composing the ring in DRDT is composed of two D-nodes. If faults occur in both of the two nodes (D-Node/U, D-Node/L) simultaneously, the path becomes blocked in the interconnection network. However if only one D-node fails at each ring node, DRDT continues its normal operation.

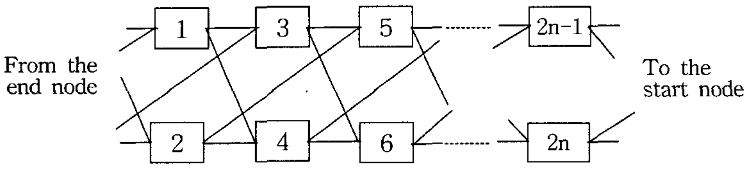


< Figure 1 > DRDT configuration

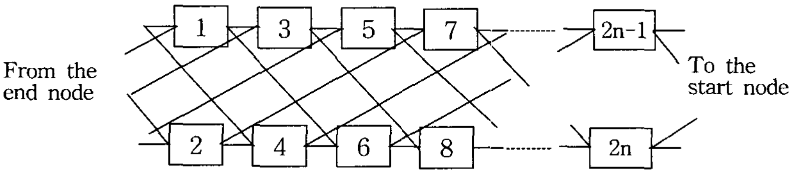
<Figure 2> and <Figure 3> show the other two types of DRDT configuration. Contrary to the basic DRDT(hereafter, DRDT-I), in DRDT-II, one of the links from the lower node bypasses one node, and in DRDT-III, one of the links from the upper node bypasses one node and one of the links from the lower node bypasses two nodes. For the purpose of convenient representation, we number nodes in up string, i.e. D-node/U, odd numbers from 1 to (2n-1) and nodes in down string, i.e. D-node/L, evens from 2 to 2n. And we consider DIFUs as source and destination in our analysis since they do not have any effect on the comparative results.

3. Some properties related to path sets

One of the most important and widely used methods for computing reliability of a network system is by either enumerating path sets or cut sets of the system. In this section, we investigate some characteristics related to path sets of DRDT networks.



< Figure 2 > DRDT-II configuration



< Figure 3 > DRDT-III configuration

Definition

A path set is a set of nodes which establishes connectivity between the source and the destination if traversed in the direction of flow of signal from the first to the latter. And the minimal path set is a minimal set of nodes whose functioning insures the functioning of the whole system.

We note that, in this paper, a path set is defined in terms of nodes while it is defined in terms of links in some other literatures.

Let i be a nonnegative integer. Let $nu(i)$ and $nl(i)$ be the number of minimal path sets including the first upper node and the first lower node, respectively, when total number of nodes are $2i$. And we assume that $nu(1) = nl(1) = 1$ and $nu(j) = nl(j) = 0$ for all $j \leq 0$. Then we have the following results on the number of minimal path sets for each DRDT network configuration.

Theorem 1

(a) For DRDT-I network configuration, $nu(i) = nu(i-1) + nl(i-1)$ and $nl(i) = nu(i-1) + nl(i-1)$. Moreover, the number of minimal path sets is given by $mp_I(i) = 2^i$.

(b) For DRDT-II network configuration, $nu(i) = nu(i-1) + nl(i-2)$ and $nl(i) = nu(i-1) + nl(i-1)$. Moreover, the number of minimal path sets is given by $mp_{II}(i) = mp_{II}(i-1) + mp_{II}(i-2) + nl(i-3)$.

(c) For DRDT-III network configuration, $nu(i) = nu(i-1) + nl(i-2)$ and $nl(i) = nu(i-3) + nl(i-1)$. Moreover, the number of minimal path sets is given by $mp_{III}(i) = mp_{III}(i-1) + mp_{III}(i-3) + nu(i-5)$.

Proof. The proof can be done by simple algebra. Hence it is omitted.

It is noted that the formulas in Theorem 1 are recursive forms. Using C programming, we calculate the number of minimal path sets of three DRDT configurations for various network sizes and the results are summarized in Table 1. The network size means the number of D-nodes paris which are D-node/U and D-node/L.

< Table 1 > The Number of Minimal Path Sets

Network Size	DRDT-I	DRDT-II	DRDT-III	Network Size	DRDT-I	DRDT-II	DRDT-III
2	4	3	2	9	512	151	40
3	8	5	3	10	1024	266	61
4	16	9	5	11	2048	465	93
5	32	16	7	12	4096	816	142
6	64	28	11	13	8192	1432	217
7	128	49	17	14	16384	2513	332
8	256	86	26	15	32768	4410	508

The reason we compute the number of minimal path sets only for up to the network size of 15 is that the possible maximum size of DRDT network is 15. We note from Table 1 that DRDT-I has more minimal path sets than DRDT-II and DRDT-III configurations. But it does not mean that DRDT-I is the most reliable configuration among three DRDT configurations. More details are discussed in Section 4.

The first step of reliability calculation by path sets approach is to obtain minimal path sets of a network. Since it is, however, not straightforward to find minimal path sets when the network size, denoted by n, is large, we introduce a tree to enumerate minimal path sets. This is called a minimal path tree (hereafter, MPT).

The MPT has the following structural characteristics. The root node is numbered 0 and always has two child-nodes, which are a D-node/U and a D-node/L. Each generation consists of a D-node/U and its dual node, a D-node/L. That is, the i-th generation has two nodes numbered $2i$ and $2i-1$. A node has a child-node, a grandchild-node or a great grandchild-node.

The rules for constructing MPT are as follows:

For DRDT-II network configuration,

Rule 1.

An upper node (i.e. D-node/U) has one child-node and one grandchild-node. For any $1 \leq j \leq n$, *a node* $(2j-1)$ *has one child-node* $(2j+1)$ *and one grandchild-node* $(2j+4)$ *provided that* $(2j+1)<2n$ *and* $(2j+4)<2n$, *respectively.*

Rule 2.

A lower node (i.e. D-node/L) has two child-nodes. For any $1 \leq j<n$, *a node* $2j$ *has two child-nodes* $(2j+1)$ *and* $2(j+1)$ *provided that* $(2j+1)<2n$ *and* $2(j+1)<2n$, *respectively.*

For DRDT-III network configuration,

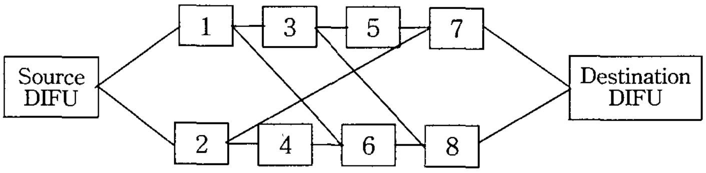
Rule 1.

An upper node has one child-node and one grandchild-node. For any $1 \leq j \leq n$, *a node* $(2j-1)$ *has one child-node* $(2j+1)$ *and one grandchild-node* $2(j+2)$ *provided that* $(2j+1)<2n$ *and* $2(j+2)<2n$, *respectively.*

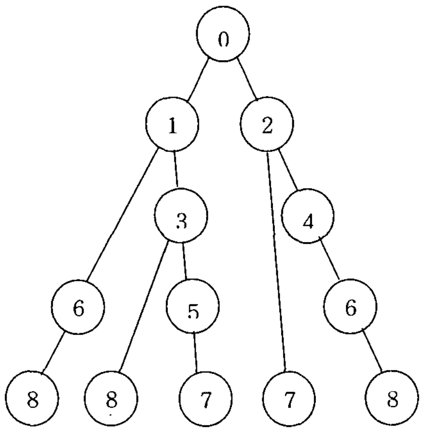
Rule 2.

A lower node (i.e. D-node/L) has one child-node and one great grandchild-node. For any $1 \leq j \leq n$, *a node* $2j$ *has one child-node,* $(2j+1)$ *and a great grandchild-node* $(2j+5)$ *provided that* $(2j+1)<2n$ *and* $(2j+5)<2n$, *respectively.*

【Example】 When the network size is 4, the configuration of DRDT-III is as follows:



Then the minimal path sets can be obtained by using MPT as shown in <Figure 4>.

i	Minimal path tree	Minimal path sets
0 1 2 3 4		{1, 6, 8}, {1, 3, 8} {1, 3, 5, 7}, {2, 7} {2, 4, 6, 8}

< Figure 4 > The MPT of DRDT-III when n = 4.

4. Reliability analysis and comparison

The concept of terminal reliability is a widely used measure for evaluating the network reliability. Utilizing this concept, we define the reliability of DRDT network as follows:

The reliability of a DRDT network is the probability that a message is successfully transmitted from a source DIFU to a destination DIFU.

Since a message from a DIFU is possibly designated for one of other DIFUs, we introduce a notation $R(\cdot;k)$ representing the reliability of DRDT network when a message passes through k D-nodes. Let $p(k;n)$ be a probability mass function which denotes the probability that the source DIFU and the destination DIFU are $(k-1)$ apart when the size of DRDT network is n. Here $k = 2, 3, \ldots, n$. Then the reliability of DRDT network of size n can be defined by

$$R(\cdot) = \sum_{k=2}^{n} R(\cdot;k)p(k,n).$$

If all DIFUs are equally likely to be selected, then $p(k;n)$ becomes an uniform distribution with the probability mass of $1/(n-1)$ and the reliability is

$$R(\cdot) = \sum_{k=2}^{n} R(\cdot;k)/(n-1).$$

In order to compute $R(\cdot;k)$ for given source and destination, we utilize the minimal path set in such a way that

$$R(\cdot;k) = P(\text{at least one of minimal path sets between the source and the destination is good}) = P(T_1 \cup T_2 \cup \ldots \cup T_m),$$

where m is the number of minimal path sets between the source and the destination and T_i represents the event that the i-th minimal path set is normally operating. We adopt the sum of disjoint products method and the algorithm proposed by Locks to evaluate $P(T_1 \cup T_2 \cup \ldots \cup T_m)$ [10, 11, 12]. In this section, we assume the followings:

(i) all D-nodes are identical and independently operating.
(ii) the events of link failure and DIFU failure are ignored.

Throughout this section, we use the following notations.

Notation	Description
$R_I(\cdot)$	The reliability of DRDT-I configuration.
$R_{II}(\cdot)$	The reliability of DRDT-II configuration.
$R_{III}(\cdot)$	The reliability of DRDT-III configuration.
$R_{IV}(\cdot)$	The reliability of SR network.
$R_V(\cdot)$	The reliability of UDLR network.

4.1 Time-dependent reliability analysis

In this section, we consider only the network configuration of the size 4. Such a network is composed of 4 D-nodes/U and 4 D-nodes/L and DRDT-III network needs at least 8 nodes to figure out its full configuration. All D-nodes are assumed to have a constant failure rate λ.

We note that the DRDT-I configuration is a parallel-series structure while the UDRL network is a series-parallel structure. And the SR network is a series structure. Using Locks algorithm, we obtain the time-dependent reliability functions for five network configurations. They are summarized as follows:

$$R_I(t)=e^{-4\lambda t}(2-e^{-\lambda t})^2(1+2e^{-\lambda t}+3e^{-2\lambda t}-4e^{-3\lambda t}+e^{-4\lambda t})/3$$

$$R_{II}(t)=e^{-2\lambda t}(4+6e^{-\lambda t}-3e^{-2\lambda t}-10e^{-3\lambda t}+5e^{-4\lambda t}+2e^{-5\lambda t}-e^{-6\lambda t})/3$$

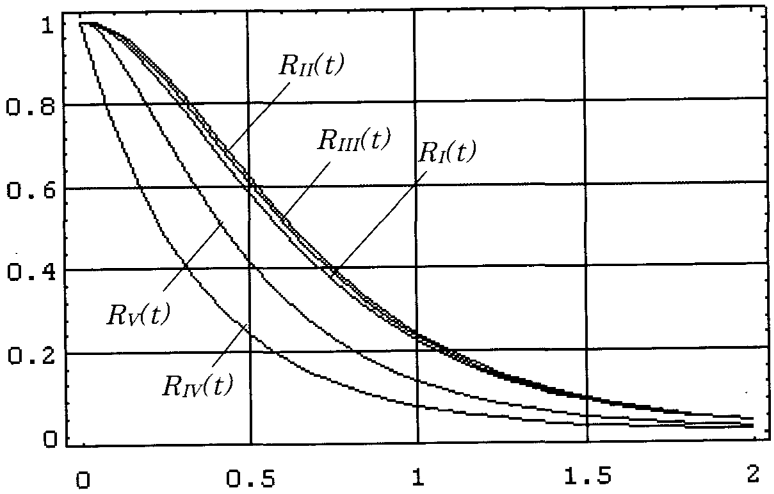
$$R_{III}(t)=e^{-2\lambda t}(4+4e^{-\lambda t}-2e^{-2\lambda t}-6e^{-3\lambda t}+3e^{-4\lambda t})/3$$

$$R_{IV}(t)=e^{-2\lambda t}(1+e^{-\lambda t}+e^{-2\lambda t})/3$$

$$R_{V(t)}=e^{-2\lambda t}(2+2e^{-\lambda t}+e^{-2\lambda t}-e^{-4\lambda t}-e^{-6\lambda t})/3$$

<Figure 5> shows the graphical representation of the time-dependent reliabilities of five configurations. It is noted that all DRDT network configurations are superior to the other configurations. The DRDT-II configuration is slightly more reliable than any other DRDT configurations. It should be, however, mentioned that this result does not hold for different network sizes. We also compute the MTTFs(Mean Time To the Failure) of five configurations by integrating corresponding reliability functions and find the relation with the MTTF of the

single-node configuration.



< Figure 5 > Time-dependent reliabilities of five network configurations

<Table 2> shows the MTTFs of five configurations when the network size is 4 and the ratios of their MTTFs to MTTF of the single-node configuration. All DRDT configurations have the MTTFs more than 1.5 times as long as the MTTF of the single-node configuration. Among DRDT configurations, DRDT-II configuration is slightly better than the other DRDT configurations in the average MTTF sense, while the most efficient configuration depends on the distance between the source DIFU and the destination DIFU.

< Table 2 > The MTTFs when the network size is 4

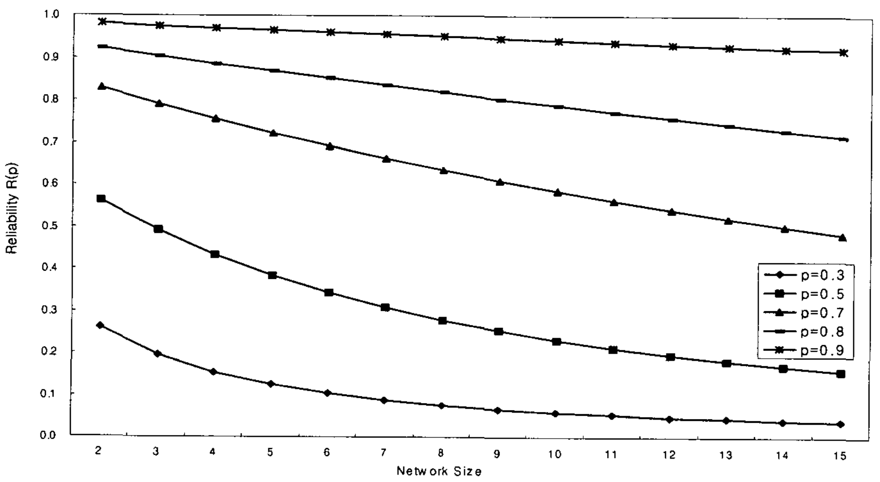
Type of Configuration	MTTF (Ratio to MTTF of SR)
DRDT-I	1847/2520λ (2.030)
DRDT-II	377/504λ (2.071)
DRDT-III	32/45λ (1.969)
DRDT-IV	13/36λ (1.000)
DRDT-V	13/24λ (1.500)

4.2 Time-independent reliability analysis

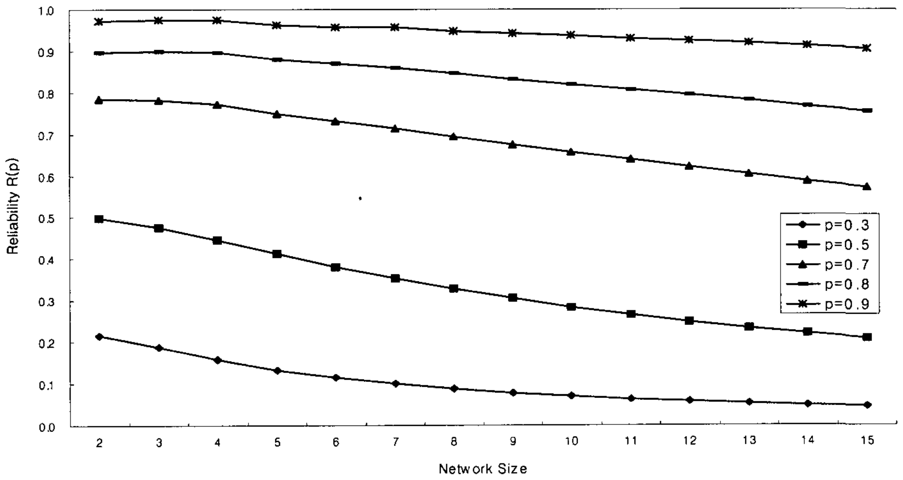
In this section, we evaluate numerically the time-independent reliabilities of five network configurations for the network sizes of 2 to 16. To this end, we use the MPT method to enumerate the minimal path sets between source and destination DIFU and also use Locks algorithm for numerical computation. All computations are done by C programming.

The reliability of each D-node is assumed to be $p \in (0,1)$. We also note that the time-independent reliabilities of DRDT-I, SR and UDLR configuration are $R_I(p) = \sum_{k=2}^{n}[p(2-p)]^k/(n-1)$, $R_{IV}(p) = p^2(1-p^{n-1})/[(1-p)(n-1)]$ and $R_V(p) = p^2(2+2p-p^2-2p^{n-1}+p^{2n})/[(1-p^2)(n-1)]$, respectively.

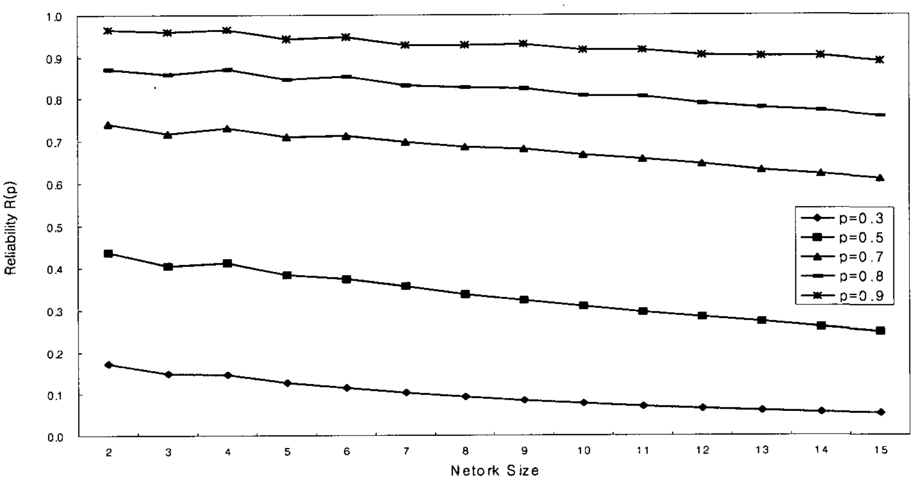
Figures 6, 7, and 8 show the graphical representations of the results on computing R(p) of DRDT-I, -II, and III for various network sizes and node reliabilities, respectively. It is expected that the global tendency of $R(p)$ decreases as the network size increases. The reliability of DRDT-I configuration is monotonically decreasing as the network size increases while the reliabilities of DRDT-II and DRDT-III are not. For DRDT-II configuration, the reliability slightly increases and then decreases as the network size increases when the node reliability is high. DRDT-III configuration shows fluctuating tendency as the network size changes. This characteristic provides an important information to the designers. For example, designers would prefer the network of the size 4 to the size 3 when they develop DRDT-III network system.



< Figure 6 > Reliability of DRDT-I configuration when the network size changes



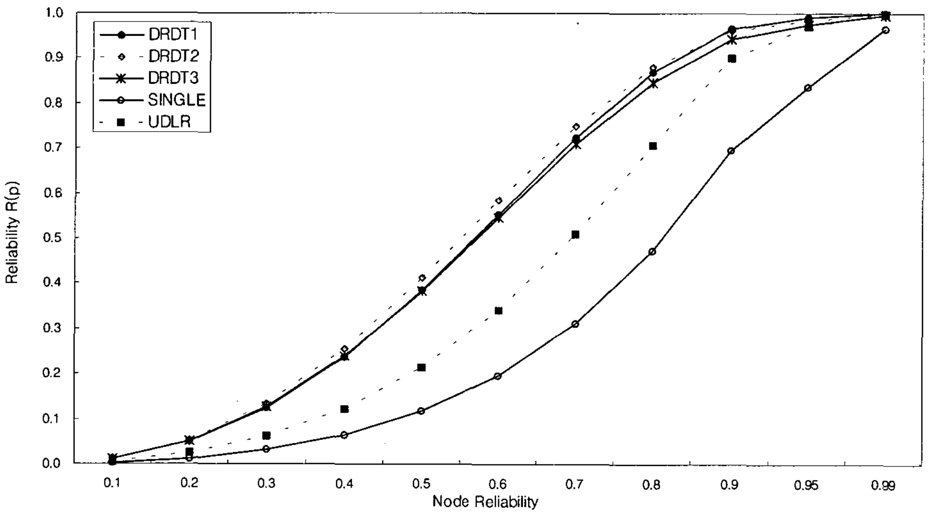
< Figure 7 > Reliability of DRDT-II configuration when the network size changes



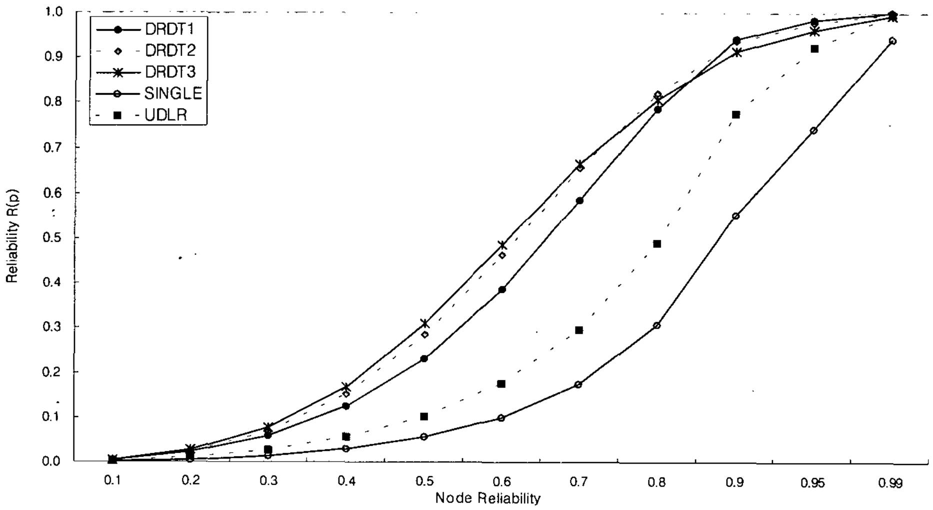
< Figure 8 > Reliability of DRDT-III configuration when the network size changes

Figures 9, 10 and 11 show the reliability of three DRDT network configurations and two comparative networks when the network sizes are 5, 10 and 15, respectively. It is shown that the reliability of all DRDT configurations dominates the other two configurations and that no DRDT configuration outperforms the other DRDTs for all network sizes and the node reliability. DRDT-II and

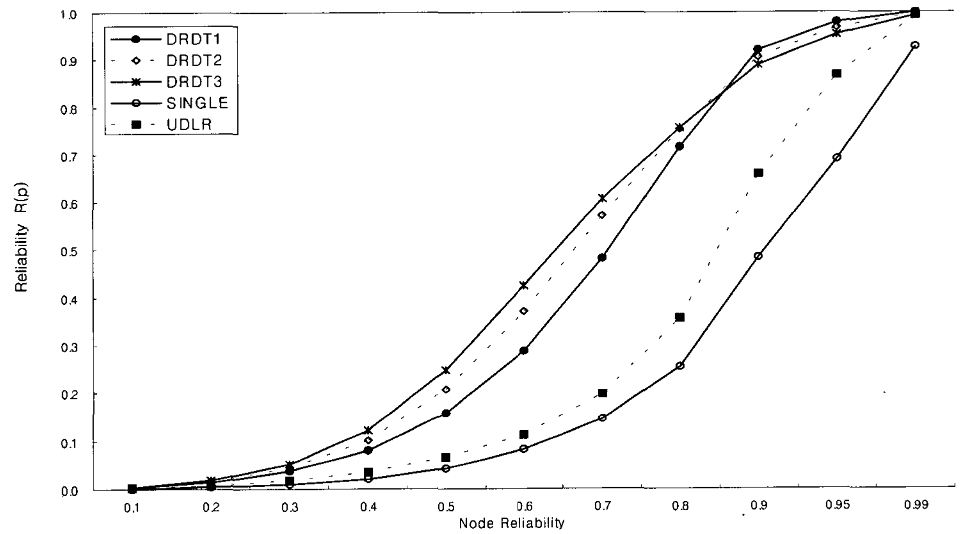
DRDT-III are slightly more reliable than DRDT-I for the node reliability less than 0.8 and DRDT-I is the most reliable when nodes are highly reliable. Figures 9, 10 and 11 enable designers to decide the level of node reliability in order to achieve a certain degree of DRDT network system.



< Figure 9 > Reliability Comparison for the network size of 5



< Figure 10 > Reliability Comparison for the network size of 10



< Figure 11 > Reliability Comparison for the network size of 15

5. Conclusion

This study is to analyze a newly proposed network configuration - DRDT network - in view of both time-dependent and time-independent reliability and to compare them with the existing network configurations such as SR network and UDLR network. We utilize the concept of the terminal reliability and the path sets approach for evaluating the reliability.

The comparison of five configurations shows that DRDT configurations are more reliable than the other network configurations. It is also noted that even if DRDT-I configuration has more minimal path sets than DRDT-II and DRDT-III, DRDT-I is not necessarily more reliable than the others. When the node reliability, however, is very high, DRDT-I is the most reliable configuration.

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